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A Higher Order B-Splines 1-D Finite Element Analysis of Lossy Dispersive Inhomogeneous Planar Layers

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ABSTRACT: In this paper, we propose an accurate and fast numerical method to obtain scattering fields from lossy dispersive inhomogeneous planar layers for both TE and TM polarizations. A new method is introduced to analyze lossy inhomogeneous Planar Layers. In this method by applying spline based Galerkin's method of the moment to scalar wave equation and imposing boundary conditions, we obtain reflection and transmission from the inhomogeneous layer. Moreover, we obtain both electric and magnetic fields in the inhomogeneous layers. The method employs a set of spline-harmonic basis functions and leads to one-dimensional integrals for system matrix elements. This fact along with the higher order nature of the basis functions provides an accurate method for the analysis of the aforementioned dispersive lossy inhomogeneous layers. The accuracy and the convergence behavior of the method are studied through several numerical examples and the results are compared with the exact solutions to establish the validity of the proposed method.

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1- Introduction

Inhomogeneous planar layers have been used in microwave filter design [1], electromagnetic shielding [2], optical devices such as narrow band high rejection filters and non-polarizing beam splitter, since inhomogeneous layers potentially provide less scattering, less stress, and better adhesion than homogeneous layers as shown, for instance, in [3]. Scalar wave equation only for few specific inhomogeneous layers has closed form solution such as exponential varying profile (e^{kz}) [4]. In recent years there has been immense interest in developing accurate and efficient numerical tools for determining reflection and transmission from inhomogeneous layers and unknown electromagnetic field in inhomogeneous layers [5-10].

One way to obtain scattered fields from the inhomogeneous layer is to express electric and magnetic fields in the inhomogeneous layer and all constitutive parameters by Taylor series [7] or Fourier series [8] then finding unknown Taylor series and Fourier series coefficients with imposing boundary conditions and using iterative method and optimization .These methods are applicable only for the inhomogeneous layer which relative permittivity, permeability and conductivity of inhomogeneous layer could be expressed by Taylor and Fourier series. Moreover, for complicated inhomogeneity, these methods will become inefficient. Another method proposed so far for analyzing inhomogeneous layer is the method of moment. In [9] using the method of the moment with rectangular pulse function as basic function and utilized point matching method so that method accuracy is less than higher order methods.

It is known that the higher order methods provide faster convergence and more accurate results for a given number of unknown electromagnetic fields in the numerical analysis of different electromagnetic problems [11].

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In this paper, we expand unknown electromagnetic fields in terms of linear combinations of the cubic splines and apply the Galerkin's method of the moment with appropriate boundary conditions and obtain reflection and transmission from the dispersive inhomogeneous layer and electromagnetic fields in the inhomogeneous layer. The proposed method is applicable to arbitrarily one-dimensional general dispersive inhomogeneous profile with continuous varying permittivity, permeability, and conductivity.

The remainder of the paper is organized as follows. In section 2, we formulate the problem and solve it numerically. In section3, the method is applied to calculate the electromagnetic field in the inhomogeneous layer and obtain reflection and transmission from the inhomogeneous layer. In section 4, a conclusion is given.

2- Theory

The geometry of the problem is shown in Fig.1. The structure consists of dispersive inhomogeneous layer where d,$\varepsilon_r(z,f)$ and $\mu_r(z,f)$ denotes the thickness and relative permittivity and permeability of inhomogeneous layer, respectively. All the layers are infinite in the transverse plane (i. e., x−y plane). The time dependence is considered as $e^{j\omega t}$ and suppressed throughout.

It is known that for the isotropic inhomogeneous layer of Fig.1 the TE and TM (to z) waves are decoupled [12,13], and the source free Maxwell equations are reduced to two second order ordinary differential equations for the TE and TM waves. Without the loss of generality, we assume that the wave propagates in the x direction and is independent of the y variable. Therefore, $E=e_y(z)e^{-jk_s x \sin\theta}\hat{y}$ and $H=h_y(z)e^{-jk_s x \sin\theta}\hat{y}$ for the TE and TM waves, respectively where $k_s=\sqrt{\varepsilon_s\mu_s}k_0$ and $k_0=\omega\sqrt{\mu_0\varepsilon_0}$ and θ is plane wave incident angle to inhomogeneous layer shown in Fig.1. The incident field is given by

$$\psi_s(z)=e^{-jk_s z\cos\theta}+Re^{jk_s z\cos\theta} \quad (1)$$

where $\psi=e_y$ in the TE mode and $\psi=h_y$ in the TM mode and R is reflection coefficient. In region (z>d), $\psi(z)$ is given by

$$\psi_c(z)=Te^{-jk_c(z-d)\cos\theta} \quad (2)$$

where $k_c=\sqrt{\varepsilon_c\mu_c}\,k_0$ and T is transmission coefficient. $\psi(z)$ is the solution of following equation in inhomogeneous layer [12]

$$\frac{d}{dz}\frac{1}{p(z)}\frac{d\psi}{dz}+\frac{k_z^2}{p(z)}\psi=0 \quad (3)$$

where $k_z^2=\varepsilon_r(z)\mu_r(z)k_0^2-(k_c\sin\theta)^2$, $p(z)=\mu_r(z)$ in the TE mode and $p(z)=\varepsilon_r(z)$ in the TM mode.

