Amirkabir University of Technology
(Tehran Polytechnic)

Amirkabir International Jounrnal of Science & Research
Electrical & Electronics Engineering
(AIJ-EEE)
Vol. 48, No. 2, Fall 2016, pp. 113-125

Stability Analysis and Robust PID Control of Cable-Driven Robots Considering Elasticity in Cables

M. A. Khosravi[1,*], H. D. Taghirad[2]

1- Assistant Professor, Department of Electrical Engineering, Amirkabir University of Technology
2- Professor, Faculty of Electrical Engineering, K. N. Toosi University of Technology

(Received 6 September 2015, Accepted 29 May 2016)

ABSTRACT

In this paper, robust PID control of fully-constrained cable driven parallel manipulators with elastic cables is studied in detail. In dynamic analysis, it is assumed that the dominant dynamics of cable can be approximated by linear axial spring. To develop the idea of control for cable robots with elastic cables, a robust PID control for cable-driven robots with ideal rigid cables is firstly designed and then this controller is modified for the robots with elastic cables. To overcome vibrations caused by the inevitable elasticity of cables, a composite control law is proposed based on singular perturbation theory. This proposed control algorithm includes robust PID control for the corresponding rigid model and a corrective term to damp the vibrations due to cables flexibility. Using the proposed control algorithm, the dynamics of the cable-driven robot is divided into slow and fast subsystems and then, based on the results of singular perturbation theory, stability analysis of the total system is performed. Finally, the effectiveness of the proposed control law is investigated through several simulations on a planar cable-driven robot.

KEYWORDS:

Cable Robots, Elasticity, Singular Perturbation, Stability Analysis

Please cite this article using:
Khosravi, M. A., and Taghirad, H. D., 2016. "Stability Analysis and Robust PID Control of Cable-Driven Robots Considering Elasticity in Cables". *Amirkabir International Journal of Electrical and Electronics Engineering*, 48(2), pp. 113–125.
DOI: 10.22060/eej.2016.821
URL: http://eej.aut.ac.ir/article_824.html
*Corresponding Author, Email: m.a.khosravi@aut.ac.ir

1- Iintroduction

Conventional robots with serial or parallel structures have some limitations to be used in large workspace applications. General serial and parallel robots use rigid links and elements in their structures and this leads to a small workspace and hence they are not suitable for long-reach robotics applications. To overcome this problem, a new trend was formed in design and implementation of the robots three decades ago. This trend is based on using cables instead of rigid links in parallel manipulators. A Cable-Driven Parallel Manipulator (CDPM) consists of an end-effector and a number of active cables connected to the end-effector. These cables are fixed on the base with actuating motors and pulleys. While the cables lengths are changing, the end-effector is forced toward the desired position and orientation. Replacing rigid links by cables inaugurates many potential applications, such as very large workspace robots [1], high-speed manipulation [2], handling of heavy materials [3], cleanup of disaster areas [4], access to remote locations, and interaction with the hazardous environment [5].

CDPMs can be classified into two types, fully-constrained and under-constrained [4-7]. In the fully-constrained type, cables can create any wrench by pulling on the end-effector [8] or equivalently, for a given set of cable lengths, the end-effector can not be moved in position and orientation [4,9]. To fully constrain a CDPM, the number of cables driving the end-effector must be at least one greater than the number of robot degrees of freedom. According to aforementioned facts, a wrench-closure pose of a CDPM is a pose at which the end-effector is fully-constrained by the cables. Based on this definition, the wrench-closure workspace of a CDPM can be defined as the set of wrench-closure poses [10]. The wrench-closure workspace only depends on the geometry of the mechanism [6]. The cable robots to be discussed in this paper are of a fully-constrained type and it is assumed that the motion control is in the wrench-closure workspace.

Using cables instead of rigid links, however, introduces new challenges in the study of CDPMs. Cables can only pull and not push, while general parallel robots have actuators that can provide bi-directional tension. Therefore, in this type of robots, the cables must be in tension in the whole workspace of the robot and as soon as the cables become slack, the structure of the cable robot collapses. The nonlinear dynamic behavior of the cables is another major challenge in mechanical and control design of this class of robots. Cables are usually elastic elements and have to encounter some unavoidable situations, such as elongation and vibration. Cables elasticity may cause position and orientation errors. Moreover, the system may lead to vibration, and cause the whole system to be uncontrollable. In applications which require high bandwidth or high stiffness of the system, vibration may be a serious concern [11]. In terms of control, proposed control algorithms for this class of robot must be designed such that they can damp vibrations and guarantee that the cables remain in tension.

Control of CDPMs has received limited attention compared to that of conventional robots. With the assumption of a massless and inextensible model for the cable, most of the common control strategies for conventional robots have been adapted for cable robots. Lyapunov-based control [2,12], computed torque method [13], sliding mode [14], robust PID control [15] and adaptive control [16,17] are some control schemes being used in the control of cable robots. The inclusion of cable dynamic characteristics in the model of the cable robot leads to a complication in the control algorithm, and research on this topic is very limited. By using elastic and massless model for the cables, authors derived a new model for the cable robot and proposed a new control algorithm [18]. This control algorithm is formed in cable length space and uses internal force concept and a damping term. The stability of the closed-loop system is analyzed through Lyapunov theory and vector closure conditions. In [19] the dynamic model of the robot is used in proposed composite controller which is formed in task space. This algorithm benefits internal force concept and a corrective term to compensate the effects of cables elasticity. However, due to its dependency to a dynamic model of the robot, this control algorithm is not robust against modeling uncertainties.

The main goal of this paper is to present a new approach in the control of CDPMs with elastic cables by using popular PID controller and singular perturbation theory [20]. Singular perturbations cause a multi-time scale behavior of dynamic systems characterized by a presence of both slow and fast transients in the response of the system [20]. Thus, dynamics of the system can be divided into two subsystems; slow and fast. These subsystems can be used in the design of efficient control algorithms. In this paper, first dynamics of cable robot with ideal rigid cables is introduced in section (2) and the robust PID control algorithm is proposed for this model. Then, UUB stability of the rigid system with proposed controller is analyzed through Lyapunov theory. Next in section (3), a dynamic model is extended for elastic cables, and a control strategy is developed using singular perturbation theory. By using Tikhonov's theorem, slow and fast variables are separated and incorporated in the stability analysis of the total closed-loop system. Finally, simulation results on a planar cable robot are given to demonstrate the effectiveness of the proposed control algorithm in practice.

2- Robust PID control of rigid cable robot

In this section, we assume that the elasticity of cables can be ignored and cables behave as massless rigid strings. Based on this assumption, the standard model for the dynamics of *n*-cable parallel robot is given as [15]:

$$\mathbf{M}_{eq}(\mathbf{x})\ddot{\mathbf{x}} + \mathbf{N}_{eq}(\mathbf{x},\dot{\mathbf{x}}) = \mathbf{J}^T\mathbf{u}_r \tag{1}$$

in which:

$$\mathbf{M}_{eq}(\mathbf{x}) = r\mathbf{M}(\mathbf{x}) + r^{-1}\mathbf{J}^T\mathbf{I}_m\mathbf{J}$$
$$\mathbf{N}_{eq}(\mathbf{x},\dot{\mathbf{x}}) = r\mathbf{N}(\mathbf{x},\dot{\mathbf{x}}) + r^{-1}\mathbf{J}^T\mathbf{I}_m\dot{\mathbf{J}}\dot{\mathbf{x}} + r^{-1}\mathbf{J}^T\mathbf{D}\mathbf{J}\dot{\mathbf{x}}$$
$$\mathbf{N}(\mathbf{x},\dot{\mathbf{x}}) = \mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{G}(\mathbf{x}) + \mathbf{F}_d\dot{\mathbf{x}} + \mathbf{F}_s(\dot{\mathbf{x}}) + \mathbf{T}_d$$
$$\mathbf{C}_{eq}(\mathbf{x},\dot{\mathbf{x}}) = r\mathbf{C}(\mathbf{x},\dot{\mathbf{x}}) + r^{-1}\mathbf{J}^T\mathbf{I}_m\dot{\mathbf{J}}$$

where $\mathbf{x}\in\mathbf{R}^6$ is the vector of generalized coordinates, $\mathbf{M}(\mathbf{x})$ is the inertia matrix, $\mathbf{I}_m$ is the diagonal matrix of actuator inertias reflected to the cable side of the gears, $\mathbf{C}(\mathbf{x},\dot{\mathbf{x}})$ represents the Coriolis and centrifugal terms, $\mathbf{G}(\mathbf{x})$ is the gravitational terms, r is the radius of pulleys and $\mathbf{u}_r$ represents the input torque. $\mathbf{J}$ represents the Jacobian matrix of the system and relates $\dot{\mathbf{x}}$ to derivative of the cable length vector by: $\dot{\mathbf{L}}=\mathbf{J}\dot{\mathbf{x}}$. Furthermore, $\mathbf{F}_d$ denotes the coefficient matrix of viscous friction, $\mathbf{F}_s$ is Coulomb friction term and $\mathbf{T}_d$ denotes disturbances which could represent any inaccuracy in a dynamic model. Although these equations are nonlinear and complex, they have some properties which are beneficial in the controller design [19].

Property 1: Inertia matrix $\mathbf{M}_{eq}(\mathbf{x})$ is symmetric and positive definite.

Property 2: Matrix $\dot{\mathbf{M}}_{eq}(\mathbf{x})-2\mathbf{C}_{eq}(\mathbf{x},\dot{\mathbf{x}})$ is skew-symmetric.

