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The effect of a charge trap in the vicinity of the quantum-dot on the charge stability diagram of a single electron transistor

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ABSTRACT: In this article, we explored the effect of a single charge trap on the charge stability diagram of the quantum-dot-based single-electron transistor. We investigated anomalies in the coulomb characteristic diagram, system energy, occupation probabilities, and quantum dot conductivity arising from the electrostatic interaction between the main dot and this charge trap. The anomalies were studied for various locations of the trap, mainly when the trap is located at the source or drain sides of the device. A significant enhancement in quantum dot conductivity was observed by bringing the main quantum dot closer to the source and drain with increased coupling capacitors. The trap, capacitively linked to the quantum dot, has two charge states, either empty or occupied by a single electron. Considering various quantum states, we solved the master equation using Fermi's golden rule to obtain tunneling rates and the matrix of tunneling coefficients. Inverting the coefficient matrix allowed us to determine the probability of each quantum state. The results of this analysis have been validated by comparing simulation results with experimental data. In conclusion, our study provides a valuable tool for detecting charge presence in a trap near a quantum dot, potentially applicable for the readout of quantum gates.

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1- Introduction

Electron transport mechanisms in single-electron systems, including quantum dots, dopant atoms, and singleelectron transistors, represent a critical resource in the study of condensed matter physics. Additionally, established nanofabrication processes facilitate the systematic production of intricate assemblies of these single-electron entities. Such advancements are currently under investigation for their applicability in the construction of quantum computing architectures, highlighting their significance in the ongoing evolution of quantum technologies [1, 2]. Transport through single-electron systems involves phenomena such as Coulomb blockade, where the transport is suppressed unless a certain energy threshold is overcome. Quantum tunneling is crucial in allowing electrons to traverse potential barriers, enabling coherent transport at the nanoscale.

Usually, quantum point contacts (QPC) or single-electron transistors (SETs) are used to read single-electron charges or spins [3] in Qubits [4, 5]. These devices are sensitive to the electrostatic environment at certain gate-voltage biases, and the single-electron charging in the dot significantly changes its conductance, enabling single-electron charge sensing [6]. QPCs are narrow channels through which electrons can flow, and they are designed to exhibit quantized conductance due to the quantization of electron energy levels in one or more

dimensions. The presence or absence of individual electrons passing through the QPC can be detected by carefully engineering the QPC and its surrounding environment. When the QPC captures an electron, it modifies the conductance of the QPC, leading to measurable changes in the electrical current. These changes can be detected and used as a signal to read out the charge state of the device [7, 8]. Using a QPC as a charge detector, the distribution function of current fluctuations in the QD can be directly measured by counting electrons. SETs have been used as very sensitive electrometers for the charge on a second quantum dot [9, 10]. Single-electron transistors (SETs) based on metallic tunnel junctions and gate-defined sensor quantum dots (SQD), conceptually equivalent to SETs, have also been widely used as proximal sensors and provide similar sensitivity and bandwidth [11-14]. In [15], mutual charge sensing between electron and hole quantum dots using a single electron transistor (SET) and a single-hole transistor (SHT) is reported. Both quantum dots sense charge displacement in the other quantum dots simultaneously. Moreover, [13] reports a real-time observation of an individual electron tunneling within a quantum dot, achieved through an integrated radiofrequency single-electron transistor (SET). Electron counting is employed to assess the quantum dot's tunneling frequency and the likelihood of its charge states being occupied.

