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Achievable Rate Region for Wireless Multiple Access Channels with UAV Relay and k Independent Transmitters

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ABSTRACT: This paper considers and characterizes a multiple access relay channel (MARC) with k transmitters, one UAV relay, and a terrestrial receiver, where we derive a new achievable rate region for the continuous alphabet wireless with an unmanned aerial vehicle (UAV) relay. The derived rate region is then evaluated numerically and compared with those of multiple access channels (MAC) and MARC with terrestrial relays. The results show that the presence of a UAV relay increases the achievable rate region of MAC systems by exploiting higher line-of-sight (LoS) channel quality and time-varying spatial link gains due to the UAV's position. This improvement in the achievable region is not merely due to the UAV's presence but arises from the different channel geometry and dynamic link conditions introduced by the UAV relay compared with terrestrial relays.

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1- Introduction

As mobile communication systems continue to expand, the development of new methods to enhance network capacity has become essential. MAC and MARC are among the key technologies that can improve coverage, achievable rate regions, and rates. On the other hand, the integration of UAVs into multiple access channels enables the establishment of fast and efficient emergency networks, offering high maneuverability (ability to be placed in any desired location and height) and low deployment costs [1, 2, 3, 4, 5], which fundamentally alters the wireless channel geometry compared to terrestrial settings. Therefore, the presence of UAVs in multiple access channels makes it possible to have a fast and efficient emergency network. Additionally, due to their low energy consumption, high mobility, wide coverage, low delay, and comparative coverage, UAVs are expected to play a significant role in the future of telecommunications [1, 2, 3, 4, 5].

In most wireless communication systems, a key characteristic is the absence of a direct line of sight between transmitters and receivers, leading to random variations in the channel coefficients, a phenomenon known as channel fading. To mitigate fading effects, more sophisticated and sometimes nonlinear signal processing techniques are often required at the transmitter and receiver, increasing system cost and latency [6, 7, 8, 9, 10]. One of the major advantages of UAVs is that they can often establish a direct line of sight between the transmitter and the receiver. As a result, the channel coefficients are often less random and more predictable than typical terrestrial fading environments, which simplifies estimation and exploitation of link quality.

Because the channel coefficients depend on the UAV's spatial position, the achievable rate region also becomes dependent on the UAV's dynamics and geometry. This change in channel coefficients motivates the need to analytically calculate the achievable rate region for multiple access channels in the presence of a UAV relay.

1- 1- Related Works

A number of articles have dealt with calculating the achievable rate region for MAC and MARC. For instance, the inner bound on the capacity region using decoding techniques and the outer bound on the capacity region derived via Fano's inequality for a non-causal memoryless MARC with two terrestrial transmitters and one terrestrial relay are presented in [11]. In [12], the authors derive achievable rate regions for MARCs with relay–source feedback and show that feedback enables cooperation among sources by allowing them to resolve residual uncertainty at the destination. This line of work is further extended in [12] and subsequent studies by incorporating feedback into both discrete memoryless and Gaussian MARC models. However, these works primarily focus on feedback-enabled cooperation and do not consider the impact of relay geometry or air-to-ground propagation characteristics.

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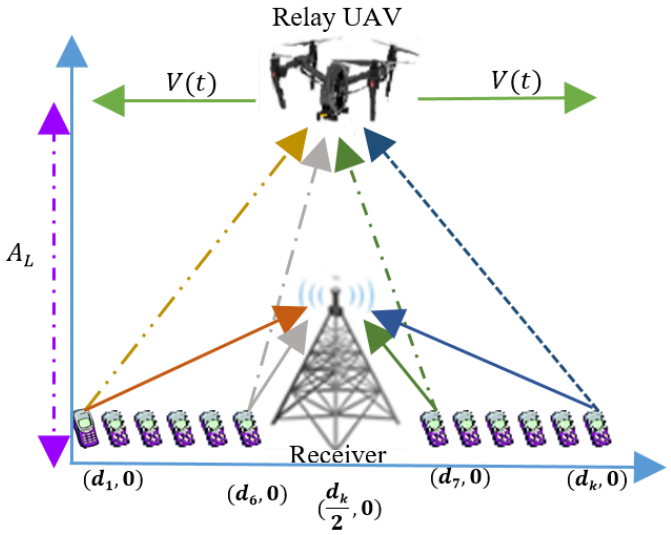

Fig. 1. A System With k Terrestrial Transmitters, Terrestrial Receiver, and UAV Relay.

The inner and outer bounds on the capacity region for Slepian-Wolf-MARC with two terrestrial transmitters, one terrestrial receiver, and one terrestrial relay over a discrete memoryless alphabet are provided in [13], with the results later extended to continuous alphabets. In [14], the authors propose a partial decode-and-forward strategy for Slepian-Wolf-MARC without memory, calculating the achievable rate region for both discrete and continuous alphabets. Unlike [13, 14], the present work does not assume fading-based opportunistic scheduling or power optimization, but instead focuses on the structural impact of a UAV relay with dominant line-of-sight (LoS) channels.

Fundamental capacity bounds for MARCs are investigated in [15], where outer bounds and achievable strategies such as decode-and-forward, compress-and-forward, and amplify-and-forward are presented for discrete memoryless and Gaussian MARCs. While [15] provides a comprehensive theoretical foundation for MARCs, it does not address scenarios involving aerial relays or phase-dependent channel alignment.

