Adaptive Optimum Notch Filter for Periodic Noise Reduction in Digital Images

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ABSTRACT

Periodic noises are unwished and spurious signals that create repetitive pattern on images and decreased the visual quality. Firstly, this paper investigates various methods for reducing the effects of the periodic noise in digital images. Then an adaptive optimum notch filter is proposed. In the proposed method, the regions of noise frequencies are determined by analyzing the spectral of noisy image. Then, the repetitive pattern of the periodic noise is produced by applying the corresponding notch pass filter. Finally, an output image with reduced periodic noise is restored by an optimum notch filter method. The results of the proposed adaptive optimum notch filter are compared by the mean and the median filtering techniques in frequency domain. The results show that the proposed filter has higher performances, visually and statistically, and has lower computational cost. In spite of the other compared methods, the proposed filter does not need to tune any parameters.

KEYWORDS

Periodic Noise, Optimum Notch Filter, Detection of Noisy Frequencies, Region Growing

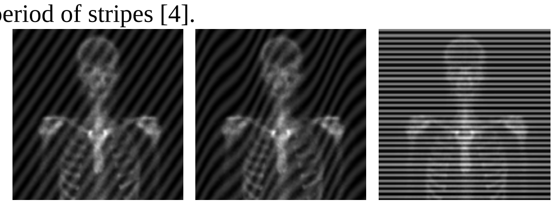
1. INTRODUCTION

Additive periodic noise is a repetitive signal which is added to the main signal. This periodic noise in a digital image is repetitive spatial pattern which effectively degrades the image quality [1].

There are some different sources for creation of periodic noises in a digital image. Electrical or electromechanical inferences in imaging systems, electrical inference in image receiver systems, and unequal sensitivity of detectors are the main sources. For example, periodic noises can be seen when an imager system is mounted on vibrated holder (for example in a non-stabilized aerial imaging), due to mechanical inference, or in a TV receiver when the receiving signal is weak, due to electrical inference between receiving signal and another periodic signal (for example inter-harmonics of power supply frequency) [2].

Additive periodic noises are usually modeled by summing several sinusoidal functions with different amplitudes and frequencies; therefore, in the frequency domain the noisy image appears like stars with high amplitude. The periodic noises in the digital images are divided into three main categories, including global, local and stripping [3-4], which are shown in Figure (1) in a typical host image.

In the global periodic noise, the noise parameters including corresponding amplitudes and frequencies are spatial-independent, while in the local periodic noise these parameters spatially vary [3]. In a multi-sensor imaging system, unequal sensitivity of detectors and corresponding electronic circuits, causes the third type of periodic noise called stripping. The number of available detectors in spatial imaging scanners determines the period of stripes [4].



(a) (b) (c)

Figure 1: Image of body scan by X-Ray contaminated by a)global b)local and c)stripping periodic noises.

Nowadays, due to image quality importance, it has been a lot attention to the periodic noise reduction algorithms. According to the location of sensors in a multi-sensor system, the location of stripping bands are predetermined, then it is possible to reduce the noise effectively using some spatial simple methods [4]. On the other hand, since the global and local periodic noises can not be simply separated from the main image in the spatial domain, the frequency-domain approaches are usually applied [1]. However, there are some reports for using spatial-domain approaches like soft morphology [5-6] in this subject.

The implementation of frequency-domain approaches

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for reducing the periodic noise effect is either fully in frequency-domain (like band reject filter), or used some extracted information from the image in frequency-domain (like optimum notch filter); therefore, the frequency-domain analysis of the noisy image and applying the filter in the frequency-domain are important in the periodic noise reduction algorithms.

In the next section, an overview on some traditional frequency-domain approaches in the periodic noise reduction subject is studied. Then the proposed optimum notch filter which is adaptively used some extracted information from the noisy image in frequency-domain is presented, and finally the results are compared with some other similar methods. Regarding to the other methods, the results of the proposed method show higher improvements in the output images, qualitatively and quantitatively. Again, the computational cost of the proposed method significantly reduces which is important in a real-time application.

2. AN OVERVIEW ON EXISTING METHODS

In this section, a brief overview on existing methods in frequency-domain for periodic noise reduction algorithms including band reject filter, notch filter, optimum notch filter, frequency-domain median filter and frequency-domain masked mean filter is studied. It is supposed that $f(x,y)$ and $g(x,y)$ are the pixel values of noise-less and noisy image in the (x,y) coordinate, and $F(u,v)$ and $G(u,v)$ are the 2-D Fourier transform of noise-less and noisy image in the (u,v) frequency, respectively.