Traps are one of the main factors that affect the electronic properties of QD-based devices [16, 17]. A system consisting *Corresponding author's email: shalchian@aut.ac.ir

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Fig. 1. Electrical model of the quantum dot in the vicinity of the trap $[20]$ (a) The trap has a capacitive coupling to The trap has a capacitive coupling to the drain (c) The trap has coupled to both source and drain **the source (b) The trap has a capacitive coupling to the drain (c) The trap has coupled to both source and drain**

of a trap connected to a quantum dot is used for various purposes, such as qubit initialization, information storage, manipulation, electron-hole recombination, and limitation due to trapping states [18]. The trap may be intentionally placed in the system to limit and control the charge state of the quantum dot, which allows precise manipulation and measurement. It might also be located due to unavoidable factors such as lattice defects and deep impurities inside the structure [19] and near the main quantum dot. In both cases, there is a need to evaluate and analyze the effect of the trap and its charge state (full or empty) on the charge stability diagram of the quantum dot. By coupling the trap to the quantum dot, it is possible to control the energy levels and interactions of the quantum dot system, which promises applications in quantum information processing and quantum computing.

 This work considers the effect of a charge trap, such as an isolated impurity, which is located in the vicinity of the main dot and capacitively coupled to it. We have taken into account the electrostatic interaction between the trap and the quantum dot. We analyzed the effect of trap position on the characteristic of the quantum dot.

There are usually two approaches to address the problem of interacting dots. One is based on classical coulomb blockade theory and capacitive coupling of the dota, which produce the principal features according to experimental results. This approach ignores quantum effects and spin interaction that slightly distort the charge stability diagram. Another approach is the quantum treatment based on the generalized Hubbard model. A generic numerical approach for solving the Master equation based on Hubbard Hamiltonian has been recently demonstrated for a double quantum dot system [3,22]. It takes into account spin-exchange, pair hopping, occupation-modulated hopping, and Zeeman splitting effects in the presence of an external magnetic field as well as the coulomb interaction. As demonstrated in [3], those effects slightly modify the shape and fade the boundaries of various regions in the charge stability diagram or create additional states within the regions such as singlet-triplet states which are beyond the scope of this work. However, the overall picture is the same as the classical approach.

In this work, we have used classical coulomb blockade theory and single-electron tunneling effect to calculate the tunneling rate from/to individual quantum states in the system using the master equation method and Fermi's golden rule. Finally, we have simulated anomalies in the characteristics of the quantum dot, which represents the charge state of the trap.

2- Device structure and parameters

Fig. 1 schematically shows the electrical model of the structure used in our simulation [20]. The main dot is between the source and drain leads, with two tunnel junctions. The gate electrode is coupled only capacitively (with zero tunneling probability) to the main dot (C_{mg}) and the trap (C_{tg}) . As shown in Fig. 1. c, GL, and GR specify the tunneling rates between the main dot and the left and right leads.

The trap has been tested in three different positions, which are, respectively, the positions where the trap is capacitively coupled to the source (C_{ts}) and the quantum dot (C_c) (Fig. 1. a), the trap is capacitively coupled to the drain (C_{td}) and the quantum dot (C_c) (Fig. 1. b), and the trap is capacitively

Value (aF)
10
11
11
0.007
0.3
0.3
0.25

Table 1. Coupling capacitances of the system used in the simulation Table 1. Coupling capacitances of the system used in the simulation

coupled to both drain and source in addition to the quantum dot (Fig. 1. c). Table 1 lists the values of coupling capacitors specified in Fig. 1, which are obtained for an experimental structure [12]. The gate electrode controls the number of electrons in the quantum dot and the trap. The source and drain electrodes bias the device and establish a current through the source and drain tunnel junctions.

3- Electrostatic energy of the system

Fig. 2.a, obtained from [12], shows the system's electrostatic energy (W) in different charge states (E_0 for zero electrons in the trap and E_1 for one electron in the trap) as a function of gate voltage (V_g) . To calculate the electrostatic energy, we assume that the number of electrons inside the quantum dot (n_m) can vary from 0 to 9, and the number of electrons inside the trap (n_i) can be 0 or 1. Therefore, there are ten different charge states as $(0,0)$, $(1,0)$, $(2,0)$, ..., $(9,0)$ in which the trap is empty and ten other charge states as $(0,1)$, $(1,1), (2,1), \ldots, (9,1)$ in which the trap is occupied by one electron and in general the system can have twenty different charge states.