The sum-capacity of degraded Gaussian MARCs is studied in [16], where the authors show that decode-and-forward strategies achieve capacity or sum-capacity under specific degradedness and SNR conditions. This work relies on polymatroid structures arising from terrestrial multiaccess links and does not account for spatial mobility or geometric flexibility of relays.

Achievable rate regions for half-degraded MARCs are considered in [17], where superposition coding and joint decoding strategies are employed, and the results are extended to Gaussian channels. Although this work broadens the class of MARCs under study, it still assumes conventional relay models and does not incorporate UAV-assisted relaying or air-to-ground channel effects.

In contrast to the above works, the present paper investigates a UAV-assisted multiple-access relay channel with a fixed aerial relay, independent transmitters, and LoS-dominated air-to-ground links. The achievable rate region is derived by explicitly accounting for the geometric configuration and phase relationships between the UAV relay and ground nodes, which are not addressed in existing MARC literature.

In [18], the outer bound on the capacity region for Slepian-Wolf-MARC is obtained through relay cooperation, considering correlated noise between the terrestrial relay and two terrestrial transmitters. Reference [19] employs a partial decode-and-forward strategy along with conventional Markov inverse encoding/decoding blocks to calculate the achievable rate region for MARC with and without memory, involving three terrestrial transmitters, one terrestrial receiver, and one terrestrial relay. The energy efficiency of MARC with a common message, two terrestrial transmitters, and one terrestrial relay is investigated in [20]. Additionally, [21] derives the upper bound on the capacity region for a multi-transmitter relay channel using a decode-and-forward strategy with two relays and two receivers. All the aforementioned studies focus on calculating the achievable rate region for channels utilizing terrestrial-based antennas.

As mentioned earlier, the integration of UAVs into modern telecommunication systems offers significant advantages. To leverage this potential, a considerable body of research has focused on optimizing UAV operational parameters in multiple access channels. These studies typically pursue objectives such as increasing data rate or system capacity through trajectory and path optimization [5,22,23], 3D positioning and deployment [24, 25, 26, 27], and joint power control and radio resource allocation [26]. Alongside these core optimization themes, attention to the inherent constraints

of UAVs - such as limited energy and processing power - has driven some studies toward designing lightweight solutions, particularly in critical areas like secure and efficient authentication protocols [28, 29, 30]. This category of work aims to minimize computational and communication overhead while ensuring security, thereby enhancing the operational sustainability of UAV-aided networks.

However, to the best of our knowledge and based on our research, the achievable rate region for these systems has not been analytically calculated. Therefore, in the following sections, we will derive the achievable rate region for a Gaussian MARC-UAV (GMARC-UAV) system.

The integration of a UAV as an aerial relay fundamentally redefines the system model compared to terrestrial [11, 13, 14, 18, 19, 21]. While the dominant Line-of-Sight (LoS) component suggests a more deterministic channel, the combined effects of the UAV's mobility, subtle scattering from the environment, and potential hovering instabilities necessitate a slow, flat-fading Rician channel model. This model accurately captures the strong LoS path alongside a weaker random component. Consequently, the channel coefficients are inherently time-variant and spatially dependent, evolving primarily with the UAV's 3D trajectory. This stands in stark contrast to the static channel geometries assumed in terrestrial MARC studies and the Rayleigh fading model typical of non-LoS environments.

The foundational works on MARCs [11, 13, 14, 18, 19, 21] have established rigorous theoretical bounds, but they are confined to a static network paradigm. In these studies, the achievable rate region is a fixed characteristic of a given, unchanging topology. Our work introduces a paradigm shift by leveraging UAV mobility not merely as a feature, but as a core controllable degree of freedom. This transforms the achievable rate region from a static consequence of the network into a dynamic and optimizable function of the UAV's trajectory. Therefore, the principal technical differentiation of our work is the transition from analyzing a fixed capacity region for a given topology to actively shaping and optimizing a spatially/temporally evolving capacity region through strategic trajectory planning a problem domain that remains largely unexplored in the existing MARC literature.

This paradigm shift directly informs our core contribution. We derive the achievable rate region for a Gaussian MARC with a UAV relay, explicitly characterizing it as a function of the relay's spatial coordinates.

1- 2- Research Gaps when Terrestrial Relays are Replaced by UAV Relays

Although the existing literature on multiple-access relay channels (MARCs) provides comprehensive insights into achievable rate regions and capacity bounds with terrestrial relays, several fundamental research gaps arise when the relay is replaced by a UAV-mounted antenna.

a) First, prior works such as [12, 13, 14, 15, 16, 17] implicitly assume fixed terrestrial relay locations with static channel statistics. When the relay is aerial, the relay–destination and source–relay links exhibit strong geometric dependence, where distance, elevation angle, and relative phase alignment directly affect the achievable rates. These geometric and phase-related effects are not captured in conventional MARC models.
b) Second, classical MARC formulations typically rely on rich-scattering or fading-based channel models. In contrast, UAV-assisted communications are dominated by line-of-sight (LoS) propagation, leading to fundamentally different channel behaviors, especially in terms of coherent signal combining and phase-sensitive rate regions. Existing terrestrial MARC analyses do not address this LoS-dominated regime.
c) Third, existing studies focus on cooperation mechanisms enabled by feedback, power allocation, or degradedness assumptions. However, when a UAV relay is employed, the spatial placement of the relay itself becomes a new degree of freedom that influences the structure of the achievable rate region, even when the UAV location is fixed. This spatial dimension is absent in terrestrial relay models.
d) Fourth, prior MARC capacity results often exploit polymatroid structures arising from conventional multiaccess links. The presence of an aerial relay with phase-aligned channels introduces additional coupling between users' signals that alters the boundary of the rate region, a phenomenon not investigated in existing terrestrial MARC literature.
e) Finally, most available results are derived for two-user or simplified relay configurations. Extending these results to a general (k)-user MARC with a UAV relay and independent transmitters remains largely unexplored.