A. Band-Reject Filters

Since band reject filters attenuate a band of frequency about the origin of the 2-D Fourier transform, these filter can be uses in periodic noise reduction applications where the general location of the noise components is approximately known in the frequency domain [1]. Unfortunately, when the distances of the periodic noise components about the origin of the 2-D Fourier transform are different, there is necessary to use either a wide band-reject filter, or several narrow band-reject filters. In both cases, the restored image may lose some important image information.

B. Notch-Reject Filters

A notch-reject filter attenuates frequencies in predefined neighborhoods about a center frequency of the 2-D Fourier transform. Therefore, this filter can be used to reduce the periodic noise effects where the main frequencies of the periodic noises are known [7]. Automatic determining of both the periodic noise main frequencies and the corresponding band-width are challenge problems in this filter. Fig. 2 shows the different steps for reducing the periodic noise containing two main frequencies in a sample image. In this example, the periodic noise frequencies are determined by trail and error.

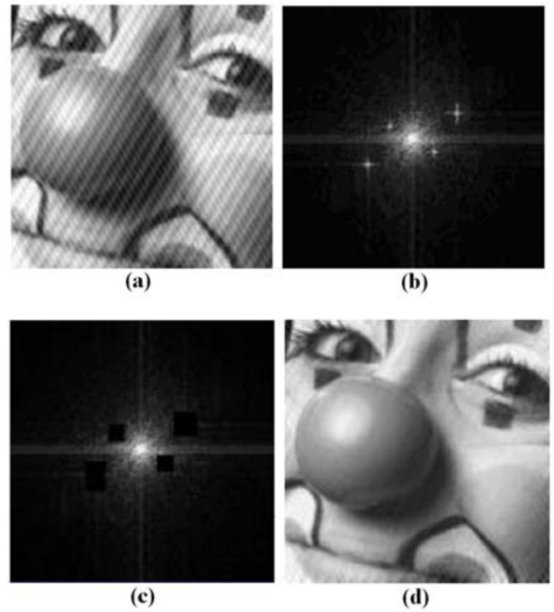

Figure 2: Different step to reduce the periodic noise effect from a sample joker image. a)Input image contaminated by additive two periodic sinusoidal signals. b) 2-D Fourier transform of the input image. c) Removing periodic noise frequencies by the proper squared type notch-reject filter and d) Restored image after applying the notch-reject filter.

C. Optimum Notch Filters

In a real system contaminated by periodic noise, the output image tends to contain 2-D periodic structure superimposed on the input image with patterns more complex than several simple sinusoidal signals. In this condition, two mentioned methods are not always acceptable because they may remove much image information in filtering process.

Optimum notch filter tries to minimize local variance of the restored image [1]. At the first stage, principal contribution of the inference repetitive pattern is extracted from the noisy image, and then the output image is restored by subtracting a variable weighted portion of the repetitive pattern from the contaminated image. The extractions of the repetitive pattern is implemented in frequency-domain by applying a proper notch-pass filter on every periodic noise frequency, and then by applying inverse 2-D Fourier transform to restore the repetitive pattern in spatial-domain.

The 2-D Fourier transform of the inference repetitive pattern, $N(u,v)$, is given by Eq. (1),

$$N(u,v) = H_{np}(u,v)G(u,v) \tag{1}$$

where $H_{np}(u,v)$ is superimposed response of all necessary notch-pass filters and $G(u,v)$ is the 2-D Fourier transform of the contaminated image. The proper selecting of $H_{np}(u,v)$ is challenge problem in the optimum notch filter.

Then, the corresponding repetitive pattern in the spatial-domain, $\eta(x,y)$, is obtained by Eq. (2),

$$\eta(x,y) = \Im^{-1}\{N(u,v)\} = \Im^{-1}\{H_{np}(u,v)G(u,v)\} \qquad (2)$$

where $\mathfrak{F}^{-1}$ is a symbolic representation of inverse 2-D Fourier transform.