The system's electrostatic energy W is calculated as a function of the charge in the quantum dot (Q_m) and the charge in the trap (Q_t) :

$$
W(Q_m, Q_t) = \frac{(Q_m + \beta_t Q_t + X)^2}{2C} + \frac{(Q_t + X_t)^2}{2(C_t + C_c)} \tag{1}
$$

(8). q_0 is the background charge in the dot.
 $n_1 = 0$,
 (n_{res}) $\mathcal{L}(\mathbf{C}_t + \mathbf{C}_c)$
The parameters of the above relation are defined in (2) to The parameters of the above relation are define

$$
Q_m(n_m) = -n_m q + q_0
$$
 (2)
$$
\log_{10} \frac{\log n}{n}
$$

CCC C i is id ig (4) $Q_i(n_i) = -n_i q$ (3) ^{tt} (3)

$$
C_i = C_{is} + C_{id} + C_{ig}
$$
\n⁽⁴⁾

to source, drain, and gate, where $(i = m$ (for main dot) or t (for trap)). *t* (for trap)). *t c* C_i is the total coupling capacitances, including coupling $\frac{1}{2}$ **c** $\frac{1}{2}$ **i** $\frac{1}{2}$

$$
\beta_t = \frac{C_c}{C_t + C_c} \tag{5}
$$

$$
C = C_m + \beta_t C_t \tag{6}
$$

$$
X_i = C_{is} V_s + C_{id} V_d + C_{ig} V_g
$$
 (7)

$$
X = X_m + \beta_t X_t \tag{8}
$$

 $\frac{H(t)}{+C_c}$ (1) electrons in the dot increases due to the gate voltage increase.

As shown in Fig. 2.b, when the system's energy is equal for () *Q n nq tt t* (3) both consecutive states, an electron is added to the quantum
dot to decrease the system's energy. Blue parabola (E_0)
efined in (2) to shows the minimum energy (ground state) of the system for Fig. 2.a shows the electrostatic energy of the system. Each rabola relates to a different value of $n_{\rm m}$. The number of of the security states, an electron is added to
other to decrease the system's energy Blue n As shown in Fig. 2.b, when the system's energy is equal for
both consecutive states, an electron is added to the quantum
dot to decrease the system's energy Blue parabola (E) $\frac{1}{2}$ ($\frac{1}{2}$ r) of the system for $n = 1$. For negative renergy and therefore, is the favorite state of er energy and, therefore, is the fav m_f =0, while black parabola (E_1) shows the minimum energy =0, while black parabola (E_1) shows the minimum energy
ground state) of the system for n_t =1. For negative V_g , E_0 has
wer energy and, therefore, is the favorite state of the system.
y increasing the gate voltage, the E_0 state increases while the energy level for the E_1 state
decreases therefore E has energy near forwards then E_1 and Fig. 2.a shows the electrostatic energy of the system. Each parabola relates to a different value of n_m . The number of electrons in the dot increases due to the gate voltage increase. ecreases; therefore, E_1 becomes more favorable
the probability of trap charging increases. Fig. probability of trap charging inc the E_0 state increases while the energy level for the E_1 state
decreases; therefore, E_1 becomes more favorable than E_0 , and
the probability of tran charging increases. Fig. 3 shows the $n_e = 0$, while black parabola (E_1) shows the minimum energy
(ground state) of the system for $n_e = 1$. For negative V_g , E_0 has
lower energy and therefore is the favorite state of the system (ground state) of the system for $n_i=1$. For negative V_g , E_0 has lower energy and, therefore, is the favorite state of the system. decreases; therefore, E_1 becomes more favorable than E_0 , and
the probability of trap charging increases. Fig. 3 shows the
total system aparexy obtained from the simulation solibrated electrons in the dot increases due to the gate voltage increase. *B B* By increasing the gate voltage, The system energy level in total system energy obtained from the simulation, calibrated shows the minimum energy (ground state) of the system for

Fig. 2. Electrostatic energy of the system in different charge states at zero bias as a function of Vg (a) results **from [20] (b) result of this work simulation in MATLAB**

Fig. 3. The ground state energies of the main dot for the empty and occupied trap **Fig. 3. The ground state energies of the main dot for the empty and occupied trap**

with experimental data [20], by setting q_0 (background charge) as a fitting parameter.