These gaps motivate the present study, which characterizes the achievable rate region of a (k)-user multiple-access channel assisted by a fixed UAV relay under LoS-dominated air-to-ground propagation.

1- 3- Our Contribution

In this paper: We extend the known rate region for discrete and memoryless -MARC to the continuous alphabet wireless MARC with UAV relay the results of which are necessary for analyzing wireless communication performances. With UAV relay, the wireless region is time varying and hence, is exploitable for improving performances.

1- 4- Paper Organization

The remainder of this paper is structured as follows:

b) Section II: System model and notation.
c) Section III: Achievable rate region derivation.
d) Section IV: Simulation results and conclusions.

2- System Model

This paper uses terrestrial and UAV antennas to calculate the achievable GMARC rate region. Since the UAV motion, location, and type of antenna and users, coding and decoding methods, and channel model affect the rate region, this section discusses the details of the network and system model.

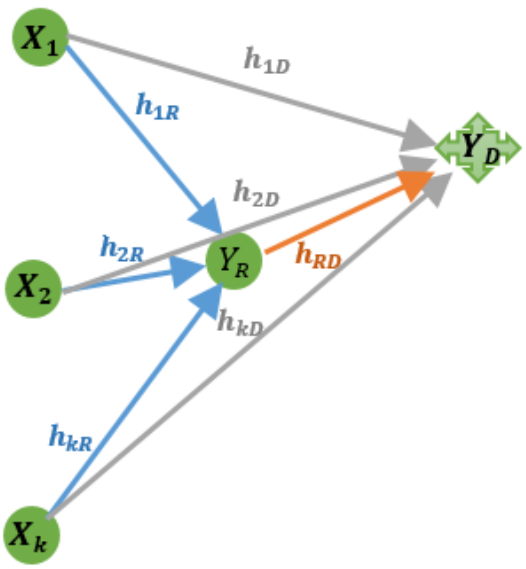

Fig. 2. A System Model With k Terrestrial Transmitters, Terrestrial Receiver, and UAV Relay.

2- 1- Network Model

The multiple-access telecommunications network discussed in this article is depicted Fig.1. As illustrated, the system comprises k terrestrial transmitters and a terrestrial receiver, all positioned along a straight line. If the transmitters are not aligned in a straight line, their relative height with respect to the UAV will vary, leading to changes in the channel coefficients. Consequently, if this assumption is modified, the simulation results must be recalculated based on the updated height values. A UAV relay, flying at a fixed altitude denoted as A_L, is deployed above this line. Clearly defining the UAV's movement strategy is essential, as it plays a critical role in determining the achievable rate region of the channel.

In this work, we adopt the movement model proposed [32]. According to this model, the UAV initiates its flight from a position above the first user and moves along the straight line at its maximum speed, $v_{\max}$, until it reaches the last transmitter. Upon reaching the last transmitter, the UAV reverses direction and returns to the first transmitter along the same trajectory. This cyclic movement, characterized by a period T, is repeated indefinitely. The UAV's position along the straight line at any time t is described by the function $r(t)$.

Furthermore, we assume the channel coefficients with the UAV relay are deterministic, as the utilization of a UAV in this configuration facilitates strong LoS wireless links. This means that the UAV relay have direct vision, and as mentioned earlier, direct vision causes the channel coefficients to be deterministic. This determinism significantly improves the reliability of communication.

2- 2- GMARC-UAV

As shown in Fig.2, we consider a specific MARC-UAV channel with a continuous Gaussian alphabet, considering a degraded channel model. In this article, we assumed k terrestrial transmitters and a UAV relay. In this model, transmitters 1 through k generate independent messages X_1 through X_k. Following the encoding scheme presented [19], each transmitter i partitions its message X_i into two parts: V_i and U_i, which V_i can be decoded at both the relay and the destination and U_i can be decodable only at the destination.

We assume V_i and U_i are independent Gaussian signals, $V_i \sim N(0,1)$ and $U_i \sim N(0,1)$.

Each transmitter $1,2,\ldots,k$ is subject to an individual power constraint $P_1,P_2,\ldots,P_k$ Let $\gamma_1,\gamma_2,\ldots,\gamma_k$, denote the fractions of the source power allocated to $V_1,V_2,\ldots,V_k$ and $\beta_1,\beta_2,\ldots,\beta_k$ represent the fractions allocated to $U_1,U_2,\ldots,U_k$. Consequently, the transmitted signals $X_1,X_2,\ldots,X_k$ can be expressed as follows:

$$\begin{cases} X_i = \sqrt{\gamma_i P_i} V_i + \sqrt{\beta_i P_i} U_i \\ \gamma_i + \beta_i = 1 \\ i = 1,2,\ldots,k \end{cases} \quad (1)$$

$$Y_R = \sum_{i=1}^{k} h_{iR}(r(t)) X_i + Z_R$$

Clearly, $X_1,X_2,\ldots,X_k$ are independent Gaussian signals, each distributed as $X_i \sim N(0,P_i)$. In typical scenarios, terrestrial transmitters and receiver often lack a direct LoS between receivers and transmitters.