For an additive noise model, if $\eta(x,y)$ is known perfectly, subtracting the repetitive pattern from the noisy image, $g(x,y)$, obtains noise-less input image, $f(x,y)$. On the other hand, the filtering procedure by applying $H_{np}(u,v)$ usually yields only an approximation of the true repetitive pattern. In order to minimize the effects of components not present in the estimate of real true repetitive pattern, subtracting a variable weighted portion of $\eta(x,y)$ from $g(x,y)$ is done to obtain an estimate of $f(x,y)$.

$$\hat{f}(x,y) = g(x,y) - w(x,y)\eta(x,y) \qquad (3)$$

Where $\hat{f}(x,y)$ is the estimate of $f(x,y)$ and $w(x,y)$, the weighting function, is to be determined such that the variance of the output is minimized over a specific neighborhood of every point (x,y). Usually, the noisy image is partitioned to several non-overlapped neighborhood of size $(2a+1)\times(2b+1)$ about points (x,y), and corresponding $w(x,y)$ in each neighborhood is considered constant. For each neighbor hood, the constant weighting function $w(x,y)$ is obtained by Eq. (4) which tries to minimize local variance of the restored image, over corresponding neighborhood [1].

$$w = w(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \overline{g}(x,y)\overline{\eta}(x,y)}{\overline{\eta^2}(x,y) - \overline{\eta}^2(x,y)} \qquad (4)$$

Where $\overline{g}(x,y)$ and $\overline{\eta}(x,y)$ are the local mean of the corrupted image and the noise pattern, respectively, $\overline{\eta^2}(x,y)$ is the local mean of square of the noise pattern, and $\overline{g(x,y)\eta(x,y)}$ is the local mean of the corrupted image multiply by the noise pattern, all of them over corresponding neighborhood.

In order to apply the optimum notch filter, firstly the noise pattern is extracted by Eq. (2), then for each neighborhood, the constant weighting function $w(x,y)$ is obtained by Eq. (4) and finally, the restored image is obtained by Eq. (3).

D. Frequency-Domain Median Filters

The frequency-domain median filter which is used to reduce the periodic noise effect contains two basic steps. In the first step, the median value of amplitudes of each frequency is computed over a predefined window. Then frequencies of the periodic noise are detected by comparing the median value of each frequency with its corresponding amplitude. In the second step, amplitude of each noisy frequency is replaced by it corresponding median value. The realization of the frequency-domain median filter for the frequency of (u,v) is summarized by Eq. (5),

$$Y(u,v) = \begin{cases} \text{Med}(X(u,v)) & \text{if} \quad \dfrac{X(u,v)}{\text{Med}(X(u,v))} > \theta \\ & \text{and} \quad (u,v) \neq (0,0) \\ X(u,v) & \text{otherwise} \end{cases} \qquad (5)$$

where $X(u,v)$ is $|G(u,v)|$, Med($X(u,v)$) is the median value of $X(u,v)$ over neighborhood frequencies of the (u,v) central frequency, θ is a predefined fixed threshold value and $Y(u,v)$ is the estimate of $|F(u,v)|$. Since $X(0,0)$ is the DC component of the contaminated image and its value is usually very large, the frequency-domain median filter should not apply on the frequency of (0,0).

The window size of 5×5, 7×7, 9×9 and 11×11 are proposed for the frequency-domain median filters. In fact, the window size is related to band-width of periodic noise, and should be selected by trial and error. Meanwhile, the predefined fixed threshold value of 3 and 6 are proposed for the window sizes of 5×5 and 7×7, respectively [8].

E. Frequency-Domain Masked Mean Filters

The basic idea of this type of periodic noise reduction filters is similar to the frequency-domain median filter. The frequency-domain masked mean filter uses the masked mean values instead of the median ones [9]. The masked mean value is defined over an $N\times N$ masked window. All values in the $N\times N$ masked window are '1', except the center which is considered '0'. It means that the center of the $N\times N$ local window is omitted in mean value computations.

Suppose that $S(u,v)$ is the masked mean value of $X(u,v)$ over $N\times N$ masked window, the (u,v) frequency is detected as noisy frequency when Eq. (6) is satisfied.

$$\frac{X(u,v)}{S(u,v)} > \theta \qquad (6)$$

where θ is a predefined fixed threshold value which is empirically set, and it depends on noise power of the periodic noise and the window size of the masked window. For a 3×3 masked window, $\theta = 4$ is proposed [9].