4- Trap occupation and charge stability diagram

A common technique for simulating transport is the master equation approach [21, 22]. The aim is to determine the probability that a system occupies a given charge state in a steady state [23]. The system can have twenty different charge states, and the electrostatic energy of each state is described by (1). To calculate the tunneling rates, we calculate all possible transitions for each of these states to neighboring states. Fig. 4 shows all possible transitions among the system's charge states. The paths that are red and blue represent the transitions between the quantum dot and the source or drain leads, respectively, when the trap is empty and occupied, the paths in green represent the transitions between the leads and trap, and the paths in Black color represents transitions between quantum dot and trap.

The Fermi–Dirac distribution describes the occupation of the electron state. In this work, we have used the Fermi distribution function (9) to calculate the tunneling rates between the quantum dot and the trap and between the trap and the leads. We have also used Fermi-Dirac auto-convolution (10) to calculate the tunneling rates between the quantum dot and the leads due to the continuous nature of energy states on both sides of the transition [24].

X CV CV CV i is s id d ig g (7) Fig. 4. All possible transitions among twenty different charge states of the system

$$
f(x) = \frac{1}{e^x + 1}
$$
 (9) $\frac{1}{f}$

$$
f^*(x) = (f^* f)(x) = \frac{x}{e^x - 1}
$$
 system to describe

For example, we assume the system is init For example, we assume the system is initially in state

(2,0). By adding an electron from the right or left leads For example, we assume the system is initially in state $G_{L,R(2,1),(2,0)} =$
N By adding an electron from the right or left leads (source or drain) to the quantum dot, the system moves to
the (3,0) state, and conversely, by removing an electron from
the quantum dot to the left or right leads, the system returns the quantum dot to the left or right leads, the system returns
to state (2,0) in which the tunneling rates are according to $G_{\text{max}} =$ the quantum dot to the fert of right reads, the system returns
to state (2,0), in which the tunneling rates are according to
relations (11) and (12) representingly **LR** \mathcal{L} , \mathcal{L} (\angle ,0). By adding an electron from the right of left leads
(source or drain) to the quantum dot, the system moves to
the $(3,0)$ state, and conversely, by removing an electron from to state (2,0), in which the tunneling rates are according to $G_{L,R(2)}$
relations (11) and (12) respectively. *XX X m tt* (8) () $\frac{1}{\sqrt{2}}$ ($\left(12\right)$ where $\left(12\right)$ respectively. *C C* (5)

$$
G_{L,R(3,0),(2,0)} = T_1 f^* (\frac{W_{(3,0)} - W_{(2,0)} + \mu_{L,R}}{K_B T})
$$
 (11)

to the (1,1) state. Both of these tur
\n
$$
G_{L,R(2,0),(3,0)} = T_1 f^* \left(\frac{W_{(2,0)} - W_{(3,0)} - \mu_{L,R}}{K_B T} \right)
$$
\nto the (1,1) state. Both of these tur
\nin reverse, that is, from the state (1)
\nelectron from the left or right elec
\nand also from the state (1,1) to (2,0)