As a result, the channels between them can be modeled as experiencing slow, flat fading with time-invariant stochastic coefficients. Additionally, we account for the presence of AWGN in these channels. As previously mentioned, the UAV benefits from a direct LoS, and thus, the channel coefficients are deterministic, time-variant, and modeled under the assumption of an AWGN channel. The received signal at the relay can be expressed as:

$$Y_R = \sum_{i=1}^{k} h_{iR}(r(t)) X_i + Z_R \quad (2)$$

Here, $h_{iR}(r(t))$, for $i = 1,2,\ldots,k$, represents the channel coefficient between the transmitter i and the relay. Eq.2, $Z_R \sim N(0,N_{iR})$ denotes the additive white Gaussian noise at the relay.

The relay first receives the signal from the transmitter i and decodes V_i. It then codes the decoded message and transmits it to the destination. The relay is subject to a power constraint P_R for transmitting the signal. Let $\alpha_1,\alpha_2,\ldots,\alpha_k$ denote the portions of P_R allocated to transmitting the encoded signals $V_1,V_2,\ldots,V_k$ respectively. The UAV relay

transmits the following signal to the terrestrial receiver:

$$\begin{cases} X_R = \sum_{i=1}^{k} \sqrt{\alpha_i P_R} V_i \\ \sum_{i=1}^{k} \alpha_i = 1 \\ X_R \sim N\left(0, P_R\right) \end{cases} \quad (3)$$

Then, the signals sent from transmitters and relay are received by the terrestrial receiver as follows:

$$Y_D = \sum_{i=1}^{k} h_{iD} X_i + Z_R + h_{RD}(r(t)) X_R + Z_D \quad (4)$$

Here, h_{iD} \$represents the channel coefficient between the transmitter i and the terrestrial receiver. These coefficients are generally random due to the absence of a direct LoS between the transmitters and the receiver. Since both the transmitters and the receiver are stationary, these coefficients are time-invariant during message transmission. Additionally, Eq.4, $h_{RD}\left(\mathrm{r}(\mathrm{t})\right)$ denotes the channel coefficient between the UAV relay and the terrestrial receiver, which is considered deterministic and time varying. Assuming an AWGN channel, $Z_D \sim N\left(0, N_D\right)$ represents the additive Gaussian noise at the receiver.

In the next section, we will derive the achievable rate of the GMARC-UAV system based on Eq.1 – Eq.4.

3- Achievable Rate Region

We use the coding and decoding method [19], Decode-and-Forward (DF) strategy, and calculate the achievable rate region for GMARC-UAV with k transmitters, a UAV relay, and a terrestrial receiver. Like reference [19], we perceive the messages independently. When we have UAV in channel, the channel coefficients change over time. In this case to calculate the achievable rate region, we need calculate the time average.

We calculate the new inner bound on the achievable rate region for Degraded discrete memoryless (DM) MARC (one transmitter) [31] to k transmitters and UAV relay. We have achievable rate for each transmitter as follows. For example, R_{r1} is achievable rate between transmitter 1 and relay, and R_{d1} is achievable rate between transmitter 1 and destination.

$$\begin{cases} R_{r1}(t) < I\left(X_1; Y_R \mid X_R, X_2, ..., X_k, V_1, V_2, ..V_k\right) \\ R_{rk}(t) < I\left(X_k; Y_R \mid X_R, X_1, X_2, ..., X_{k-1}, V_1, V_2, ..V_k\right) \\ \sum_{i=1}^{k} R_{ri}(t) < I\left(X_1, X_2, ..., X_k; Y_R \mid X_R, V_1, V_2, ..V_k\right) \end{cases} \quad (5)$$

Also, the achievable rate in the receiver is:

$$\begin{cases} R_{d1}(t) < I\left(X_1, X_R; Y_D \mid X_2, ..., X_k, V_2, V_3, ..V_k\right) \\ R_{dk}(t) < I\left(X_k, X_R; Y_D \mid X_1, X_2, ..., X_{k-1}, V_1, V_2, ..V_{k-1}\right) \\ \sum_{i=1}^{k} R_{di}(t) < I\left(X_1, X_2, ..., X_k, X_R; Y_D\right) \end{cases} \quad (6)$$

where, $I(.\ ;.|\ .)$ represents conditional mutual information.

In the presented system model, where the UAV serves as the receiver, it is crucial to recognize that the UAV's spatial position and corresponding channel coefficients vary over time. This temporal variation leads to time-dependent modifications in the achievable rate region. Consequently, all equation parameters become functions of time, transforming the achievable rate region expressions as follows:

Compute the time average of the instantaneous rate expressions given Eq.5, Eq.6.

Select the minimum values following multi-user information theory principles

Then the achievable rate region in the channel is $min\left[(5),(6)\right]$ and time average in UAV relay, as below:

$$\begin{cases} R_1 < \min\left(R_{r1}, R_{d1}\right) \\ R_k < \min\left(R_{rk}, R_{dk}\right) \\ \sum_{i=1}^{k} R_i < \min\left(\sum_{i=1}^{k} R_{ri}, \sum_{i=1}^{k} R_{di}\right) \end{cases} \quad (7)$$

Where, R_i, R_{rk}, and R_{dk} respectively, are the rate of channel, relay and transmitter i.

Therefore, the distance between UAV and transmitters is a function of time and depends on the scenario of the UAV movement. We perceive the motion scenarios [32] and calculate the distance, channel coefficients, and achievable rate region in the GMARC-UAV with the system model in the previous section.