In the design of robust PID controller, it is assumed that all dynamical terms, such as $\mathbf{M}_{eq}(\mathbf{x})$ and $\mathbf{C}_{eq}(\mathbf{x},\dot{\mathbf{x}})$ are uncertain and only some information about their bounds is available. As it is demonstrated in [15], in spite of uncertainties in all parameters, the following relations hold for dynamic terms of the cable robot.

$$\begin{cases} \underline{m} \le \left\| M_{eq} \right\| \le \overline{m} \\ \left\| \mathbf{C}_{eq}(\mathbf{x},\dot{\mathbf{x}}) \right\| \le \beta_3 + \beta_4 \left\| \mathbf{y} \right\| \\ \left\| \mathbf{G}_{eq}(\mathbf{x}) \right\| \le \xi_{g_{eq}} \\ \left\| \mathbf{N}_{eq}(\mathbf{x},\dot{\mathbf{x}}) \right\| \le \beta_0 + \beta_1 \left\| \mathbf{y} \right\| + \beta_2 \left\| \mathbf{y} \right\|^2 \end{cases} \tag{2}$$

in which, $\mathbf{e}=\mathbf{x}_d-\mathbf{x}$ and $y=[\int_0^t \mathbf{e}^T(s)ds\,\mathbf{e}^T\dot{\mathbf{e}}^T]^T$. The control law is designed based on these bounds and assumptions to satisfy robust stability conditions. Recall dynamic model of system (1), and choose a PID controller for the system as follows:

$$\mathbf{u}_1 = \mathbf{J}^T\mathbf{u}_r = \mathbf{K}_V\dot{\mathbf{e}} + \mathbf{K}_P\mathbf{e} + \mathbf{K}_I\int_0^t \mathbf{e}(s)ds \tag{3}$$

or

$$\mathbf{u}_r = \mathbf{J}^\dagger\left[\mathbf{K}_V\dot{\mathbf{e}} + \mathbf{K}_P\mathbf{e} + \mathbf{K}_I\int_0^t \mathbf{e}(s)ds\right] + r\mathbf{Q} \tag{4}$$

$\mathbf{J}^\dagger$ is pseudo-inverse of $\mathbf{J}^T$, which achieves a minimum norm response, and $\mathbf{Q}$ which is called the internal forces, spans null space of $\mathbf{J}^T$ and must satisfy

$$\mathbf{J}^T\mathbf{Q} = 0 \tag{5}$$

It is important to note that the vector $\mathbf{Q}$ does not contribute to the motion of the end-effector and only causes internal forces in the cables. This term ensures that all cables remain in tension in the whole workspace. In this paper, it is assumed that the motion is always within the wrench-closure workspace and, as a consequence, at all times, positive internal forces can be produced such that the cables remain in tension.

2- 1- Stability analysis

Implement the control law $\mathbf{u}_r$ in (1) to get:

$$\dot{\mathbf{y}} = \mathbf{A}\mathbf{y} + \mathbf{B}\Delta\mathbf{A} \tag{6}$$

where,

$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{I}_6 & 0 \\ 0 & 0 & \mathbf{I}_6 \\ -\mathbf{M}_{eq}^{-1}\mathbf{K}_I & -\mathbf{M}_{eq}^{-1}\mathbf{K}_P & -\mathbf{M}_{eq}^{-1}\mathbf{K}_V \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \mathbf{M}_{eq}^{-1} \end{bmatrix},\ \Delta\mathbf{A} = \mathbf{N}_{eq}(\mathbf{x},\dot{\mathbf{x}}) + \mathbf{M}_{eq}\ddot{\mathbf{x}}_d$$

To analyze the robust stability of the system, consider the following Lyapunov function.

$$V_R(\mathbf{y}) = \mathbf{y}^T\mathbf{P}\mathbf{y}$$

in which,

$$\mathbf{P} = \frac{1}{2}\begin{bmatrix} \mu\mathbf{K}_P + \mu\mathbf{K}_I + \mu^2\mathbf{M}_{eq} & \mu\mathbf{K}_V + \mathbf{K}_I + \mu^2\mathbf{M}_{eq} & \mu\mathbf{M}_{eq} \\ \mu\mathbf{K}_V + \mathbf{K}_I + \mu^2\mathbf{M}_{eq} & \mu\mathbf{K}_V + \mathbf{K}_P + \mu^2\mathbf{M}_{eq} & \mu\mathbf{M}_{eq} \\ \mu\mathbf{M}_{eq} & \mu\mathbf{M}_{eq} & \mathbf{M}_{eq} \end{bmatrix}$$

Now for simplicity choose, $\mathbf{K}_P=\mathbf{k}_p\mathbf{I}$, $\mathbf{K}_V=\mathbf{k}_V\mathbf{I}$ and $\mathbf{K}_I=\mathbf{k}_I\mathbf{I}$. Then we may prove the following results.

Lemma 1: Assume the following inequalities hold:

$$\mu > 0 \quad , \quad 2\mu < 1$$

$$s_1 = \mu(k_P - k_V) - (1-\mu)k_I - \mu\overline{m} > 0$$

$$s_2 = k_P - k_I - \mu\overline{m} > 0$$

Then $\mathbf{P}$ is positive definite and satisfies the following condition:

$$\underline{\lambda}(\mathbf{P})\|\mathbf{y}\|^2 \le V_R(\mathbf{y}) \le \overline{\lambda}(\mathbf{P})\|\mathbf{y}\|^2 \qquad (7)$$

in which,

$$\underline{\lambda}(\mathbf{P}) = min\{\frac{1-2\mu}{2}\underline{m}, \frac{s_1}{2}, \frac{s_2}{2}\}$$

$$\overline{\lambda}(\mathbf{P}) = max\{\frac{1+2\mu}{2}\overline{m}, \frac{s_3}{2}, \frac{s_4}{2}\}$$

and

$$s_3 = \mu(k_P + k_v) + (1+\mu)k_I + (1+2\mu)\mu\overline{m}$$

$$s_4 = \mu\overline{m}(1+2\mu) + 2\mu k_V + k_P + k_I$$

$\underline{m}$ and $\overline{m}$ are defined as before in (2).

The proof is based on Gershgorin theorem and is similar to that in [21]. Now, since $\mathbf{P}$ is positive definite, by using skew-symmetry of $\dot{\mathbf{M}}_{eq}$-2$\mathbf{C}_{eq}$ and some manipulations, one may write

$$\dot{V}_R(\mathbf{y}) = -\mathbf{y}^T\mathbf{H}\mathbf{y} + \frac{1}{2}\mathbf{y}^T\begin{bmatrix}\mu\mathbf{I}\\ \mu\mathbf{I}\\ \mathbf{I}\end{bmatrix}(\mathbf{C}_{eq} + \mathbf{C}_{eq}^T)[\mu\mathbf{I} \quad \mu\mathbf{I} \quad \mathbf{I}]\mathbf{y}$$

$$+\mathbf{y}^T\begin{bmatrix}\mu\mathbf{I}\\ \mu\mathbf{I}\\ \mathbf{I}\end{bmatrix}\Delta\mathbf{A} +$$

$$\frac{1}{2}\mathbf{y}^T\begin{bmatrix} 0 & \mu^2\mathbf{I} & \mu^2\mathbf{I} \\ \mu\mathbf{I} & 2\mu^2\mathbf{I} & (\mu^2+\mu)\mathbf{I} \\ \mu^2\mathbf{I} & (\mu^2+\mu)\mathbf{I} & \mu\mathbf{I} \end{bmatrix}\begin{bmatrix} \mathbf{M}_{eq} & 0 & 0 \\ 0 & \mathbf{M}_{eq} & 0 \\ 0 & 0 & \mathbf{M}_{eq} \end{bmatrix}\mathbf{y}$$

where,

$$\mathbf{H} = \begin{bmatrix} \mu k_I\mathbf{I} & 0 & 0 \\ 0 & (\mu k_P - \mu k_V - k_I)\mathbf{I} & 0 \\ 0 & 0 & k_V\mathbf{I} \end{bmatrix}$$

Hence, we have

$$\dot{V}_R(\mathbf{y}) \le$$

$$-\rho\|\mathbf{y}\|^2 + \lambda_1\|\mathbf{C}_{eq}\|\|\mathbf{y}\|^2 + \mu^{-1}\lambda_1\|\mathbf{y}\|\|\Delta\mathbf{A}\| + \lambda_2\overline{m}\|\mathbf{y}\|^2$$

in which,

$$\rho = min\{\mu k_I, \mu(k_P - k_V) - k_I, k_V\}$$

Using inequalities (2), one may write $\dot{V}_R(\mathbf{y})$ as

$$\dot{V}_R(\mathbf{y}) \le \|\mathbf{y}\|(\xi_0 - \xi_1\|\mathbf{y}\| + \xi_2\|\mathbf{y}\|^2) \qquad (8)$$

in which,

$$\begin{cases} \xi_0 = \mu^{-1}\lambda_1\beta_0 + \mu^{-1}\lambda_1\lambda_3\overline{m} \\ \xi_1 = \rho - \lambda_1\beta_3 - \frac{1}{2}\lambda_2\overline{m} - \mu^{-1}\lambda_1\beta_1 \\ \xi_2 = \lambda_1\beta_4 + \mu^{-1}\lambda_1\beta_2 \end{cases} \qquad (9)$$

where $\lambda_1=\lambda_{max}(\mathbf{R}_1)$, $\lambda_2=\lambda_{max}(\mathbf{R}_2)$ and $\lambda_3=\lambda_{max}(\mathbf{R}_3)$, and λ_{max} denotes the largest eigenvalue of the corresponding matrix, and

$$\mathbf{R}_1 = \begin{bmatrix} \mu^2\mathbf{I} & \mu^2\mathbf{I} & \mu\mathbf{I} \\ \mu^2\mathbf{I} & \mu^2\mathbf{I} & \mu\mathbf{I} \\ \mu\mathbf{I} & \mu\mathbf{I} & \mu\mathbf{I} \end{bmatrix}$$

$$\mathbf{R}_2 = \begin{bmatrix} 0 & \mu^2\mathbf{I} & \mu^2\mathbf{I} \\ \mu\mathbf{I} & 2\mu^2\mathbf{I} & (\mu^2+\mu)\mathbf{I} \\ \mu^2\mathbf{I} & (\mu^2+\mu)\mathbf{I} & \mu\mathbf{I} \end{bmatrix}$$

According to the result obtained so far, we may state the stability conditions for the error system based on the following theorem.