LR LR , , *qV* (13) In the above relations, the indices L and R represent the shown in left and right electrodes; K_B is Boltzmann's constant, T_1 is a constant coefficient, which is 1000 times larger than T_2 , considering that the capacitors connecting the quantum dot
to the source and drain are much larger than the capacitors $G_{L,R(1,0),(2,0)} =$ to the source and drain are much larger than the capacitors connecting the trap to the source and quantum dot and μ_{LR} is the variation of the electrostatic energy of the system due to removing/adding a single electron from/to the left/right lead
specified as (13): specified as (13): be source and drain are much larger the source and drain are much larger moving/adding a single electron from

becified as (13): $\frac{1}{2}$ a single electron from/to the left/right lead

$$
\mu_{L,R} = -qV_{L,R} \tag{13}
$$

X EXSIGN 115100 **i** *CCC C i is id ig* (4) In another case, an electron can be added to the trap from the left or right electrode, and as a result, the system moves from the $(2,0)$ state to the $(2,1)$ state, or vice versa, the electron inside the trap moves to the left or right electrodes, and the system returns from the state $(2,1)$ to state $(2,0)$, which are described in relations (14) and (15) respectively.

$$
G_{L,R(2,1),(2,0)} = T_2 f\left(\frac{W_{(2,1)} - W_{(2,0)} + \mu_{L,R}}{K_B T}\right)
$$
(14)

$$
G_{L,R(2,0),(2,1)} = T_2 f\left(\frac{W_{(2,0)} - W_{(2,1)} - \mu_{L,R}}{K_B T}\right)
$$
 (15)

 $T_1 f^* \left(\frac{W_{(3,0)} - W_{(2,0)} + \mu_{L,R}}{K_B T} \right)$ (11) Finally, in the (2,0) state, an electron can tu

quantum dot to the left or right electrode, an

moves to the (1,0) state or an electron can turn to the (1,1) state. Both of these tunnelings are also possible
in reverse, that is, from the state (1,0) to (2,0) by adding an $T = T_1 f'(-\frac{(3,0)}{(2,0)}, \frac{(2,0)}{(2,0)}, \ldots)$ (11) quantum dot to the left or right electrode, and the system
moves to the (1,0) state, or an electron can tunnel from the
dot to the tran, and the system transits from the (2,0) s Finally, in the (2,0) state, an electron can tunnel from the Finally, in the $(2,0)$ state, an electron can tunnel from the quantum dot to the left or right electrode, and the system to the (1,1) state. Both of these tunnelings are also possible shown in relations $(16-19)$ respectively. and also from the state $(1,1)$ to $(2,0)$ by tunneling an electron from the trap to the quantum dot, all these tunnelings are shown in relations $(16-19)$ respectively. *L R* moves to the $(1,0)$ state, or an electron can tunnel from the dot to the trap, and the system transits from the $(2,0)$ state to the $(1,1)$, that Γ R the of the a transition are also possible electron from the left or right electrode to the quantum dot
and also from the state $(1,1)$ to $(2,0)$ by tunneling an electron *M* When the left or right electrode to the quantum dot and also from the state (1,1) to (2,0) by funneling an electron $(10-19)$ respectively. ot to the trap, and the system tran
 b the (1,1) state. Both of these tunn

$$
G_{L,R(1,0),(2,0)} = T_1 f^* (\frac{W_{(1,0)} - W_{(2,0)} - \mu_{L,R}}{K_B T})
$$
(16)

$$
G_{(1,1),(2,0)} = T_2 f\left(\frac{W_{(1,1)} - W_{(2,0)}}{K_B T}\right)
$$
 (17)

Fig. 5. Characteristics of source-couple **Fig. 5. Characteristics of soul** Fig. 5. Characteristics of source-coupled trap occupancy probability as a Fig. 5. Characteristics of soul Fig. 5. Characteristics of source-coupled trap occupancy probability as a Fig. 5. Characteristics of source-coupled trap occupancy probability as a function of gate a , (2,1),(2,0) 2 () *L R L R* nction of g *G* The *G* The *G* The *K* Th (17) **Fig. 5. Characteristics of source-coupled trap occupancy probability as a function of gate and drain voltage**

$$
G_{L,R(2,0),(1,0)} = T_1 f^* \left(\frac{W_{(2,0)} - W_{(1,0)} + \mu_{L,R}}{K_B T} \right)
$$
 of $(P_{(0,0)}, P_{(1,0)}, P_{(1$

$$
G_{(2,0),(1,1)} = T_2 f\left(\frac{W_{(2,0)} - W_{(1,1)}}{K_B T}\right)
$$
\n
$$
G_{(2,0),(1,1)} = T_2 f\left(\frac{W_{(2,0)} - W_{(1,1)}}{K_B T}\right)
$$
\n(19)