According to Eq.1, Eq.2, Eq.3, Eq.5 and time average in UAV relay, the achievable rate region in the relay, is as follows:

$$\begin{aligned} R_{r1}(t) &< I\left(X_1; Y_R \mid X_R, X_2, ..., X_k, V_1, V_2, ..V_k\right) = \\ &H\left(Y_R \mid X_R, X_2, ..., X_k, V_1, V_2, ..V_k\right) \\ &- H\left(Y_R \mid X_1, X_R, X_2, ..., X_k, V_1, V_2, ..V_k\right) \\ &= H\left(\sum_{i=1}^{k} h_{iR}(r(t)) X_i + Z_R \mid X_R, X_2, ..., X_k, V_1, V_2, ..V_k\right) \\ &- H\left(\sum_{i=1}^{k} h_{iR}(r(t)) X_i + Z_R \mid X_1, X_R, X_2, ..., X_k, V_1, V_2, ..V_k\right) \\ &= H\left(h_{1R}(r(t)) X_1 + Z_R \mid X_R, X_2, ..., X_k, V_1, V_2, ..V_k\right) - H\left(Z_R\right) = \end{aligned} \quad (8)$$

$$
\begin{aligned}
&H\left(h_{1R}(r(t))\sqrt{\beta_1 P_1}U_1+Z_R\,|\,X_R,X_2,...,X_k,V_1,V_2,..V_k\right)-H\left(Z_R\right)\\
&=\frac{1}{T}\int_0^T\frac{1}{2}\log\left(2\pi e\left(N_R+\left(h_{1R}(r(t))\sqrt{\beta_1 P_1}U_1\right)^2\right)\right)dt\\
&-\frac{1}{2}\log\left(2\pi eN_R\right)=\frac{1}{T}\int_0^T\frac{1}{2}\log\left(1+\frac{|h_{1R}(r(t))|^2\,\beta_1 P_1}{N_R}\right)dt
\end{aligned}
\tag{8}
$$

Also, such as (8) we have:

$$
\begin{cases}
R_{r2}<\dfrac{1}{T}\displaystyle\int_0^T\frac{1}{2}\log\left(1+\frac{|h_{2R}(r(t))|^2\,\beta_2 P_2}{N_R}\right)dt\\
.\\
.\\
R_{rk}<\dfrac{1}{T}\displaystyle\int_0^T\frac{1}{2}\log\left(1+\frac{|h_{1R}(r(t))|^2\,\beta_1 P_1}{N_R}\right)dt
\end{cases}
\tag{9}
$$

We will calculate $\sum_{i=1}^{k}R_{ri}$ such as R_{r1}:

$$
\begin{aligned}
&\sum_{i=1}^{k}R_{ri}(t)<I\left(X_1,X_2,...,X_k;Y_R\,|\,X_R,V_1,V_2,..V_k\right)=\\
&H\left(Y_R\,|\,X_R,V_1,V_2,..V_k\right)-H\left(Y_R\,|\,X_R,X_1,X_2,...,X_k,V_1,V_2,..V_k\right)\\
&=H\left(\sum_{i=1}^{k}h_{iR}(r(t))X_i+Z_R\,|\,X_R,V_1,V_2,..V_k\right)\\
&-H\left(\sum_{i=1}^{k}h_{iR}(r(t))X_i+Z_R\,|\,X_R,X_1,X_2,...,X_k,V_1,V_2,..V_k\right)\\
&=H\left(\sum_{i=1}^{k}h_{iR}(r(t))\left(\sqrt{\gamma_i P_i}V_i+\sqrt{\beta_i P_i}U_i\right)+Z_R\,|\,X_R,V_1,V_2,..V_k\right)\\
&-H\left(Z_R\right)=H\left(\sum_{i=1}^{k}h_{iR}(r(t))\sqrt{\beta_i P_i}U_i+Z_R\right)-H\left(Z_R\right)\\
&=\frac{1}{T}\int_0^T\frac{1}{2}\log\left(2\pi e\left(\sum_{i=1}^{k}\left(h_{iR}(r(t))\sqrt{\beta_i P_i}\right)^2+N_R\right)\right)dt\\
&-\frac{1}{2}\log\left(2\pi eN_R\right)=\frac{1}{T}\int_0^T\frac{1}{2}\log\left(1+\frac{|h_{iR}(r(t))|^2\,\beta_i P_i}{N_R}\right)dt
\end{aligned}
\tag{10}
$$

According to Eq.1, Eq.3, Eq.4, Eq.6, the achievable rate region in the receiver, is as follows:

$$
\begin{aligned}
&R_{d1}(t)<I\left(X_1,X_R;Y_D\,|\,X_2,...,X_k,V_2,V_3,..V_k\right)=\\
&H\left(Y_D\,|\,X_2,...,X_k,V_2,V_3,..V_k\right)\\
&-H\left(Y_D\,|\,X_R,X_1,X_2,...,X_k,V_2,V_3,..V_k\right)\\
&=H\left(\begin{matrix}\sum_{i=1}^{k}h_{iD}X_i+Z_R\\ +h_{RD}(r(t))X_R+Z_D\end{matrix}\,|\,X_2,...,X_k,V_2,V_3,..V_k\right)-H\left(Z_D+Z_R\right)\\
&=H\left(\begin{matrix}h_{1D}X_1+Z_R\\ +h_{RD}(r(t))X_R+Z_D\end{matrix}\,|\,V_2,V_3,..V_k\right)-H\left(Z_D+Z_R\right)\\
&=H\left(\begin{matrix}h_{1D}\left(\sqrt{\gamma_1 P_1}V_1+\sqrt{\beta_1 P_1}U_1\right)+Z_R\\ +h_{RD}(r(t))\sum_{i=1}^{k}\sqrt{\alpha_i P_R}V_i+Z_D\end{matrix}\,|\,V_2,V_3,..V_k\right)-H\left(Z_D+Z_R\right)
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
&=H\left(\begin{matrix}h_{1D}\sqrt{\beta_1 P_1}U_1+Z_R\\ +\left(h_{1D}\sqrt{\gamma_1 P_1}+h_{RD}(r(t))\sqrt{\alpha_1 P_R}\right)V_1\\ +Z_D\,|\,V_2,V_3,..V_k\end{matrix}\right)-H\left(Z_D+Z_R\right)\\
&=\frac{1}{T}\int_0^T\frac{1}{2}\log\left(2\pi e\left(\begin{matrix}N_D+|h_{1D}|^2\,\beta_1 P_1\\ +|h_{RD}(r(t))|^2\,\alpha_1 P_R+|h_{1D}|^2\,\gamma_1 P_1\\ +2h_{1D}h_{RD}(r(t))\sqrt{P_1 P_R\alpha_1\gamma_1}\cos\Delta\theta_1\end{matrix}\right)\right)dt\\
&-\frac{1}{2}\log\left(2\pi e\left(N_R+N_D\right)\right)=\\
&\frac{1}{T}\int_0^T\frac{1}{2}\log\left(2\pi e\left(\begin{matrix}|h_{1D}|^2\,\beta_1 P+|h_{RD}(r(t))|^2\,\alpha_1 P_R\\ +|h_{1D}|^2\,\gamma_1 P_1\\ 1+\dfrac{+2h_{1D}h_{RD}(r(t))\sqrt{P_1 P_R\alpha_1\gamma_1}\cos\Delta\theta_1}{N_R+N_D}\end{matrix}\right)\right)dt
\end{aligned}
$$

We know $\Delta\theta_1$ is the angle between h_{RD} and h_{1D}. Also, we have R_{dk} such as R_{d1}:

$$
R_{dk}<\frac{1}{T}\int_0^T\frac{1}{2}\log\left(2\pi e\left(\begin{matrix}|h_{kD}|^2\,\beta_k P_k+|h_{RD}(r(t))|^2\,\alpha_k P_R\\ +|h_{kD}|^2\,\gamma_k P_k\\ 1+\dfrac{+2h_{kD}h_{RD}(r(t))\sqrt{P_k P_R\alpha_k\gamma_k}\cos\Delta\theta_k}{N_R+N_D}\end{matrix}\right)\right)dt
\tag{12}
$$

We will calculate $\sum_{i=1}^{k}R_{di}$ such as R_{d1}:

$$
\begin{aligned}
&\sum_{i=1}^{k}R_{di}(t)<I\left(X_1,X_2,...,X_k,X_R;Y_D\right)=\\
&H\left(Y_D\right)-H\left(Y_D\,|\,X_1,X_2,...,X_k,X_R\right)=\\
&H\left(\sum_{i=1}^{k}h_{iD}X_i+Z_R+h_{RD}(r(t))X_R+Z_D\right)-H\left(Z_R+Z_D\right)\\
&=H\left(\begin{matrix}\sum_{i=1}^{k}h_{iD}\left(\sqrt{\gamma_i P_i}V_i+\sqrt{\beta_i P_i}U_i\right)+Z_R\\ +h_{RD}(r(t))\sum_{i=1}^{k}\sqrt{\alpha_i P_R}V_i+Z_D\end{matrix}\right)-H\left(Z_R+Z_D\right)\\
&=H\left(\begin{matrix}\sum_{i=1}^{k}h_{iD}\sqrt{\beta_i P_i}U_i+Z_R\\ +\left(\sum_{i=1}^{k}\left(\begin{matrix}h_{iD}\sqrt{\gamma_i P_i}\\ +h_{RD}(r(t))\sqrt{\alpha_i P_R}\end{matrix}\right)V_i\right)+Z_D\end{matrix}\right)-H\left(Z_R+Z_D\right)\\
&=\frac{1}{T}\int_0^T\frac{1}{2}\log\left(2\pi e\left(\begin{matrix}N_D+N_R\\ +\sum_{i=1}^{k}\left(\begin{matrix}P_i\left(\gamma_i\beta_i\right)|h_{iD}|^2\\ +|h_{RD}(r(t))|^2\,P_R\alpha_i\end{matrix}\right)\\ +2h_{iD}h_{RD}(r(t))\sqrt{P_i P_R\alpha_i\gamma_i}\cos\Delta\theta_i\end{matrix}\right)\right)dt
\end{aligned}
\tag{13}
$$

$$-\frac{1}{2}\log\left(2\pi e\left(N_R+N_D\right)\right)$$
$$=\frac{1}{T}\int_0^T\frac{1}{2}\log\left(2\pi e\left(1+\frac{\sum_{i=1}^{k}\left(P_i\left(\gamma_i\beta_i\right)|h_{iD}|^2\right)+|h_{RD}(r(t))|^2 P_R\alpha_i}{N_R+N_D}+\frac{+2h_{iD}h_{RD}(r(t))\sqrt{P_iP_R\alpha_i\gamma_i}\cos\Delta\theta_i}{N_R+N_D}\right)\right)dt \quad (13)$$