Theorem 2.1: The error system (6) is uniformly ultimately bounded (UUB), if ξ_1 is chosen large enough.

Proof: According to Eqs. (7) and (8) and lemma 3.5 from [22], if the following conditions hold, the system is UUB stable with respect to $\mathbf{B}(0,d)$, where

$$d = \frac{2\xi_0}{\xi_1 + \sqrt{\xi_1^2 - 4\xi_0\xi_2}}\sqrt{\frac{\overline{\lambda}(\mathbf{P})}{\underline{\lambda}(\mathbf{P})}}$$

The conditions are:

$$\xi_1 > 2\sqrt{\xi_0\xi_2}$$

$$\xi_1^2 + \xi_1\sqrt{\xi_1^2 - 4\xi_0\xi_2} > 2\xi_0\xi_2(1+\sqrt{\frac{\overline{\lambda}(\mathbf{P})}{\underline{\lambda}(\mathbf{P})}})$$

$$\xi_1 + \sqrt{\xi_1^2 - 4\xi_0\xi_2} > 2\xi_2\|\mathbf{y}_0\|\sqrt{\frac{\overline{\lambda}(\mathbf{P})}{\underline{\lambda}(\mathbf{P})}}$$

These conditions can be simply met by choosing large enough ξ_1. According to (9), this can be met by choosing appropriate large control gains $\mathbf{K}_p$, $\mathbf{K}_V$ and $\mathbf{K}_I$.

3- Robot with elastic cables

3- 1- Dynamic model

In a CDPM vibration caused by inevitable elasticity in cables may be a major concern for some applications which require high accuracy or high bandwidth. New research results have shown that in fully-constrained cable robots, dominant dynamics of cables correspond to longitudinal vibrations [11,19,23] and therefore, axial spring model can suitably describe the effects of dominant dynamics of cable.

In order to model a general cable-driven robot with n cables assume that: L_{1i}:i=1,2,...,n denotes the length of i-th cable with tension which can be measured by a string pot. L_{2i}:i=1,2,...,n denotes the cable length corresponding to the i-th actuator and may be measured by the motor shaft encoder. If the system is rigid, then $L_{1i}=L_{2i}$, $\forall i$. Let us denote:

$L=(L_{11},L_{12},...,L_{1n},L_{21},L_{22},...,L_{2n})=(L_1^T,L_2^T)^T$

With this notation, the final equations of motion are derived in [19], which may be written as follows:

$$\mathbf{M}(\mathbf{x})\ddot{\mathbf{x}}+\mathbf{N}(\mathbf{x},\dot{\mathbf{x}})=\mathbf{J}^T\mathbf{K}(\mathbf{L}_2-\mathbf{L}_1) \quad (10)$$

$$\mathbf{I}_m\ddot{\mathbf{q}}+\mathbf{D}\dot{\mathbf{q}}+r\mathbf{K}(\mathbf{L}_2-\mathbf{L}_1)=\mathbf{u} \quad (11)$$

in which,

$$\mathbf{N}(\mathbf{x},\dot{\mathbf{x}})=\mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}}+\mathbf{G}(\mathbf{x})+\mathbf{F}_d\dot{\mathbf{x}}+\mathbf{F}_s(\dot{\mathbf{x}})+\mathbf{T}_d$$

$$\mathbf{L}_2-\mathbf{L}_0=r\mathbf{q} \quad , \quad \dot{\mathbf{L}}_1=\mathbf{J}\dot{\mathbf{x}}$$

In these equations, $\mathbf{L}_0$ denotes the initial cables length vector at $\mathbf{x}$=0, $\mathbf{q}$ is the motor shaft position vector, and other parameters are defined as before. For the notational simplicity, we assumed that all cable stiffness coefficients are equal[1]. Furthermore, assume that the stiffness values of the cables are in order of magnitude larger than other system parameters. To idealize this assumption, assume that $\mathbf{K}=O(1/\varepsilon^2)$ where ε is a small parameter.

Eqs. (10) and (11) represent CDPM as a nonlinear and coupled system. This representation includes both rigid and flexible subsystems and their interactions. It can be shown that the model of cable driven parallel robot with elastic cables is reduced to (1), if the cable stiffness $\mathbf{K}$ tends to infinity. Furthermore, this model has inherited the properties of rigid dynamics (1), such as the positive definiteness of inertia matrix and skew symmetricity of $\dot{\mathbf{M}}_{eq}$-2$\mathbf{C}_{eq}$.

[1] This assumption does not reduce the generality of the problem, and for the general case this can be easily reached by variable scaling.

3- 2- Control

In this section, we will show that the control law (4) developed for a rigid robot can be modified for the robot with elastic cables. First, consider a composite control law by adding a corrective term to the control law (4) in the form of

$$\mathbf{u}=\mathbf{u}_r+\mathbf{K}_d(\dot{\mathbf{L}}_1-\dot{\mathbf{L}}_2) \quad (12)$$

where $\mathbf{u}_r$ is given by (4) in terms of , $\mathbf{x}$ and $\mathbf{K}_d$ is a constant and positive diagonal matrix whose diagonal elements are in order of $O(1/\varepsilon)$. Notice that:

$$\mathbf{L}_2=r\mathbf{q}+\mathbf{L}_0\Rightarrow\dot{\mathbf{L}}_2=r\dot{\mathbf{q}},\ddot{\mathbf{L}}_2=r\ddot{\mathbf{q}} \quad (13)$$

Substitute control law (12) in (11) and define variable $\mathbf{z}$ as

$$\mathbf{z}=\mathbf{K}(\mathbf{L}_2-\mathbf{L}_1) \quad (14)$$

The closed loop dynamics reduces to

$$r^{-1}\mathbf{I}_m\ddot{\mathbf{z}}+\mathbf{K}_d\dot{\mathbf{z}}+r\mathbf{K}\mathbf{z}=\mathbf{K}(\mathbf{u}_r-r^{-1}\mathbf{I}_m\ddot{\mathbf{L}}_1) \quad (15)$$

By the assumption on $\mathbf{K}$ and our choice for $\mathbf{K}_d$ we may write

$$\mathbf{K}=\frac{\mathbf{K}_1}{\varepsilon^2}\,;\,\mathbf{K}_d=\frac{\mathbf{K}_2}{\varepsilon} \quad (16)$$

where $\mathbf{K}_1$ and $\mathbf{K}_2$ are of $O(1)$. Therefore (15) can be written as

$$\varepsilon^2r^{-1}\mathbf{I}_m\ddot{\mathbf{z}}+\varepsilon\mathbf{K}_2\dot{\mathbf{z}}+r\mathbf{K}_1\mathbf{z}=\mathbf{K}_1(\mathbf{u}_r-r^{-1}\mathbf{I}_m\ddot{\mathbf{L}}_1) \quad (17)$$

Now Eqs. (10) and (17) can be written together as:

$$\mathbf{M}(\mathbf{x})\ddot{\mathbf{x}}+\mathbf{N}(\mathbf{x},\dot{\mathbf{x}})=\mathbf{J}^T\mathbf{z} \quad (18)$$

$$\varepsilon^2r^{-1}\mathbf{I}_m\ddot{\mathbf{z}}+\varepsilon\mathbf{K}_2\dot{\mathbf{z}}+r\mathbf{K}_1\mathbf{z}=\mathbf{K}_1(\mathbf{u}_r-r^{-1}\mathbf{I}_m\ddot{\mathbf{L}}_1) \quad (19)$$

The variable $\mathbf{z}$ and its time derivative $\dot{\mathbf{z}}$ may be considered as the fast variables while the end-effector position variable $\mathbf{x}$ or $\mathbf{L}_1$ and its time derivative $\dot{\mathbf{x}}$ are considered as the slow variables. Using the results of singular perturbation theory, the elastic system (18) and (19) can be approximated by the quasi–steady state system or slow subsystem and the boundary layer system or fast subsystem [20]. With ε=0, equation (19) becomes

$$\bar{\mathbf{z}}=r^{-1}(\bar{\mathbf{u}}_r-r^{-1}\mathbf{I}_m\ddot{\bar{\mathbf{L}}}_1) \quad (20)$$

in which, the overbar variables represent the values of variables when ε=0. Substitute (20) into (18)

$$\mathbf{M}(\bar{\mathbf{x}})\ddot{\bar{\mathbf{x}}}+\mathbf{N}(\bar{\mathbf{x}},\dot{\bar{\mathbf{x}}})=r^{-1}\mathbf{J}^T(\bar{\mathbf{u}}_r-r^{-1}\mathbf{I}_m\ddot{\bar{\mathbf{L}}}_1) \quad (21)$$