(2,0) , (2,0),(1,0) (1,0) (2,0),(1,1) (1,1) *L R* state and all the tunneling events entering the (2,0) state, as $(W_0)^T$ at V_g <indicated in (20). state solution of the master equation. In other words, it is the suggests the difference between all the tunneling events leaving the $(2,0)$ (E)" at V difference between all the tunneling events leaving the $(2,0)$
state and all the tunneling events entering the $(2,0)$ state as The net tunneling rate of $(2,0)$ is obtained from the steady-
the solution of the master equation. In other words, it is the varies in the
trap occupation of the trap of

$$
\Gamma_{(2,0)} = G_{L,R(2,0),(1,0)} P_{(1,0)} + G_{(2,0),(1,1)} P_{(1,1)}
$$
\n
$$
+ G_{L,R(2,0),(2,1)} P_{(2,1)} + G_{L,R(2,0),(3,0)} P_{(3,0)}
$$
\n
$$
- P_{(2,0)} (G_{L,R(1,0),(2,0)} + G_{(1,1),(2,0)})
$$
\n
$$
+ G_{L,R(2,1),(2,0)} + G_{L,R(3,0),(2,0)})
$$
\n
$$
I = -q \sum_{n=0}^{n=0} \binom{m}{n} \bin
$$

In (20), $P(i,j)$ is the probability of charge state (i,j) . After writing an the net tunnering rates for each state, equations are written in matrix form using (21): *m* t *m* t mt *m* t *m* t *m m m n m* \overline{C} writing all the net tunneling rates for each state, the tunneling and (21) .

$$
C = P A. \tag{21}
$$

 $T_1 f^* \left(\frac{W_{(2,0)} - W_{(1,0)} + \mu_{L,R}}{K_p T} \right)$ (18) of $(P_{(0,0)}, P_{(1,0)}, P_{(2,0)}, \dots, P_{(9,0)}, P_{(0,1)}, P_{(2,1)}, \dots, P_{(9,1)}, P_{(1,1)}, P_{(2,1)}, \dots, P_{(9,1)}$). A is the matrix of tunneling rates, which is obtained by taking into In relation (21) , P is the vector of probabilities in the form the matrix of tunneling rates, which is obtained by taking into account the exchange of all states as we briefly demonstrated for (2,0) state, and C is a vector in the form of $(c=0, c=0)$ for (2,0) state, and C is a vector in the form of (c₁=0, c₂=0, c₁=0, c₁=0, c₂=0, c₁=0, c₂=0, c $c_3 = 0, \ldots, c_{20} = 1$). The last equation indicates that the sum of probabilities of all charge states of the system should be unity.

 $\frac{1}{3}$ (18), modeling
calculations as a
reling rate of (2.0) is obtained from the steady-
This, figure, show Fig. 5 shows the collective probability for all states with one electron in the trap, which is coupled to the source (Fig. *GPP)* 1. a), including {(0,1),(1,1), ...(9,1)} obtained from our calculations as a function of gate voltage and drain voltage. This figure shows a close correlation with Fig. 3. which suggests that the ground state of the system is an "Empty trap (E_0) " at V_g <-50 mV. It switches to "Occupied Trap" at $V_g > 50$ mV, while the ground state oscillates between E_0 and E_1 as V_g varies in this range. Therefore, we expect the probability of trap occupation to also vary from 0 to 1 and show significant variations between 0 and 1 as we increase V_g . The tunneling current of the system is calculated by (22) :

$$
I = -q \sum_{n_m, n_t} (G_{L(n_m+1, n_t)(n_m, n_t)} P_{(n_m, n_t)} + G_{L(n_m, n_t+1)(n_m, n_t)} P_{(n_m, n_t)} \qquad \qquad + G_{L(n_m-1, n_t)(n_m, n_t)} P_{(n_m, n_t)} - G_{L(n_m, n_t-1)(n_m, n_t)} P_{(n_m, n_t)} \tag{22}
$$