The realization of the frequency-domain masked mean filter for the frequency of (u,v) is summarized by Eq. (7),

$$Y(u,v) = \begin{cases} X(u,v)/\delta & \text{if} \quad \dfrac{S(u,v)}{X(u,v)} > \theta \\ & \text{and} \quad (u,v) \neq (0,0) \\ X(u,v) & \text{otherwise} \end{cases} \qquad (7)$$

where δ is selected based on the periodic noise reduction power of this filter. Like the previous filter, the frequency of (0,0) should be unchanged [9].

F. Discussion about Existing Methods

For band reject, notch and optimum notch filters, it supposed that the periodic noise frequencies and corresponding bands are predetermined, where the optimum notch filters show the best results [1]. Detection of the periodic noise frequencies is slightly solved in the frequency-domain mean and median filters, but there are some extra parameters in these filters including window size and threshold that degrade the output quality if they choose incorrectly. The frequency-domain median filter show higher quality than frequency-domain masked mean filter, but the corresponding computational cost is higher.

3. The Proposed Method

The proposed method which is based on the optimum notch filter, the accurate periodic pattern in the spatial domain is adaptively detected. In the proposed method, the spectral of the contaminated image is supposed as a gray level image. Figure (2-b) shows the spectral of a sample image contaminated by the periodic noises which frequencies are highlighted as bright points. It means that the corresponding main frequencies of periodic noises can be detected by applying a proper thresholding after masking the low frequency of the input image, as shown in Figure (3).

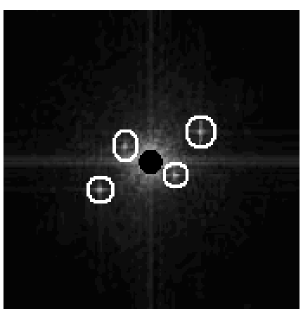

Figure 3: The low frequency region of spectral of the input image is firstly masked which is shown by the black circle in the center then by applying a proper thresholding, the corresponding frequencies of the periodic noises can be detected which are highlighted by the white circles.

In order to accurately detect the corresponding pattern of the periodic noise in spatial domain, it is necessary to consider all contaminated frequencies around the main frequencies of the periodic noise. In the proposed method, these contaminated frequencies are detected by a simple proposed region growing around each main frequency of the periodic noise. After providing the pattern of the periodic noise in spatial domain by applying the inverse 2-D Fourier transform on all frequencies of the periodic noises, the input image is restored by the conventional optimum notch filter.

A. Detecting of the Main Frequencies of Periodic Noises

As mentioned previously, the amplitude of frequencies of the periodic noise is locally greater than its neighborhood. Therefore, they can be detected by applying a proper threshold in the spectral of image. On the other hand, the low frequency region of ordinary images contains most of image information image and corresponding amplitude is greater than other frequencies. It means that it is necessary to mask the low frequency region before applying the threshold. We consider the radius of this mask as R_{mask}.

In order to determine the R_{mask} at the first step, suppose that there are non-overlapped concentric circles at frequency center, with radius of R and width of *w*, in the image spectra as shown in Figure 4.

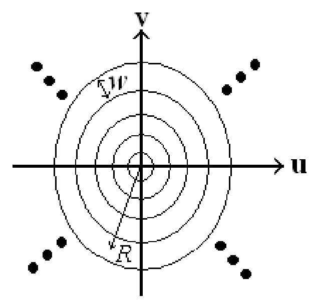

Figure 4: The non-overlapped concentric circles at frequency center, with radius of R and width of *w*.

For spectra of Figure (2-a) which is shown in Figure (2-b), the plot of mean of sum of amplitudes in each circle versus radius of circles is shown in Figure (5). The R_{mask} can be computed by detecting the first local minimum of the plot.

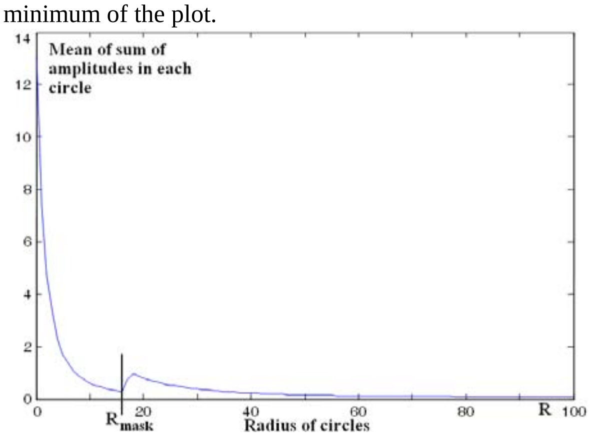

Figure 5: Plot of mean of sum of amplitudes of circles for Figure (2-b) in term of R. The corresponding radius of the first local minimum is computed as radius of mask, R_{mask}.