Substitute $\ddot{\bar{\mathbf{L}}}_1=\mathbf{J}\ddot{\bar{\mathbf{x}}}+\dot{\mathbf{J}}\dot{\bar{\mathbf{x}}}$ in above equation:

$$(r\mathbf{M}(\bar{\mathbf{x}})+r^{-1}\mathbf{J}^T\mathbf{I}_m\mathbf{J})\ddot{\bar{\mathbf{x}}}+(r\mathbf{C}(\bar{\mathbf{x}},\dot{\bar{\mathbf{x}}})\dot{\bar{\mathbf{x}}}+r^{-1}\mathbf{J}^T\mathbf{I}_m\dot{\mathbf{J}}\dot{\bar{\mathbf{x}}})$$
$$+r\mathbf{G}(\bar{\mathbf{x}})=\mathbf{J}^T\bar{\mathbf{u}}_r$$

or

$$\mathbf{M}_{eq}(\bar{\mathbf{x}})\ddot{\bar{\mathbf{x}}}+\mathbf{C}_{eq}(\bar{\mathbf{x}},\dot{\bar{\mathbf{x}}})\dot{\bar{\mathbf{x}}}+\mathbf{G}_{eq}(\bar{\mathbf{x}})=\mathbf{J}^T\bar{\mathbf{u}}_r \quad (22)$$

Eq. (22) is called quasi-steady state system. Note that (22) is the rigid model (1) in terms of $\bar{\mathbf{x}}$. Using

Tikhonov's theorem [20], for t>0 the elastic force $\mathbf{z}(t)$ and the end-effector position $\mathbf{x}(t)$ satisfy

$$\mathbf{z}(t)=\overline{\mathbf{z}}(t)+\eta(\tau)+O(\varepsilon),\ \mathbf{x}(t)=\overline{\mathbf{x}}(t)+O(\varepsilon),$$
$$\mathbf{L}_1(t)=\overline{\mathbf{L}}_1(t)+O(\varepsilon)$$

where $\tau=t/\varepsilon$ is the fast time scale and η is the fast state variable that satisfies the following boundary layer equation,

$$r^{-1}\mathbf{I}_m\frac{d^2\eta}{d\tau^2}+\mathbf{K}_2\frac{d\eta}{d\tau}+r\mathbf{K}_1\eta=0 \tag{23}$$

Considering these results, elastic system (18) and (19) can be approximated up to $O(\varepsilon)$ as

$$\mathbf{M}(\mathbf{x})\ddot{\mathbf{x}}+\mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}}+\mathbf{G}(\mathbf{x})=\mathbf{J}^T(\overline{\mathbf{z}}+\eta(\tau))$$
$$r^{-1}\mathbf{I}_m\frac{d^2\eta}{d\tau^2}+\mathbf{K}_2\frac{d\eta}{d\tau}+rK_1\eta=0$$

According to (20)

$$\mathbf{M}_{eq}(\mathbf{x})\ddot{\mathbf{x}}+\mathbf{N}_{eq}(\mathbf{x},\dot{\mathbf{x}})=\mathbf{J}^T(\mathbf{u}_r+r\eta(\tau)) \tag{24}$$

$$r^{-1}\mathbf{I}_m\frac{d^2\eta}{dt^2}+\mathbf{K}_d\frac{d\eta}{dt}+r\mathbf{K}\eta=0 \tag{25}$$

Notice that the controller gain $\mathbf{K}_d$ can be suitably chosen such that the boundary layer system (23) becomes asymptotically stable. By this means, with sufficiently small values of ε, the response of the elastic system (10) and (11) with the composite control (12) consisting of the rigid control $\mathbf{u}_r$ given by (4) and the corrective term $\mathbf{K}_d(\dot{\mathbf{L}}_1\text{-}\dot{\mathbf{L}}_2)$ will be nearly the same as the response of rigid system (1) with the rigid control $\mathbf{u}_r$ alone. This will happen after some initially damped transient oscillation of the fast variables $\eta(t/\varepsilon)$.

3- 3- Stability analysis of total system

Control of rigid model and its stability analysis were discussed in the previous section. Furthermore, it is demonstrated that the boundary layer or the fast subsystem (23) is asymptotically stable if the corrective term is used in the control law. However, in general, the individual stability of the boundary layer and that of quasi-steady state subsystems does not guarantee the stability of the total closed-loop system. In this section, the stability of the total system is analyzed in detail. Recall the dynamic equations of elastic system (24) and (25), and apply the control law (4) from the previous section. Consider $\mathbf{y}=[\int_0^t\mathbf{e}^T ds\ \ \mathbf{e}^T\ \ \dot{\mathbf{e}}^T]^T$ and $\mathbf{h}=[\eta^T\ \ \ \dot{\eta}^T]^T$, in which $\mathbf{e}=\mathbf{x}_d\text{-}\mathbf{x}$. Then the dynamics equations can be rewritten as,

$$\dot{\mathbf{y}}=\mathbf{A}\mathbf{y}+\mathbf{B}\Delta\mathbf{A}+\mathbf{C}[\mathbf{I}\ 0]\mathbf{h} \tag{26}$$

$$\dot{\mathbf{h}}=\tilde{\mathbf{A}}\mathbf{h} \tag{27}$$

in which,

$$\mathbf{A}=\begin{bmatrix}0 & \mathbf{I}_6 & 0\\ 0 & 0 & \mathbf{I}_6\\ -\mathbf{M}_{eq}^{-1}\mathbf{K}_I & -\mathbf{M}_{eq}^{-1}\mathbf{K}_P & -\mathbf{M}_{eq}^{-1}\mathbf{K}_V\end{bmatrix}$$

$$\mathbf{B}=\begin{bmatrix}0\\0\\\mathbf{M}_{eq}^{-1}\end{bmatrix},\ \mathbf{C}=\begin{bmatrix}0\\0\\-r\mathbf{M}_{eq}^{-1}\mathbf{J}^T\end{bmatrix}$$

and

$$\Delta\mathbf{A}=\mathbf{N}_{eq}(\mathbf{x},\dot{\mathbf{x}})+\mathbf{M}_{eq}\ddot{\mathbf{x}}_d,\ \tilde{\mathbf{A}}=\begin{bmatrix}0 & \mathbf{I}\\ -r^2\mathbf{I}_m^{-1}\mathbf{K} & -r\mathbf{I}_m^{-1}\mathbf{K}_d\end{bmatrix}$$

The stability of this system may be analyzed by the following Lemma and Theorem.

Lemma 3.2: There is a positive definite matrix $\mathbf{K}_d$ such that the closed-loop system (27) is asymptotically stable.

Proof: Consider the following Lyapunov function candidate:

$$V_F=\mathbf{h}^T\mathbf{W}\mathbf{h},\ \mathbf{W}=\frac{1}{2}\begin{bmatrix}r^2(\mathbf{K}_d+\mathbf{K}) & r\mathbf{I}_m\\ r\mathbf{I}_m & \mathbf{I}_m\end{bmatrix} \tag{28}$$

According to Shur complement, in order to have positive definite $\mathbf{W}$, it is sufficient to have $\mathbf{K}_d$>$\mathbf{I}_m$. Differentiate V_F along trajectories of (27):

$$\dot{V}_F=-\mathbf{h}^T\mathbf{S}\mathbf{h},\ \mathbf{S}=\begin{bmatrix}r^3\mathbf{K} & 0\\ 0 & r(\mathbf{K}_d-\mathbf{I}_m)\end{bmatrix} \tag{29}$$

Since, $\mathbf{K}$, $\mathbf{K}_d$ and $\mathbf{I}_m$ are diagonal positive definite matrices, $\dot{V}_F$ becomes negative definite if $\mathbf{K}_d$>$\mathbf{I}_m$. If this condition holds, the closed-loop system (27) is asymptotically stable.

Theorem 3.3: The closed-loop system (26) and (27) is UUB stable if ξ_1 and $\mathbf{K}_d$ are chosen suitably large.