Considering the equations (1) and (2) and applying the continuity of the transverse component of fields at z=d and at z=0, we obtain the following boundary conditions of the third kind for e_y and h_y at z=d and at z=0

$$\left(\frac{1}{p}\frac{d\psi}{dz}\right)_{z=d}=\frac{-jTk_c\cos\theta}{p_c} \quad (4)$$

$$\left(\frac{1}{p}\frac{d\psi}{dz}\right)_{z=d}=\frac{-jk_s\cos\theta}{p_s}(1-R) \quad (5)$$

To solve (3), we make use of a B-spline finite-element method by expanding the unknown field $\psi(z)$ in in homogeneous layer in terms of a linear combination of cubic spline basis functions as follows

$$\psi(z)=\sum_{n=1}^{seg+3}a_nS_n(z) \quad (6)$$

where a_n's are the unknown coefficients that should be determined. S_n's are the cubic spline basis functions in the interval [0-d], [14], seg is the number of segments for the construction of the cubic B-splines.

By imposing boundary condition of continuity of transverse component of electric and magnetic field at z=0 and z=d and using the fact that $\psi(z=d)=a_{seg+3}$ and $\psi(z=0)=a_1$ we obtain:

$$T=a_{seg+3} \quad (7)$$

$$R=a_1-1 \quad (8)$$

Inserting (7) and (8) in (3), applying Galerkin's method and making use of the boundary conditions at (z=0) and (z=d), we obtain the following matrix equations for the unknown coefficients a_n

$$[A+B][a_1...a_{seg+3}]^T=\left[\frac{-2jk_s\cos\theta}{p_s}0...0\right]^T \quad (9)$$

where T represents a transpose. Explicit expressions for the matrix elements in (10) are

$$A_{mn}=-\int_0^d\frac{S_n'S_m'}{p}dz+\int_0^d k_z^2\frac{S_nS_m}{p}dz \quad (10)$$

$$\begin{bmatrix}\frac{-jk_s\cos\theta}{p_s} & .... & 0\\ \vdots & 0 & \vdots\\ 0 & .... & \frac{-jk_s\cos\theta}{p_s}\end{bmatrix} \quad (11)$$

After the matrix equations in (9) are solved, the values of ψ for every ($0\le z\le d$) can be determined from (6) and we can obtain transmission and reflection coefficient from equations (7) and (8), respectively.

3- Numerical Result

In this section, the accuracy and efficiency of the proposed method through some numerical examples is demonstrated. The proposed method can be used to convert the scalar wave equation to a system of linear algebraic equations for lossy or lossless, homogeneous or non-homogeneous and dispersive or non-dispersive layer with different upper and lower half space. In Fig.2 and Fig.3 transverse component of electric field (e_y) and transmission and reflection coefficients are shown, respectively in exponential inhomogeneous layer using proposed method, where ($\varepsilon_r(z)=4e^{Kz}$,$\mu_r=1$) and are compare with the obtained results from exact solution [7] Calculation time in this example is 0.1 seconds. Calculation time of this example with proposed method in [7] is 1 second.

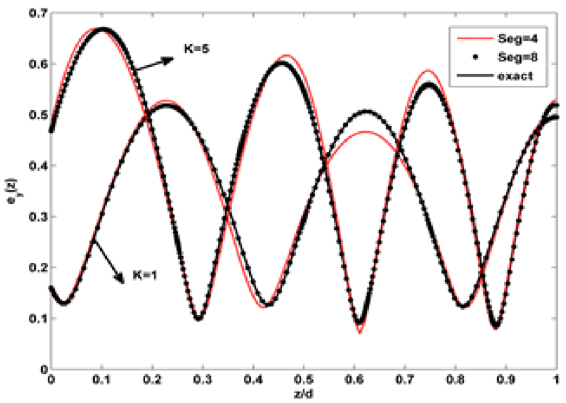

Fig. 2.Magnitude of transverse component of electric field e_y in TE polarization at f=1GHz and incident angle θ=π/3 and thickness of layer is d=20cm

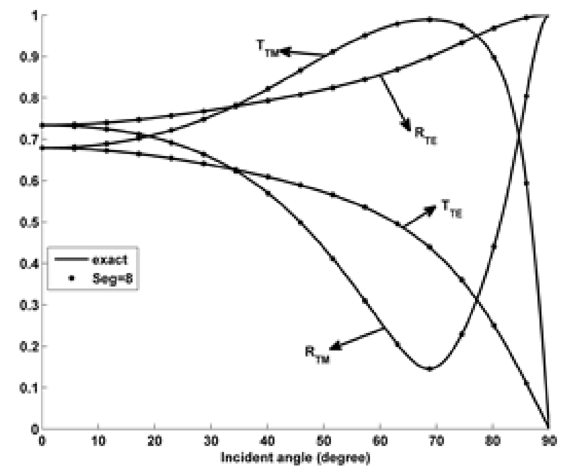

Fig. 3. Amplitude of transmission and reflection coefficient for both TE and TM polarization at f=1GHz and thickness of layer is d=20cm and $\varepsilon_r(z)=4e^{5z}$, $\mu_r=1$

In Fig.4, we consider dispersive inhomogenous layer and obtained reflection coefficient in a very wide band frequency and compare results with those obtained from exact solution. Calculation time in this example is 2.6 seconds. In Fig.4 after 20GHz for acceptable accuracy, we use 32 segmentations thus when frequency increases, we need more segmentation to achieve the same precision and 16 segmentation before 20GHz are needed.