Finally, the achievable rate region in the GMARC-UAV is as follows:

$$\begin{cases} R_k<\min\left(\frac{1}{T}\int_0^T\frac{1}{2}\log\left(1+\frac{|h_{kR}(r(t))|^2\beta_kP_k}{N_R}\right)dt,\ \frac{1}{T}\int_0^T\frac{1}{2}\log\left(2\pi e\left(1+\frac{|h_{kD}|^2P_k+|h_{RD}(r(t))|^2\alpha_kP_R}{N_R+N_D}+\frac{+2h_{kD}h_{RD}(r(t))\sqrt{P_kP_R\alpha_k\gamma_k}\cos\Delta\theta_k}{N_R+N_D}\right)\right)dt\right) \\ \sum_{i=1}^{k}R_i<\min\left(\frac{1}{T}\int_0^T\frac{1}{2}\log\left(1+\frac{\sum_{i=1}^{k}|h_{iR}(r(t))|^2\beta_iP_i}{N_R}\right)dt,\ \frac{1}{T}\int_0^T\frac{1}{2}\log\left(2\pi e\left(1+\frac{\sum_{i=1}^{k}|h_{iD}|^2P_i+|h_{RD}(r(t))|^2\alpha_iP_R}{N_R+N_D}+\frac{+2h_{iD}h_{RD}(r(t))\sqrt{P_iP_R\alpha_i\gamma_i}\cos\Delta\theta_i}{N_R+N_D}\right)\right)dt\right) \end{cases} \quad (14)$$

4- Simulations and Results

In this section:

a) We first simulated the achievable rate region for a MARC-UAV system with k (14) terrestrial transmitters and independent Gaussian signals, and a UAV receiver with white Gaussian noise.

b) In the next step, we used the channel coefficients [32], for $h_{iR}\left(\mathrm{r(t)}\right)$ and $h_{RD}\left(\mathrm{r(t)}\right)$ (15).

c) We removed the relay and calculate achievable rate region GMAC-UAV (in this case, the transmitters and receiver are terrestrial).

d)Then, we assumed terrestrial relay and used coefficients channel [19], for h_{iD}.

e) Then achievable rate region of GMARC-UAV, GMAC-UAV, and terrestrial GMRAC are compared.

Reference [32] we have:

$$h_i(t)=\frac{\lambda}{4\pi d_i(t)} \quad (15)$$

Where:

$$\begin{cases} \lambda=\frac{C}{f} \\ d_i(t)=\sqrt{\|d(r(t))-W_i\|^2+A_L{}^2} \\ W_i=\begin{cases}0\\ d\end{cases} \end{cases} \quad (16)$$

Also, we deem:

$A_L=250m$, $V_{max}=20\,m/s$, $P_1=P_2=P_R=30dBm$, $N_R=N_D=10^{-7}$, $T=100s$, $\gamma_i=\beta_i=0.5$, $h_{ir}=1.6$, $\alpha_i=\frac{1}{k}$, $f=1.2GH$.

The notation of this coefficients channel is shown in Table 1.

We put Eq.15 and Eq.16 in Eq.14, and performed the simulation in the MATLAB (GMARC-UAV). If we delete parameters of relay, the Eq.14 is changed to:

$$R_1<\frac{1}{2}\log\left(2\pi e\left(1+\frac{P_1|h_{1D}|^2}{N_D}\right)\right) \quad (17)$$

Table 1. Notation of coefficients channel UAV.

Notation	Corresponding Meaning
λ	Wavelength
C	Speed of light
$d(r(t))$	Horizontal location of terrestrial transmitter k
A_L	UAV's flight altitude
$d_i(t)$	Distance between the UAV and transmitter k at time t
$h_i(r(t))$	Channel power gain between the UAV and transmitter k at time t
h_{ir}	Channel power gain between the terrestrial antenna and transmitter k

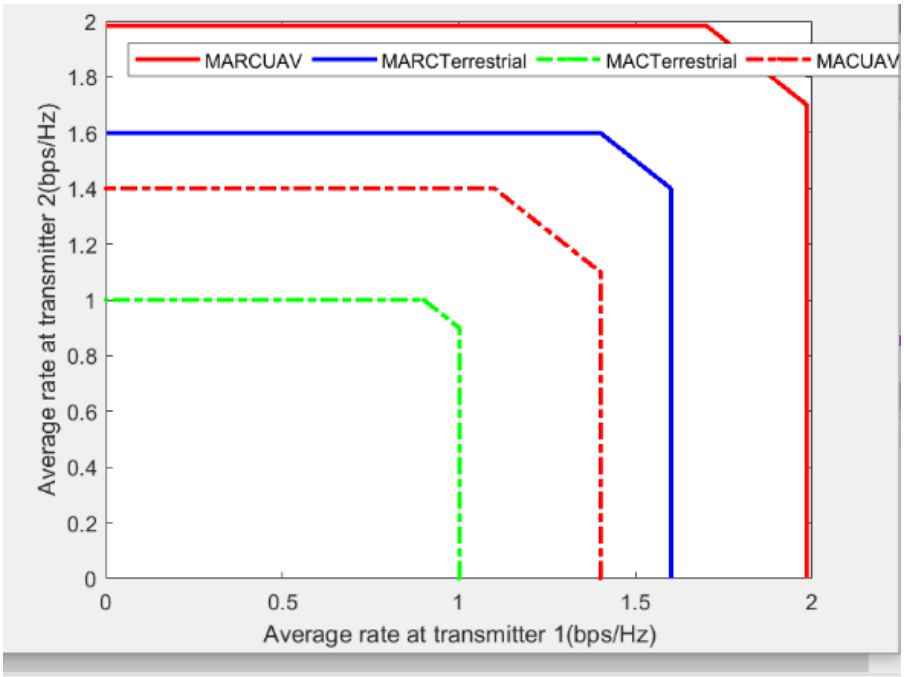

Fig. 3. Comparison of the achievable rate regions.