Proof: Consider the following composite Lyapunov function candidate

$$V(\mathbf{y},\mathbf{h})=V_R+V_F=\mathbf{y}^T\mathbf{P}\mathbf{y}+\mathbf{h}^T\mathbf{W}\mathbf{h} \tag{30}$$

in which, $\mathbf{y}^T\mathbf{P}\mathbf{y}$ denotes the Lyapunov function candidate for the rigid subsystem and $\mathbf{h}^T\mathbf{W}\mathbf{h}$ denotes that for the fast subsystem (23). According to Rayleigh-Ritz inequality:

$$\begin{cases}\underline{\lambda}(\mathbf{P})\|\mathbf{y}\|^2\le\mathbf{y}^T\mathbf{P}\mathbf{y}\le\overline{\lambda}(\mathbf{P})\|\mathbf{y}\|^2\\ \underline{\lambda}(\mathbf{W})\|\mathbf{h}\|^2\le\mathbf{h}^T\mathbf{W}\mathbf{h}\le\overline{\lambda}(\mathbf{W})\|\mathbf{h}\|^2\end{cases}$$

in which $\underline{\lambda}$ and $\overline{\lambda}$ are the smallest and largest eigenvalues of the matrices, respectively. By adding these inequalities, one can write,

$$\begin{gathered}[\|\mathbf{y}\| \quad \|\mathbf{h}\|]\begin{bmatrix}\underline{\lambda}(\mathbf{P}) & 0\\ 0 & \underline{\lambda}(\mathbf{W})\end{bmatrix}\begin{bmatrix}\|\mathbf{y}\|\\ \|\mathbf{h}\|\end{bmatrix} \le V(\mathbf{y},\mathbf{h}) \le \\ [\|\mathbf{y}\| \quad \|\mathbf{h}\|]\begin{bmatrix}\bar{\lambda}(\mathbf{P}) & 0\\ 0 & \bar{\lambda}(\mathbf{W})\end{bmatrix}\begin{bmatrix}\|\mathbf{y}\|\\ \|\mathbf{h}\|\end{bmatrix}\end{gathered} \tag{31}$$

Define $\mathbf{Z}_t = \begin{bmatrix}\|\mathbf{y}\|\\ \|\mathbf{h}\|\end{bmatrix}$ and apply Rayleigh-Ritz inequality, it can be written as:

$$\underline{\lambda}\|\mathbf{Z}_t\|^2 \le V(\mathbf{y},\mathbf{h}) \le \bar{\lambda}\|\mathbf{Z}_t\|^2 \tag{32}$$

in which,

$$\underline{\lambda} = min(\underline{\lambda}(\mathbf{P}), \underline{\lambda}(\mathbf{W}))$$

$$\bar{\lambda} = max(\bar{\lambda}(\mathbf{P}), \bar{\lambda}(\mathbf{W}))$$

Differentiate $V(\mathbf{y},\mathbf{h})$ along trajectories of (26) and (27). Hence,

$$\dot{V}(\mathbf{y},\mathbf{h}) = 2\mathbf{y}^T\mathbf{P}\dot{\mathbf{y}} + \mathbf{y}^T\dot{\mathbf{P}}\mathbf{y} + 2\mathbf{h}\mathbf{W}\dot{\mathbf{h}} = -\mathbf{h}^T\mathbf{S}\mathbf{h} + \left[2\mathbf{y}^T\mathbf{P}(\mathbf{A}\mathbf{y} + \mathbf{B}\Delta\mathbf{A}) + \mathbf{y}^T\dot{\mathbf{P}}\mathbf{y}\right] + 2\mathbf{y}^T\mathbf{P}\mathbf{C}[\mathbf{I} \quad 0]\mathbf{h}$$

Using Rayleigh-Ritz inequality,

$$-\mathbf{h}^T\mathbf{S}\mathbf{h} \le -\lambda_{min}(\mathbf{S})\|\mathbf{h}\|^2 \tag{33}$$

According to (8), it can be concluded that,

$$2\mathbf{y}^T\mathbf{P}(\mathbf{A}\mathbf{y} + \mathbf{B}\Delta\mathbf{A}) + \mathbf{y}^T\dot{\mathbf{P}}\mathbf{y} \le \|\mathbf{y}\|(\xi_0 - \xi_1\|\mathbf{y}\| + \xi_2\|\mathbf{y}\|^2)$$

Furthermore,

$$2\mathbf{y}^T\mathbf{P}\mathbf{C}[\mathbf{I} \quad 0]\mathbf{h} \le 2r\bar{m}\bar{\lambda}(\mathbf{P})\sigma_{max}(\mathbf{J}^T)\|\mathbf{y}\|\|\mathbf{h}\|$$

in which, λ_{min} and σ_{max} denote the smallest eigenvalue and largest singular value of the corresponding matrices, respectively. By using the above inequalities, one may write

$$\dot{V}(\mathbf{y},\mathbf{h}) \le -\mathbf{Z}_t^T\mathbf{R}\mathbf{Z}_t + \xi_0\|\mathbf{Z}_t\| + \xi_2\|\mathbf{Z}_t\|^3 \tag{34}$$

in which,

$$\mathbf{R} = \begin{bmatrix}\xi_1 & -r\bar{m}\bar{\lambda}(\mathbf{P})\sigma_{max}(\mathbf{J}^T)\\ -r\bar{m}\bar{\lambda}(\mathbf{P})\sigma_{max}(\mathbf{J}^T) & \lambda_{min}(\mathbf{S})\end{bmatrix} \tag{35}$$

R is the positive definite if

$$\lambda_{min}(\mathbf{S}) > \frac{r^2\bar{m}^2\bar{\lambda}^2(\mathbf{P})\sigma_{max}^2(\mathbf{J}^T)}{\xi_1} \tag{36}$$

This condition is met by a suitable choice of $\mathbf{K}_d$ for the fast subsystem. Therefore, one can write,

$$\dot{V}(\mathbf{y},\mathbf{h}) \le \|\mathbf{Z}_t\|(\xi_0 - \lambda_{min}(\mathbf{R})\|\mathbf{Z}_t\| + \xi_2\|\mathbf{Z}_t\|^2) \tag{37}$$

Now, according to (37) and (32), and Lemma 3.5 of [22], if these conditions are met, then the closed-loop system (26) and (27) is UUB stable with respect to $Y(0,d')$ where:

$$d' = \frac{2\xi_0}{\lambda_{min}(\mathbf{R}) + \sqrt{\lambda_{min}^2(\mathbf{R}) - 4\xi_0\xi_2}}\sqrt{\frac{\bar{\lambda}(\mathbf{P})}{\underline{\lambda}(\mathbf{P})}}$$

and the stability conditions are:

$$\lambda_{min}(\mathbf{R}) > 2\sqrt{\xi_0\xi_2}$$

$$\lambda_{min}^2(\mathbf{R}) + \lambda_{min}(\mathbf{R})\sqrt{\lambda_{min}^2(\mathbf{R}) - 4\xi_0\xi_2} > 2\xi_0\xi_2\left(1 + \sqrt{\frac{\bar{\lambda}(\mathbf{P})}{\underline{\lambda}(\mathbf{P})}}\right)$$

$$\lambda_{min}(\mathbf{R}) + \sqrt{\lambda_{min}^2(\mathbf{R}) - 4\xi_0\xi_2} > 2\xi_2\|\mathbf{Z}_{t0}\|\sqrt{\frac{\bar{\lambda}(\mathbf{P})}{\underline{\lambda}(\mathbf{P})}}$$

These conditions are satisfied by increasing $\lambda_{min}(\mathbf{R})$, through appropriate choice of large ξ_1, and $\lambda_{min}(\mathbf{S})$. Note that, ξ_1 is a function of the robust PID control gains $\mathbf{K}_I$, $\mathbf{K}_P$ and $\mathbf{K}_V$ and $\lambda_{min}(\mathbf{S})$ is affected by the control gain $\mathbf{K}_d$ for the fast subsystem. Therefore, the robust stability of the closed-loop system is guaranteed by suitable choice of the controller gains such that the above conditions are met.

4- Simulations

To show the effectiveness of the proposed control algorithm, a simulation study has been performed on a planar cable robot. Our model of a planar cable robot [24], consists of a moving platform that is connected by four cables to the base platform, as shown in Fig. (1). In this figure, A_i denote the fixed base points of the cables, and B_i denote points of connection of the cables on the moving platform. The position of the center of the mass of the moving platform **P** is denoted by **P**=$[x_P,y_P]$, and the orientation of the moving platform is denoted by φ with respect to the fixed coordinate frame. Hence, the manipulator poses three degrees of freedom **x**=$[x_P,y_P,\varphi]$, with one degree of actuator redundancy.

The equations of motion can be written in the following form,

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{G} + \mathbf{F}_d\dot{\mathbf{x}} + \mathbf{F}_s(\dot{\mathbf{x}}) + \mathbf{T}_d = \mathbf{J}^T\mathbf{K}(\mathbf{L}_2 - \mathbf{L}_1)$$

$$\mathbf{I}_m\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + r\mathbf{K}(\mathbf{L}_2 - \mathbf{L}_1) = \mathbf{u}$$

in which, **x**=$[x_P,y_P,\varphi]$, and

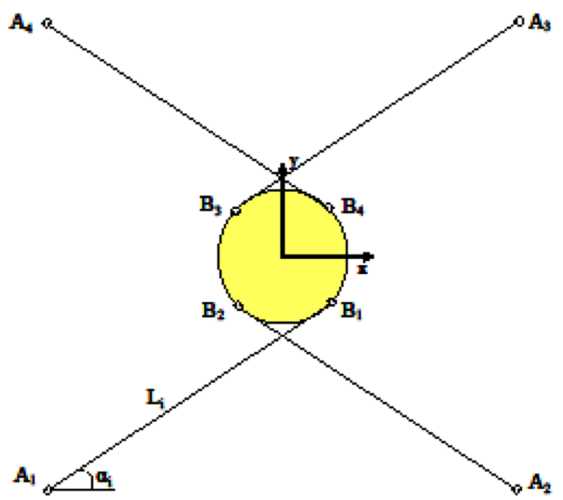

Fig. 1. The schematics of planar cable mechanism

$$\mathbf{M}=\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_z \end{bmatrix} \quad and \quad \mathbf{G}=\begin{bmatrix} 0 \\ mg \\ 0 \end{bmatrix}$$

The following parametric values in SI units are used in the simulations; $\mathbf{I}_m$=0.6$\mathbf{I}_{4\times4}$, r=0.035, $\mathbf{K}$=1000$\mathbf{I}$, m=2.5 and I_z=0.1. In order to demonstrate a highly flexible system, $\mathbf{K}$ is intentionally chosen very low. To show the effectiveness of the proposed composite control algorithm, suppose that the system is at the origin and has to track the following smooth reference trajectories in x, y and φ coordinates,

$$\begin{cases} x_d = 0 \\ y_d = 0.4 + 2e^{-t} - 2.4e^{-t/1.2} \\ \varphi_d = 0.2\sin(0.1\pi t) \end{cases} \quad (38)$$

The controller is based on (12) and consists of rigid control $\mathbf{u}_r$ given by (4) and the corrective term. Controller gain matrices are chosen as $\mathbf{K}_P$=250$\mathbf{I}_{3\times3}$, $\mathbf{K}_V$=40$\mathbf{I}_{3\times3}$, $\mathbf{K}_I$=20$\mathbf{I}_{3\times3}$ and $\mathbf{K}_d$=350$\mathbf{I}_{4\times4}$ to satisfy the stability conditions. In the first step, only rigid control law $\mathbf{u}_r$ is applied to the manipulator. As is illustrated in Fig. (2), the manipulator experiences instability if the rigid control $\mathbf{u}_r$ is solely applied to the system. The main reason for instability is the divergence of its fast variables.