Fig. 6 shows the tunneling conductance (a derivative of the tunneling current) as a function of V_d and V_g . The conductance curve is usually referred to as the charge stability diagram, and our simulation results (Fig 6. b) show excellent

Fig. 6. Conductance characteristic of a quantum dot in the vicinity of an occupied source-coupled trap (a) **Experimental results [12] (b) result of this work simulation in MATLAB**

Fig. 7. Characteristics of drain-coupled trap occupancy probability as a function of gate and drain voltage **Fig. 7. Characteristics of drain-coupled trap occupancy probability as a function of gate and drain voltage**

agreement with the experimental data (Fig 6. a [12]). The anomalies of the charge stability diagram are compared with the diagram of an ideal single-electron transistor (simple diamond shape) and are the footprint of the charged trap.

When the trap is located at the drain side and coupled to the drain, the probability of trap occupation increases by increasing the drain voltage. This feature is demonstrated in Fig. 7, which shows horizontal mirror characteristics compared to Fig. 5, where the trap is located at the source side.

Another distinctive feature for source-side and drainside traps is extra teeth, demonstrated in the charge stability diagram of Fig. 6.b for the source-side trap and Fig. 8 for the drain-side trap, highlighted with white boxes. According to Fig. 6. b, when the traps are at the source side, these extra

teeth are more visible if the gate voltage and the drain voltage are both positive or negative, but this feature is mirrored again for the drain-side trap.

Fig. 9 shows the trap's occupation probability when capacitively connected to both the source and the drain (Fig. 1.c). This structure equals the intersection of the probabilities of occupation for the trap connected to the source (Fig. 5) and the trap connected to the drain (Fig.7). This causes the creation of triangular shapes (symmetric in the y-direction), which become brighter with increasing the gate voltage, as the probability of occupancy increases.

Fig. 10 shows the conductance characteristics of a trap coupled to the source, drain, and gate. It shows that the probability of the trap filling is almost uniform for positive and negative drain biases; therefore, extra teeth are found throughout the characteristic.

Fig. 8. Conductance characteristic of a quantum dot in the vicinity of an occupied drain-coupled trap **Fig. 8. Conductance characteristic of a quantum dot in the vicinity of an occupied drain-coupled trap**

Fig. 9. Characteristics of source and drain-coupled trap occupancy probability as a function of gate and **drain voltage**

drain-coupled trap Fig. 10. Conductance characteristic of a quantum dot in the vicinity of an occupied source and

Fig. 11. Conductivity characteristic of a quantum dot in the vicinity of a trap (a) Quantum dot has strong coupling with source and drain and weak coupling with trap (b) Quantum dot has weak coupling with source and **drain and strong coupling with trap**

5- Dot-Lead-Trap coupling strength

The stronger coupling enhances the interaction between the quantum dot and the leads or trap, facilitating the transfer of electrons, resembling the dot's movement toward the leads or trap. Conversely, when the coupling capacitors are decreased, the interaction between the quantum dot and the leads or trap reduces, resembling the dot's movement away from the leads or trap.