After masking the low frequency region, it is necessary to compute the proper threshold which is determined by Eq. (8) for detecting the main frequency of each periodic noise.

$$A_{Thr} = (A_{\max} + A_{mean})/2 \tag{8}$$

A_{Thr} is the proposed threshold, $A_{\max}$ and A_{mean} are the maximum and mean of amplitudes of spectra, respectively, after masking the low frequency region. This simple threshold has been already used for effectively detecting the high temperature point targets in infrared image [10].

B. Detecting Other Frequencies of Periodic Noise

Applying the computed threshold by Eq. (8) detects all main frequencies of periodic noises, but for accurate extraction of the periodic noise pattern in spatial domain, the other frequencies of the periodic noise should be detected and considered. In this step, a simple region growing algorithm [11] which considers a 3×3 window around each main frequency as primary region is proposed in the spectral domain. At the first step of region growing, a surrounding 5×5 window is considered and the proposed region growing algorithm tries to find which frequencies in perimeter of surrounding 5×5 window are contaminated by the periodic noise by comparing the amplitude of neighborhood frequencies as shown in Figure (6).

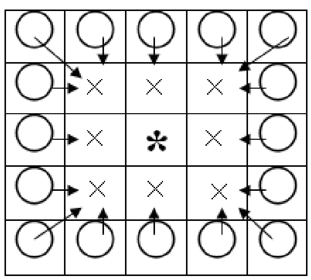

Figure 6: The star sign shows the main frequency of periodic noise. The 3×3 window around the main frequency is also considered as corresponding frequencies of periodic noise which is shown by multiply sign. The corresponding neighbors of each frequencies in surrounding 5×5 window are shown by arrows.

Each frequency in perimeter of 5×5 window is considered as periodic noise, if the corresponding amplitude is less than the amplitude of neighborhood frequencies in 3×3 window. If the number of frequencies contaminated by periodic noise in perimeter of 5×5 window is greater than the predefined threshold, N_{Thr}, the region growing process repeats for a surrounding 7×7 window. In the proposed region growing algorithm, the N_{Thr} is set to half of pixels in perimeter of surrounding window which is $(2n\text{-}2)$ where n is surrounding window size and n is always odd number. After stopping the proposed region growing algorithm, all frequencies of the grown region are considered as frequencies of the periodic noise. Figure (7) shows the output of applying the proposed region growing algorithm on a sample part of spectra which is contaminated by a periodic noise.

Again, for each detected main frequency of periodic noise in the thresholding stage, the proposed region growing algorithm is repeated to find all contaminated frequencies by periodic noises.

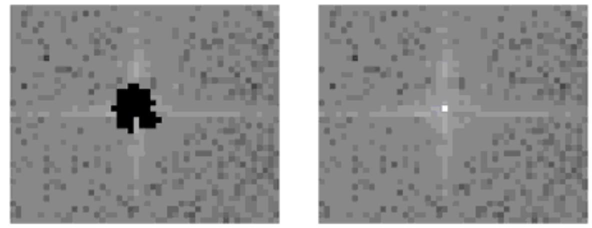

Figure 7: A part of spectra contaminated by periodic noise in the right side, the main frequency of periodic noise is highlighted by white. The region of detected frequencies of periodic noise is masked by black in the left side.

C. Applying the Optimum Notch Filter

The output of applying the proposed thresholding and region growing algorithm on the spectra of the input noisy image is a proper notch pass filter, $H_{np}(u,v)$, which can be applied on spectra of the input noisy image to extract the spatial pattern of the periodic noise based on Eq. (2). Then the restored image can be computed by using Eqs. (3) and (4).

4. EXPERIMENTAL RESULTS

The proposed adaptive optimum notch filter as well as the frequency-domain median and masked mean filters are implemented under Matlab environment [12]. The results on different images and periodic noises are quantitatively and qualitatively compared. In order to compare the results quantitatively, Mean of Absolute Error (MAE) between the noiseless image and restored image is presented. Moreover, to compare the output images qualitatively, the histogram equalization algorithm is applied on the restored image to visually highlight the differences. Two different experiments are reported, at the first one, a sinusoidal periodic noise with variable amplitude is applied and the results are quantitatively and qualitatively compared. At the second experiment, all algorithms are applied on a sample TV image contaminated by a complex real periodic noise and the results are qualitatively compared. It is necessary to tune the parameters of the mean and median filters by trail and error to achieve the best results. In this experiment, the window sizes and the thresholds for mean and median filters are set to 3×3 and 15, and 9×9 and 6, respectively.