Fig. (3) illustrates dynamic behavior of the closed-loop system with the proposed control algorithm. Internal forces $\mathbf{Q}$ are used whenever, at least, one cable becomes slack (or L_{1i}<L_{2i},i=1,...,4), in order to ensure that the cables remain in tension. Although the system is very flexible, the proposed control algorithm can suitably stabilize the system. As it is seen in this figure, position and orientation outputs track the desired values pretty well, and the steady state errors are very small, while as it is shown in Fig. (4), all cables are in tension for the whole maneuver.

To investigate the robustness of the proposed controller, another simulation with a maximum mass of the end-effector m=3 kg and $\mathbf{T}_d$=0.1$\mathbf{G}$ is performed. As it is shown in Fig. (5), in spite of uncertainties in the dynamics of the manipulator, the desired trajectories (38) are suitably tracked and the steady state errors can be ignored. Moreover, according to Fig. (6), all cables remain in tension.

To compare resulting performance of the proposed control algorithm with traditional (rigid) one, a more realistic case with $\mathbf{K}$=1000$\mathbf{I}_{4\times4}$ is considered for simulation. In this test, suppose that the home position for the end-effector is zero and the desired

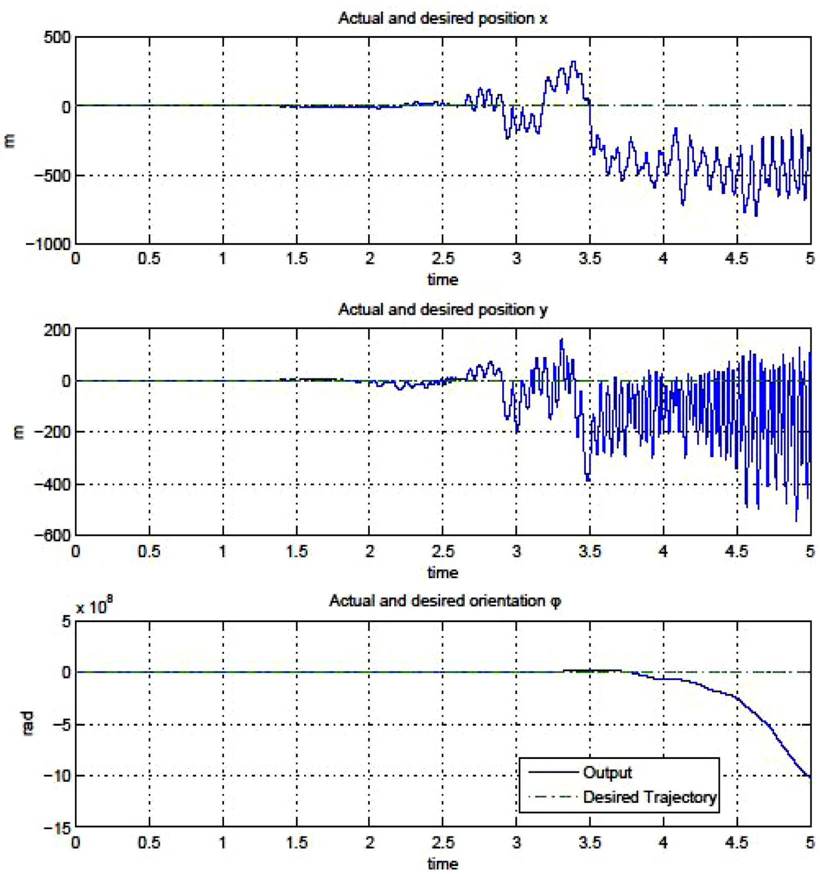

Fig. 2. The closed-loop system experiences instability if only rigid controller $\mathbf{u}_r$ is applied

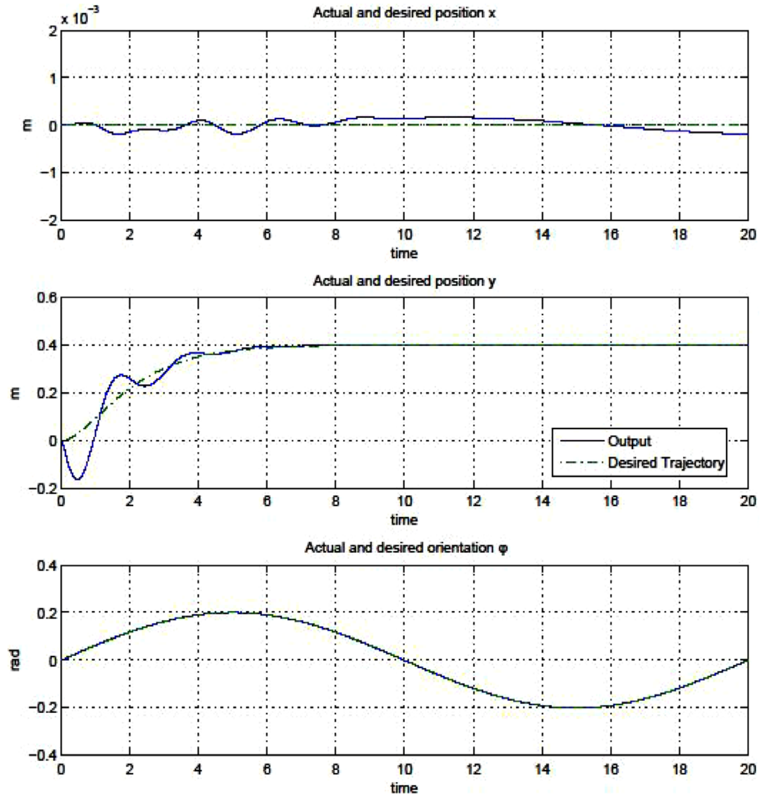

Fig. 3: Suitable tracking performance of the closed-loop system to smooth reference trajectories; Proposed control algorithm

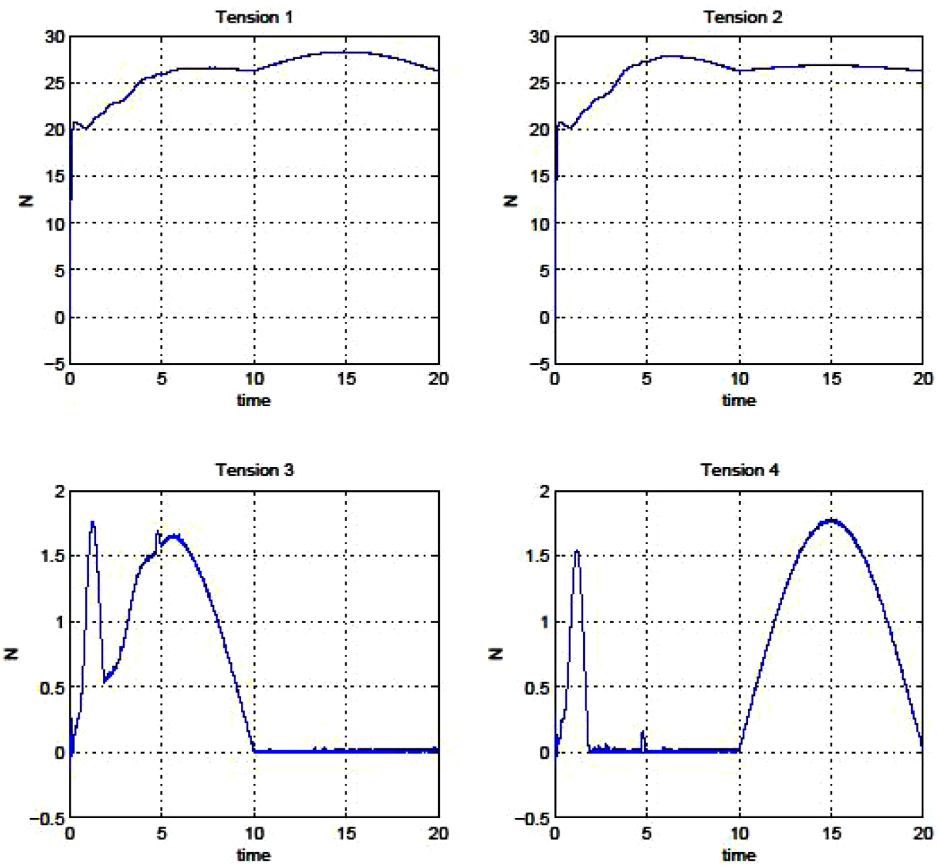

Fig. 4. Simulation results showing the cables tension for smooth reference trajectories