Fig. 11 shows the quantum dot charge stability diagram

near the trap in different states. In the first case (Fig. 11.a), we increased the coupling capacitor between the quantum dot and the source and drain leads to 18 aF and decreased the coupling capacitor between the quantum dot and the trap to 0.08 aF. In this case, the conductivity has increased (in the range of 0.6 to 1 mA), and the anomaly (extra teeth) of the diagram due to trap perturbation is reduced. In the second case (Fig. 11.b), we reduced the coupling capacitors between the quantum dot and the leads to 5 aF and increased the

Fig. 12. Coulomb characteristic diagram of a quantum dot in the vicinity of a trap (a) When the trap is empty $\left(\mathbf{b}\right)$ When the trap is equantion (b) When the trap is occupied

coupling between the quantum dot and the trap to 0.35 aF. As can be seen, the conductivity decreased (in the range of 0 2 to 0.6 mA), and additional teeth were observed due to the disturbance induced by the trap.

6- Shift of the dot coulomb blockade characteristic due to trap charging

Fig. 12 shows the boundaries of neighboring charge states of the main dot in the V_g and V_d plane for two scenarios: a) empty trap and b) occupied trap. As can be seen, there is a horizontal shift of 5.4mV in Fig 12. b compared to Fig 12.a. This shift can be attributed to the coulombic repulsion between the charged trap and the electrons in the main dot, which leads to additional energy (gate voltage) required to add a single electron to the main dot. By increasing the capacitor between the trap and the gate (C_{t_0}) , the horizontal shift decreases because by increasing the coupling between the trap and the gate and by increasing the gate voltage, the effect of the gate on the presence of electrons in the trap rises, and as a result, the impact of electron repulsion at the main point with the trap decreases, and less shift is created.

7- Conclusion

This work has studied the effect of a charge trap near the quantum dot on the charge stability diagram. Several scenarios have been investigated, including the situation when the two-state trap is coupled with source, drain, and both source and drain. Tunneling rates were calculated using the master equation for all permitted transitions with neighboring states, and the matrix of tunneling coefficients was obtained. The current diagram, conduction, and Coulomb blockade

characteristics were simulated in MATLAB for both empty and occupied traps, and the simulation results are in good agreement with the experimental results. The presence of the trap with different charge states results in the anomaly of the pure SET diamond characteristics in the form of extra teeth and a shift of stability diagram; these features might be used to estimate the location and coupling strength between the dot, trap, and the leads. Furthermore, this analysis might intentionally position a trap near a qubit for manipulation and readout of the charge state of the main dot for quantum computing applications.

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9- Nomenclature

Cmg The capacitor between the quantum dot and the gate, F

Cmd The capacitor between the quantum dot and the drain, F

Cms The capacitor between the quantum dot and the source, F

 C_{tg} The capacitor between the trap and the gate, F

 C_{ts} The capacitor between the trap and the source, F

 C_{td} The capacitor between the trap and the drain, F

C_c The capacitor between the trap and the quantum dot, F

- *C_t* Capacitors connected to the trap, F
- *C* Total system capacitors, F
- *W* Electrostatic energy, J
- *Qm/t* The charge in the quantum dot/ trap, C
- nm/t The number of electrons in the quantum dot/ trap
- *X* Charge stored in all system capacitors, J
- *X_t* Charge stored in capacitors connected to the trap, J
- *X_m* Charge stored in capacitors connected to the quantum dot, J
- *q*₀ The background charge in the dot, C
- *q* absolute value of the electron charge, 1.602.10-19 C
- $V_g' V_s' V_d$ Gate/ Source/ Drain voltage, V
- *e* Euler's number, 2.718
- G_{LR} Tunneling rate between two consecutive states to/ from left or right leads
- T_p, T_q Constant coefficients 1, 1000
- K_B Boltzmann factor, 1.381.10⁻²³ J/k
- *T* Temperature, K
- V_{LR} The voltage of the left or right leads, V
- *P* The vector of probabilities
- *A* The matrix of tunneling rate

Greek symbols

- β _t Structure of the trap signature
- μ_{LR} Chemical potential in left or right leads, J
- Γ The net tunneling rate of a state

Subscript

- *m* Quantum dot
- *t* Trap
- *L* Left
- *R Right*

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