A. Sinusoidal Periodic Noise with Variable Amplitude

The two dimensional sinusoidal periodic noise, $\eta(x,y)$ which is given by Eq. (9), is added to a sample 256×256 gray level image.

$$\eta(x,y) = 1 + a.\sin(x/2, y/1.5) \tag{9}$$

At the first part of this experiment, a is fixed to 0.1. Figure (8) shows the noisy image and the restores images by the mean and median filters in the frequency domain as well as the proposed adaptive optimum notch filter. In order to present a better comparison between methods, the histogram equalization is applied on all restored images. As shown in Figure (8), the improvement of the proposed method is higher than the other compared method, qualitatively.

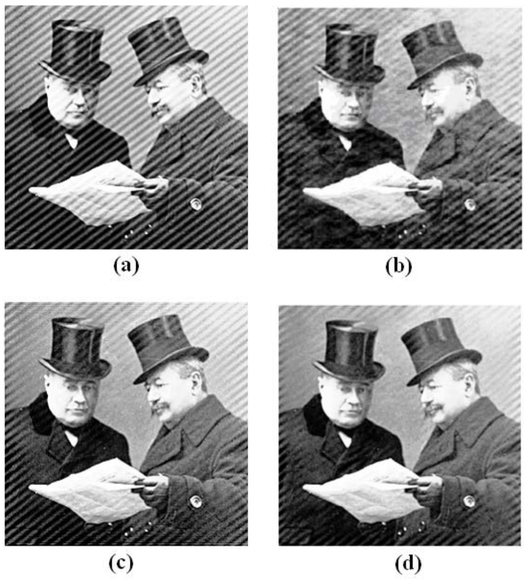

Figure 8: The visually comparison between the output of different restoration methods, after applying the histogram equalization. The image contaminated by the periodic noise of Eq. (8) with a=0.1 (a), and the restored images by the masked mean filter (b), the median filter (c) and finally the proposed adaptive optimum notch filter (d).

At the second part of this experiment, a in Eq. (9) is varied between to 0.0 to 1.0, and three methods including mean and median filters both in frequency domain and the proposed adaptive optimum notch filter are used to restore the noiseless image. The parameters of the mean and median filters in the frequency domain are selected as the corresponding parameters in the previous experiments. In order to quantitatively compare the results, MAE of each method is computed and plotted in Figure (9).

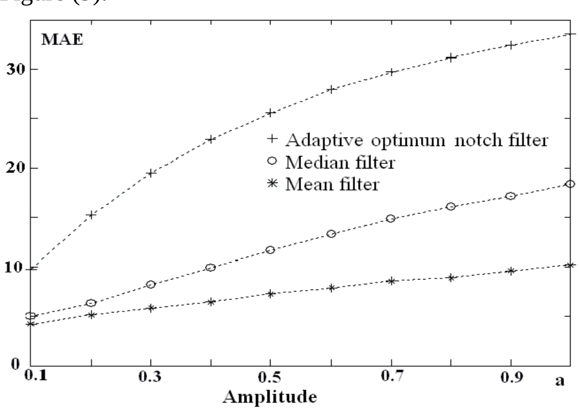

Figure 9: The plot of variation of MAE in term of amplitude of periodic noise, a, in Eq. (9).

Figure (9) shows that the MAE of the proposed adaptive optimum notch filter is always higher than MAE of the other compared methods. Moreover, when the amplitude of the sinusoidal periodic noise increases, the quantitative difference between the proposed method and two other compared ones increases, because the proposed adaptive method could adapt itself in different noise amplitudes.

B. Complex Real Periodic Noise

At the second experiment which results are shown in Figure (10), all compared methods are applied on a sample 640×580 TV image contaminated by a complex real periodic noise. As shown in Figure (10), the visual quality of the restored image by the proposed adaptive optimum notch filter is a little better than two other compared methods.