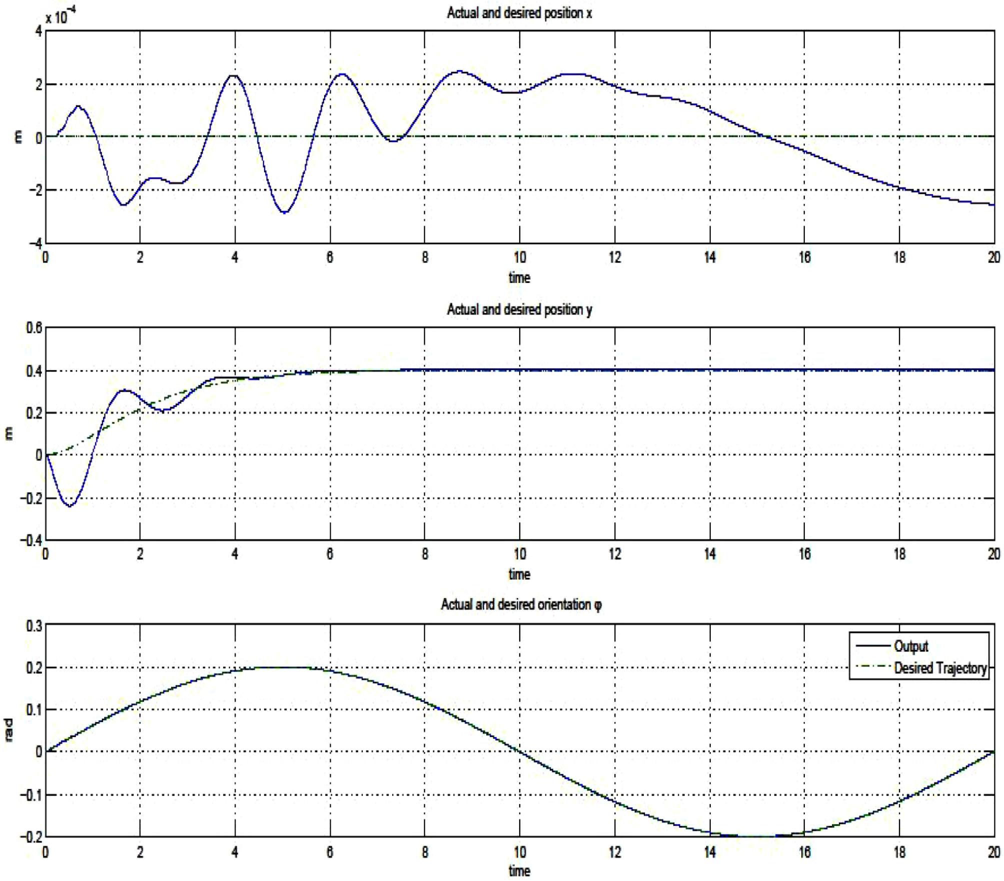

Fig. 5. Suitable tracking performance of the closed-loop system for the model with uncertainties

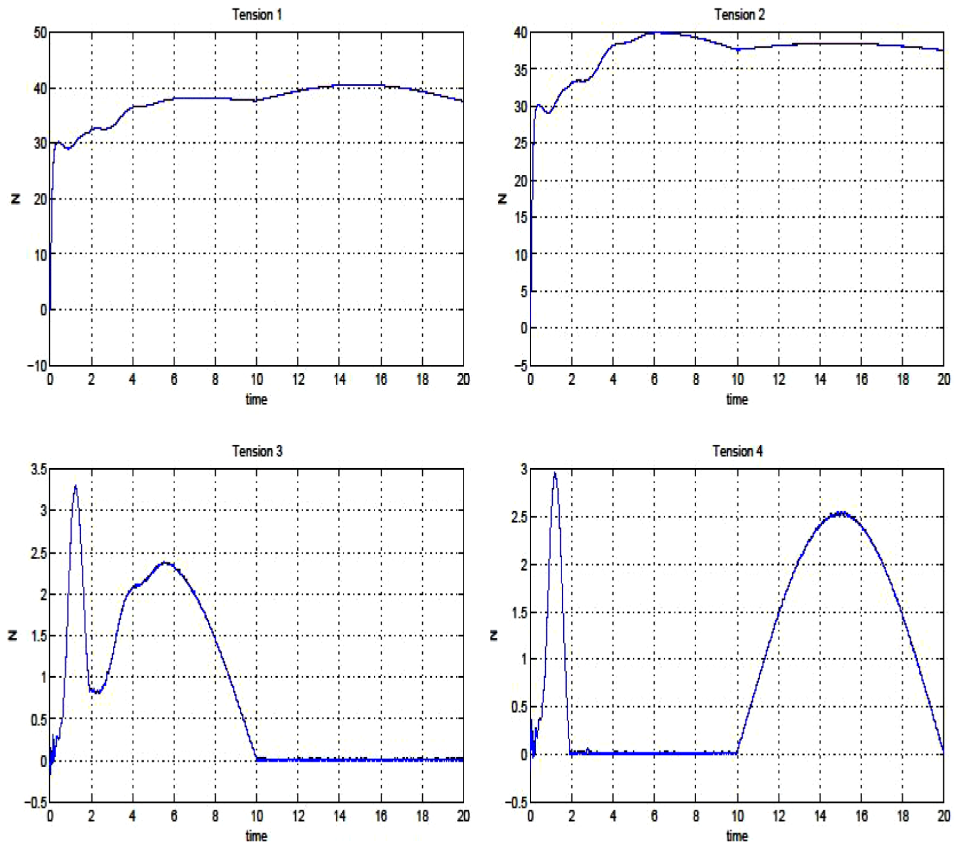

Fig. 6. Simulation results showing the cables tension for the model with uncertainties

end-effector position and orientation are

$x_d = 0$

$y_d = 0.4 + 2e^{-t} - 2.4e^{-t/1.2}$

$\varphi_d = 0$

As it is illustrated in Fig. (7), the closed-loop system becomes stable but there exist vibrations in the output, provided that only the corresponding rigid control effort $\mathbf{u}_r$ is applied to the system. These vibrations limit the absolute accuracy and the bandwidth of the mechanism which are very important in many applications such as high-speed manipulation. However, as illustrated in Fig. (8) the system becomes stable and the desired trajectories are well tracked, implementing the proposed control algorithm on the system.

5- Conclusions

In this paper, a robust PID control of CDPMs with elastic cables is examined in detail. Initially, Robust PID control of CDPMs with ideal cables is studied and it is proved that the proposed controller stabilize the system in the presence of dynamic uncertainties. Then, by using singular perturbation theory, this algorithm is modified to be applicable in the case of CDPMs with elastic cables. The proposed composite control algorithm consists of a robust PID control according to the corresponding rigid model and a corrective term to stabilize the fast subsystem. The stability of the closed-loop system is analyzed through Lyapunov second method, and it is shown that the proposed composite controller is capable of stabilizing the system in the presence of flexible cables. Finally, the performance of the proposed composite controller is examined through simulations.

6- References

[1] Taghirad, H. D. and Nahon, M.; "Kinematic Analysis of A Macro–Micro Redundantly Actuated Parallel Manipulator," *Advanced Robotics*, Vol. 22, No. 6–7, pp. 657–87, 2008.

[2] Kawamura, S.; Kino, H. and Won, C.; "High-Speed Manipulation by Using Parallel Wire-Driven Robots," *Robotica*, Vol. 18, No. 3, pp. 13–21, 2000.

[3] Bostelman, R.; Albus, J.; Dagalakis, N. and Jacoff, A.; "Applications of the NIST Robocrane," *Proceedings of the 5th International Symposium on Robotics and Manufacturing*, pp. 403–410, 1994.