In Figs.5 and 6 reflection coefficient and transverse electric field for the lossy inhomogeneous layer is shown, respectively. When a layer is lossy, calculation time is the same with the lossless layer.

If the thickness of the layer is much larger than the wavelength, the proposed method will become inefficient. With proposed method, we can determine scattering field from multilayer structures and anisotropic layer.

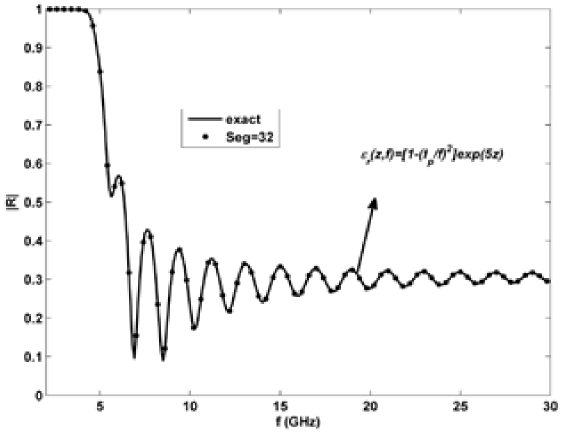

Fig. 4. The amplitude of the reflection coefficient for TE polarization .Thickness of layer is d=10cm and $\varepsilon_r(z,f)=(1-(f_p/f)^2)e^{5z}$ and f_p=3GHz, μ_r=1

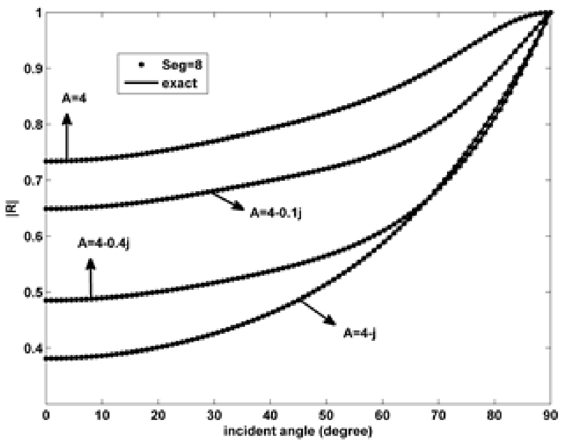

Fig. 5. Amplitude of reflection coefficient for TE polarization at f=1GHz and thickness of layer is d=20cm and $\varepsilon_r(z)=Ae^{5z}$ and μ_r=1

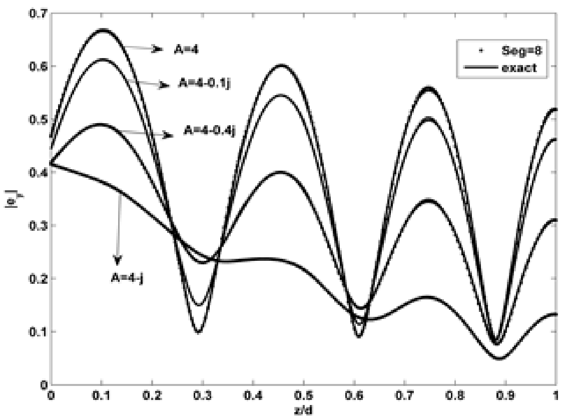

Fig. 6. Magnitude of transverse component of the electric field e_y in TE polarization at f=1GHz and incident angle θ=π/3 and thickness of layer is d=20cm and $\varepsilon_r(z)=Ae^{5z}$ and μ_r=1

4- Conclusion

We applied spline-based 1-D Galerkin's Finite element method for calculating transmission and reflection coefficient from the dispersive inhomogeneous layer and determined unknown electrical and magnetic field in the inhomogeneous layer. In the proposed method with Galerkin's method of moment, we discretized scalar wave equation in inhomogeneous medium then by imposing boundary condition, we obtained transmission and reflection coefficient from the dispersive inhomogeneous layer and determined unknown electrical and magnetic field in the inhomogeneous layer. The proposed method is applicable for lossy inhomogeneous layer; moreover, the layer could be dispersive. The accuracy of the proposed method depends on a number of segments and order of B-splines. In all examples, we utilized cubic splines as a basis function. In high frequency and large layer we need more segmentation to reach acceptable precision. The method has been applied to several inhomogeneous and dispersive profiles and the calculated results have been compared with the results of exact solution and the validity of the proposed method has been established. The proposed method is very fast since we solved all the examples less than 0.5 seconds.

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