$$R_k < \frac{1}{2}\log\left(2\pi e\left(1+\frac{P_k\,|\,h_{kD}\,|^2}{N_D}\right)\right) \tag{18}$$

$$\sum_{i=1}^{k} R_i < \frac{1}{2}\log\left(2\pi e\left(1+\frac{\sum_{i=1}^{k} P_i\,|\,h_{iD}\,|^2}{N_D}\right)\right) \tag{19}$$

We used Eq.17- Eq.19 to simulate achievable rate region MARC in MATLAB. If we assume a terrestrial relay, we have:

$$\begin{cases} R_k < \min\left(\begin{array}{l} \frac{1}{T}\int_0^T \frac{1}{2}\log\left(1+\frac{|h_{kR}(r(t))|^2\,\beta_k P_k}{N_R}\right)dt, \\ \frac{1}{T}\int_0^T \frac{1}{2}\log\left(2\pi e\left(1+\frac{|h_{kD}|^2 P_k+|h_{RD}(r(t))|^2 P_R}{N_R+N_D}+\frac{2h_{kD}h_{RD}(r(t))\sqrt{P_k P_R \alpha_k \gamma_k}\cos\Delta\theta_k}{N_R+N_D}\right)\right)dt \end{array}\right) \\ \sum_{i=1}^{k} R_i < \min\left(\begin{array}{l} \frac{1}{T}\int_0^T \frac{1}{2}\log\left(1+\frac{\sum_{i=1}^{k}|h_{iR}(r(t))|^2\,\beta_i P_i}{N_R}\right)dt, \\ \frac{1}{T}\int_0^T \frac{1}{2}\log\left(2\pi e\left(1+\frac{\sum_{i=1}^{k}|h_{iD}|^2 P_i+|h_{RD}(r(t))|^2 P_R}{N_R+N_D}+\frac{2h_{iD}h_{RD}(r(t))\sqrt{P_i P_R \alpha_i \gamma_i}\cos\Delta\theta_i}{N_R+N_D}\right)\right)dt \end{array}\right) \end{cases} \tag{20}$$

We utilized Eq.20 to simulate the achievable rate region of a MARC with a terrestrial relay in MATLAB. The results, depicted in Fig.3, demonstrate that the inclusion of the relay significantly enhances the achievable rate region. Furthermore, transitioning the relay from a terrestrial antenna to an UAV does not diminish the achievable rate region. However, it is noteworthy that employing UAV relays introduces LoS conditions, where the channel coefficients become deterministic and time-variant. This characteristic facilitates the estimation of channel coefficients and simplifies the practical realization of the achievable rate region.

Due to the direct LoS between the relay and the receiver and transmitters, the channel coefficient is non-random and, in many cases, exceeds the coefficient of the terrestrial antennas which are random and the achievable rate region increases. The result is shown in Fig.3.

If direct LoS between the relay and the receiver and transmitters is Changed to non LoS(NLos), the achievable rate region is decrease (like terrestrial antenna). Fig.4, demonstrate that the inclusion of the relay significantly enhances the achievable rate region with decrementation channel coefficients in NLoS.

5- Conclusion

In this paper, we first extended the known rate region for DMC-MARC to the continuous alphabet version of wireless MARC with a UAV relay. Then, we calculated the obtained achievable rate region for the MARC with a UAV relay, involving k transmitters and a terrestrial receiver, by simulating the channel conditions. The simulation results

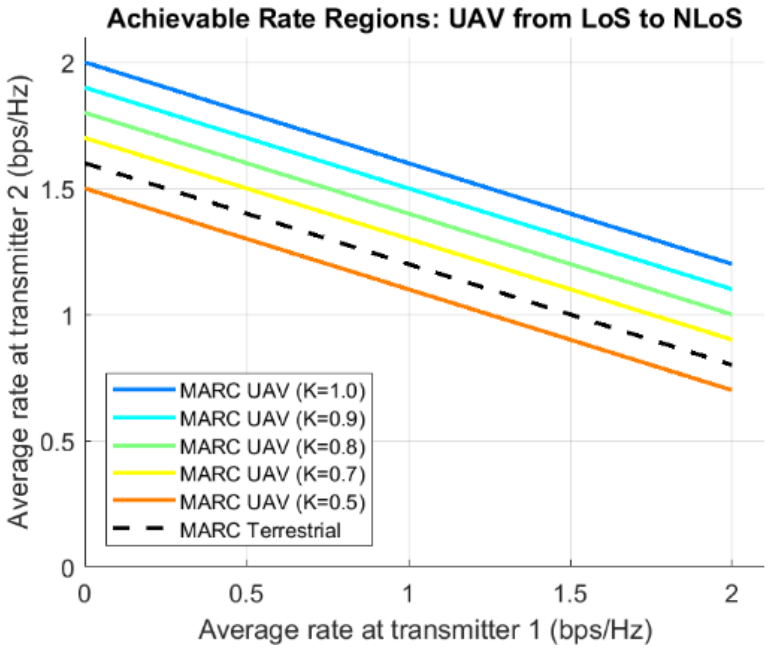

Fig. 4. Comparison of the achievable rate regions between LoS and NLoS.

indicate that the introduction of a relay significantly expands the achievable rate region of the channel.

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