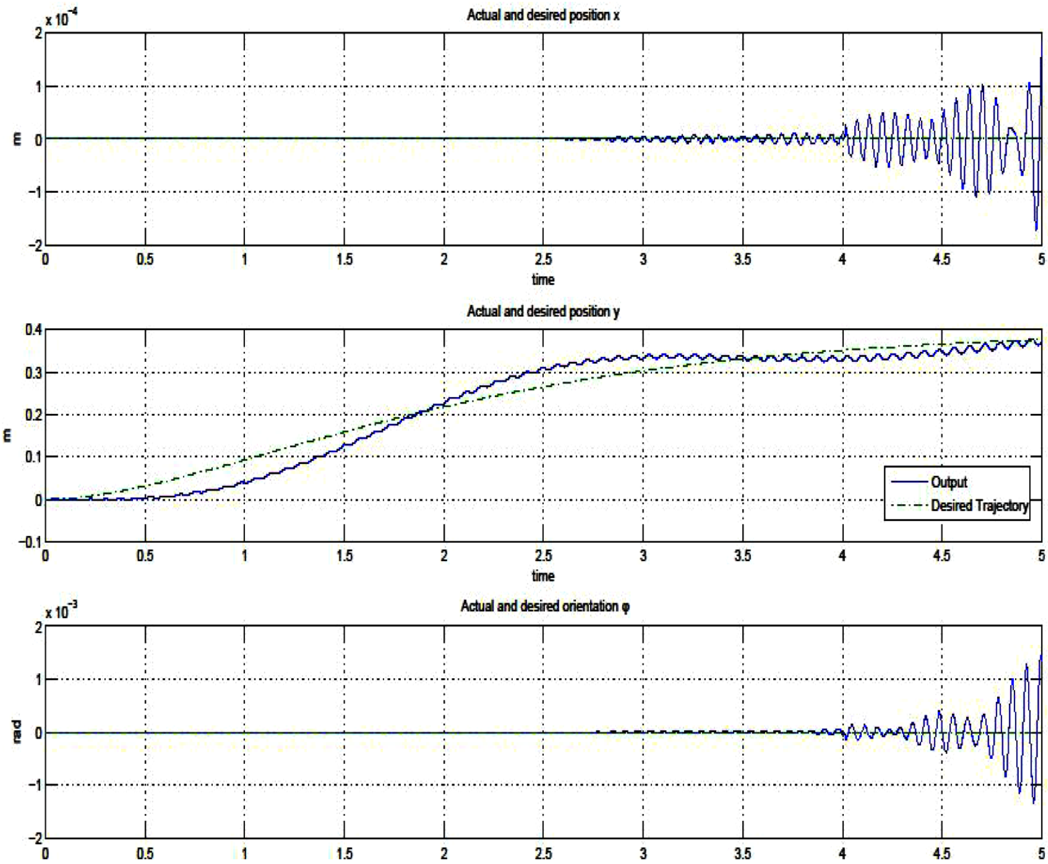

Fig. 7. Tracking performance of the closed-loop system with K=1000$\mathbf{I}_{4\times4}$, if only rigid controller $\mathbf{u}_r$ is applied

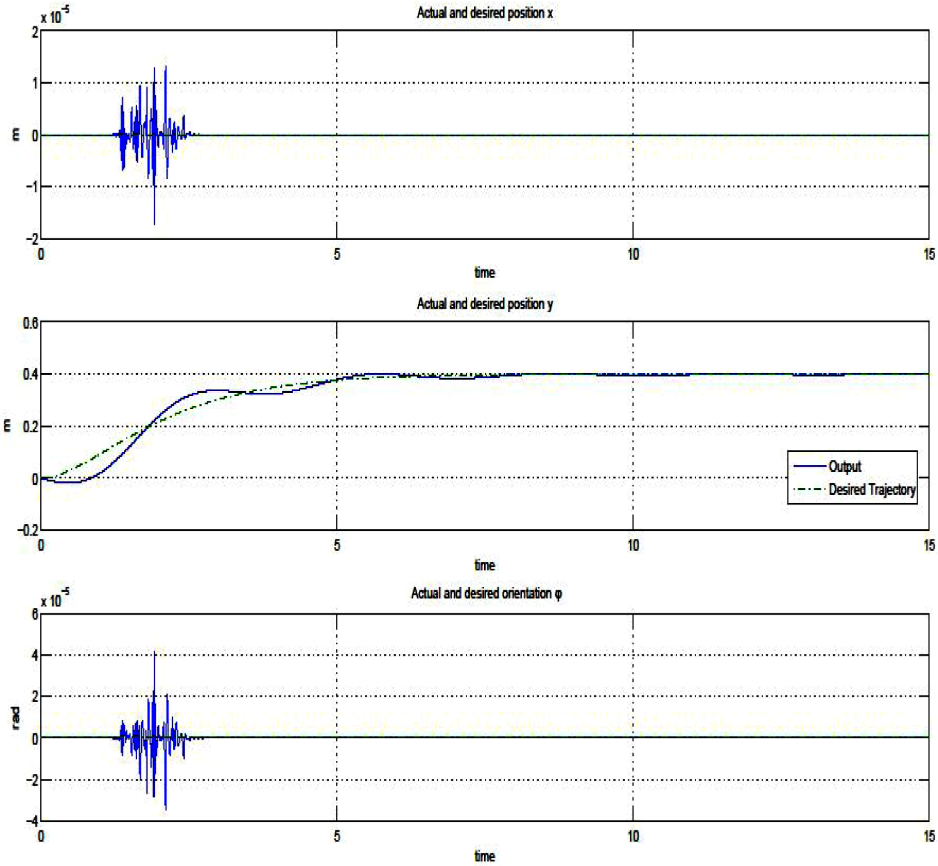

Fig. 8. Suitable tracking performance of the closed-loop system with K=1000I$_{4\times4}$; proposed control algorithm

[4] Roberts, R.; Graham, T. and Lippitt, T.; “On the Inverse Kinematics, Statics, and Fault Tolerance of Cable-Suspended Robots,” *Journal of Robotic Systems*, Vol. 15, No. 10, pp. 649–661, 1998.

[5] Riechel, A.; Bosscher, P.; Lipkin, H. and Ebert-Uphoff, I.; “Concept Paper: Cable-Driven Robots for Use in Hazardous Environments,” *Proceedings of 10th Int. Topical Meet. Robot Remote Syst. Hazardous Environ.*, Gainesville, pp. 310–317, 2004.

[6] Lim, W. B.; Yang, G.; Yeo, S. H. and Mustafa, S. K.; “A Generic Force-Closure Analysis Algorithm for Cable-Driven Parallel Manipulators,” *Mechanism and Machine Theory*, Vol. 46, No. 9, pp. 1265–1275, 2011.

[7] Bosscher, P. M.; “Disturbance Robustness Measures and Wrench-Feasible Workspace Generation Techniques for Cable-Driven Manipulators,” *Ph.D. Thesis*, Georgia Institute of Technology, 2004.

[8] Gouttefarde, M.; “Characterizations of Fully-Constrained Poses of Parallel Cable-Driven Robots: A Review,” *Proceedings of the ASME International Design Engineering Technical Conferences*, pp. 3–6, 2008.

[9] Bosscher, P. M.; Riechel, A. T. and Ebert-Uphoff, J.; “Wrench-Feasible Workspace Generation for Cable-Driven Robots,” *IEEE Trans. on Robotics*, Vol. 22, No. 5, pp. 890–902, 2006.

[10] Gouttefarde, M. and Gosselin, C.; “Analysis of the Wrench-Closure Workspace of Planar Parallel Cable-Driven Mechanisms,” *IEEE Trans. on Robotics*, Vol. 22, No. 3, pp. 434–445, 2006.

[11] Diao, X. and Ma, O.; “Vibration Analysis of Cable-Driven Parallel Manipulators,” *Multibody Syst Dyn*, Vol. 21, No. 4, pp. 347–360, 2009.

[12] Alp, A. and Agrawal, S.; “Cable Suspended Robots: Feedback Controllers with Positive Inputs,” *Proceedings of the American Control Conference*, Vol. 1, pp. 815–820, 2002.

[13] Williams, R. L.; Gallina, P. and Vadia, J.; “Planar Translational Cable Direct Driven Robots,” *Journal of Robotic Systems*, Vol. 20, No. 3, pp. 107–120, 2003.

[14] Ryoek, S. and Agrawal, S.; “Generation of Feasible Set Points and Control of a Cable Robot,”

IEEE Trans. on Robotics, Vol. 22, No. 3, pp. 551–558, 2006.

[15] Khosravi, M. A. and Taghirad, H. D.; "Robust PID Control of Fully-Constrained Cable Driven Parallel Robots," *Mechatronics*, Vol. 24, No. 2, pp. 87–97, 2014.

[16] Kino, H.; Yahiro, T. and Takemura, F.; "Robust PD Control Using Adaptive Compensation for Completely Restrained Parallel-Wire Driven Robots," *IEEE Trans. on Robotics*, Vol. 23, No. 4, pp. 803–812, 2007.

[17] Babaghasabha, R.; Khosravi, M. A. and Taghirad, H. D.; "Adaptive Robust Control of Fully-Constrained Cable Driven Parallel Robots," *Mechatronics*, Vol. 25, pp. 27–36, 2015.

[18] Khosravi, M. A. and Taghirad, H. D.; "Dynamic Analysis and Control of Cable Driven Robots with Elastic Cables," *Trans. Can. Soc. Mech. Eng.*, Vol. 35, No. 4, pp. 543–557, 2011.

[19] Khosravi, M. A. and Taghirad, H. D.; "Dynamic Modeling and Control of Parallel Robots with Elastic Cables: Singular Perturbation Approach," *IEEE Trans. on Robotics*, Vol. 30, No. 3, pp. 694–704, 2014.

[20] Kokotovic, P.; Khalil, H. K. and O'Reilly, J.; "Singular Perturbation Methods in Control: Analysis and Design," *Academic Press*, 1986.

[21] Qu, Z. and Dorsey, J.; "Robust PID Control of Robots," *International Journal of Robotics and Automation*, Vol. 6, No. 4, pp. 228–235, 1991.

[22] Qu, Z. and Dawson, D. M.; "Robust Tracking Control of Robot Manipulators," *IEEE Press*, 1996.

[23] Ottaviano, E. and Castelli, G.; "A Study on the Effects of Cable Mass and Elasticity in Cable-Based Parallel Manipulators," *ROMANSY 18 Robot Design, Dynamics and Control*, Vol. 524, Chapter 1, pp. 149–156, 2010.

[24] Khosravi, M. A. and Taghirad, H. D.; "Experimental Performance of Robust PID Controller on a Planar Cable Robot," *Proceedings of the First International Conference on Cable-Driven Parallel Robots*, pp. 337–352, 2012.