C. Execution Times

Complexity is another aspect of a restoration algorithm. The implementation cost of low complexity algorithm is lower than more complex algorithm. In order to compare the complexity of the compared algorithm, the execution times of the compared algorithms are computed. Since the execution time of the masked mean filter in the frequency domain is lower than the median one, the execution time of the masked mean filter is considered as execution time unit. Table (1) reports the execution times of all compared restoration algorithms for Figures (8) and (10).

TABLE 1

EXECUTION TIMES OF THE COMPARED RESTORATION METHODS

Restoration Filters	Figure (8)	Figure (10)
Mean	1.0	1.0
Median	20.0	94.5
Adaptive Optimum Notch	0.57	0.73

Table (1) shows that the execution time of the proposed adaptive optimum notch filter is lower than the masked mean filter which execution time is very lower than the median one.

5. CONCLUSION

In this paper, an adaptive optimum notch filter is proposed for reducing the effect of periodic noise in digital image. Firstly, the proposed algorithm determines the main frequency of the periodic noise by applying the proposed thresholding algorithm on the spectra of noisy image. Then, by applying the proposed region growing on each main frequency of the periodic noise, related all contaminated frequencies by periodic noise are separately determined. Finally the periodic noise pattern is computed by applying inverse 2-D Fourier transform, and then the restored image is obtained by applying the optimum notch filter method.

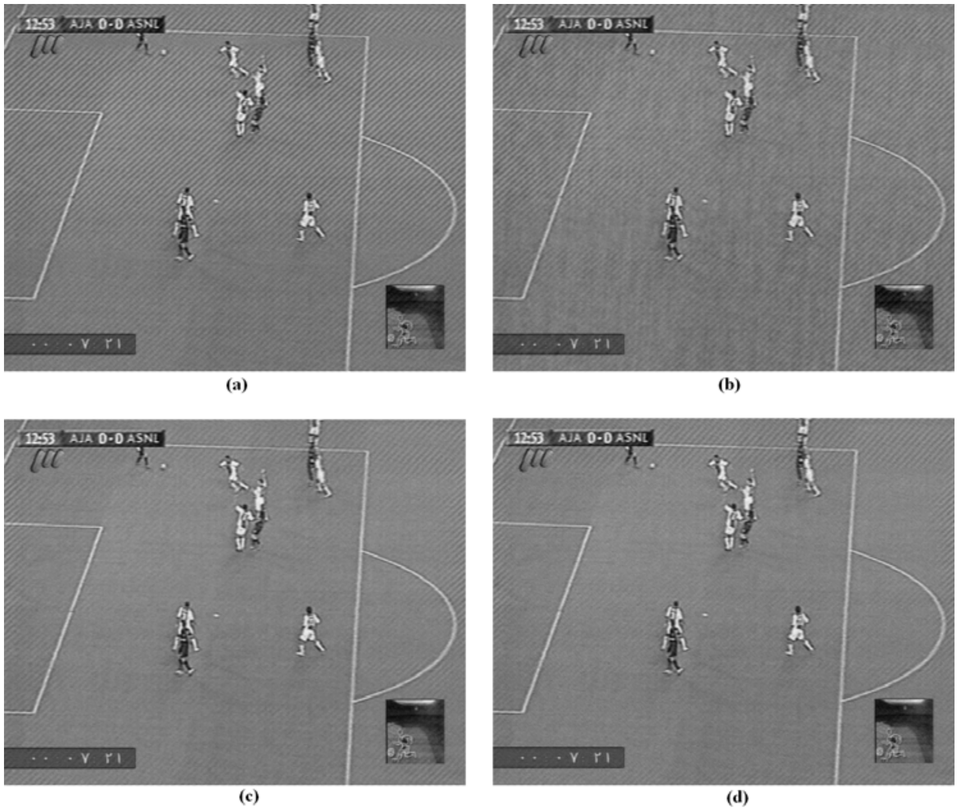

Figure 10: The visually comparison between the output of different restoration methods for restoring the TV image contaminated by a real complex periodic noise. a) Noisy image, and the restored images by b) The masked mean filter, c) The median filter and finally d) The proposed adaptive optimum notch filter.

The proposed adaptive optimum notch filter, the mean and median filters in the frequency domain are used to reduce the effects of different periodic noises in different sample images. The results show that not only the quality of restored image by the proposed method is higher than two other compared methods, but also its execution time is lower. In spite of the other compared methods, the proposed filter does not need to tune any parameters.

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