



Neighborhood-Based Event-Triggered Distributed Fault Estimation Observer for Multi-Agent Systems

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ABSTRACT: This study addresses the distributed Fault Estimation control problem for linear multi-agent systems with an event-triggered communication mechanism. In multi-agent systems, a substantial challenge is to find out the size and shape of the occurred faults and how to reduce the wastage of communication bandwidth and unnecessary executions. In order to address these concerns, we proposed a distributed Fault Estimation observer, where each agent employs an augmented system based on a predefined communication graph, and with consideration of its neighbors, to estimate the fault and states both in itself as well as its neighbors, simultaneously. In addition, an event-triggering scheme was implemented in this approach in order to effectively reduce unnecessary signal transmission between the agents and attain a reasonable allocation of resources. Sufficient conditions are presented to guarantee that the closed-loop system is asymptotically stable with prescribed disturbance attenuation, and the parameter matrices of the event-triggered mechanism and observer can be obtained simultaneously by solving a set of Linear Matrix Inequalities (LMIs). Eventually, some simulations are included to demonstrate the performance of the introduced Fault Estimation and effectively of the event-triggered mechanism.

Review History:

Received: May, 03, 2022

Revised: Jun. 26, 2022

Accepted: Jul. 12, 2022

Available Online: Jan. 01, 2022

Keywords:

Distributed Fault Estimation
event-triggered mechanism
neighborhood-based observer
multi-agent systems,

1- Introduction

In the past few decades, there have been significant progress in information technology, task scales, and distributed actuator networked systems. All these improvements have led to multi-agent systems receiving a surge of attention, due to their wide potential applications in many domains, including multi-robots for water distribution systems [1], Kalman filter [2], unmanned aerial vehicles [3], formation and flight control systems [4, 5], etc. A comprehensive review of the most recent advances in multi-agent systems and their applications are provided in [6, 7]. A major challenge in such systems is to design the controller so that the reliability and the sustainability of the system are maintained. Due to information interconnection between agents through communication networks, large-scale systems are exceedingly vulnerable to losing their performance. For instance, when the fault occurs in one of these agents, the fault information first propagates to its neighbors and then throughout the system, and finally the system performance and reliability would be destroyed [8, 9]. Faults are inevitable because physical events or occurring failures may always happen in some sensors. Therefore, it is necessary for the controller in a multi-agent system to be able to monitor the system behavior at any time to find out the location and the occurrence of the fault.

By taking a closer look at the literature, it can be found that the most relevant works, with regard to the problem of

occurring fault, focus on the centralized structures whilst these structures are not suitable for employment in networked systems. Fault diagnosis schemes contain three main topics: fault isolation, fault detection, and Fault Estimation [10-12]. In the case of designing the fault diagnosis schemes for multi-agent systems, fruitful outcomes were achieved over the recent years [13-15]. The main aim of the fault detection and isolation approaches lies in determining whether or not a fault has occurred, and finding in which component a fault has occurred [16]. The main aim of the Fault Estimation problem is to acquire accurate information about the size and the shape of the occurred faults in the system. However, the difficulties of the Fault Estimation method are appreciably more than the fault detection and isolation methods.

Recently, efficient approaches for Fault Estimation problems were devoted to multi-agent systems [17-20]. For instance, the authors in [21] proposed a distributed Fault Estimation approach for linear multi-agent systems with switching communication topologies. In [22], a distributed Fault Estimation approach was provided for linear multi-agent systems, where the constructed observer on each agent could estimate the occurred faults of the entire network. When the fault information was obtained in the system, a controller could compensate the destructive effects of the fault using this information. The efficient approaches for the distributed Fault Estimation and accommodation problem for multi-agent systems were proposed in the literature [23-25].

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The authors of [23] proposed a method to consider Fault Estimation and fault-tolerant control for a class of nonlinear multi-agent systems. In [25], a Fault Estimation (FE) and accommodation approach were provided for general interconnected systems with both nonlinear and linear dynamics. The mentioned works have some design difficulties and specific applications; thus, the issues of FE in multi-agent systems is needed to be more investigated.

It is worth noting that it is necessary in multi-agent systems that each agent continuously communicates and exchanges information (sensor reading and sending to its neighbors or its observer) with other agents. However, communication resources like bandwidth or capacity are often limited in reality. In regards to this challenge, different event-triggered scheduling mechanisms were introduced in the literature in order to reduce the number of signals transmission and save the computing cost of the controller while ensuring the desired control performance [26]. Recently, the problem of event-triggered Fault Estimation for networked control systems was investigated in [9, 27, 28]. The authors in [28] proposed an approach for the problem of the Fault Estimation control for networked systems through an integral-based event-triggered mechanism. In [27], the authors studied the event-triggered Fault Estimation problem and consensus control for time-delayed multi-agent systems subject to Markov switching topologies.

It is also worth noting that in the aforementioned works, the employed observers are constructed in all agents, while it is not possible to assemble an observer in all agents of a networked system. Meanwhile, there sometimes may be a situation in real engineering systems, in which it is necessary to construct the observer in only some agents instead of all agents to reduce the cost. Such approaches are reported in [29]. However, the proposed observers do not consider the constrained communication resources during exchanging information between agents in these works. Therefore, we are motivated to propose an event-triggered Fault Estimation observer in a more straightforward way that each agent would be able to estimate the fault and the states at the same time, both within itself and in its neighbors.

Based on the above observations, this paper is allocated to present a novel distributed Fault Estimation with an event-triggered communication scheme for linear multi-agent systems. The main contributions of this paper can be highlighted as follows:

(1) A distributed robust Fault Estimation observer with an event-triggered strategy for linear multi-agent systems is proposed, where each agent, based on a predefined communication graph, can simultaneously estimate the occurred faults and state within itself and in its neighbors. In this approach, unlike the existing methods of fault/state estimation with an event-triggered mechanism for multi-agent systems, observers can be constructed in some specific agents, instead of each agent, to estimate all possible faults in the whole system.

(2) Sufficient conditions were presented in the form of LMIs to guarantee the derived observer utilizing Lyapunov function, where the parameters of the event-triggered mech-

anism and observer can be computed by solving the established LMIs.

The rest of this work proceeds as follows: In section 2, the necessary background materials are called from literature. The problem of distributed Fault Estimation observer subject to an event-triggered communication scheme for linear multi-agent systems is modeled in Section 3. Subsequently, the proposed observer is presented. Next, the stability of the proposed observer in the framework of the H_∞ problem is analyzed. In Section 4, simulation results are provided to verify the efficiency of the introduced approach. Ultimately, the conclusion is presented in section 5.

Notations: The notations employed in this paper are standard. The Kronecker product between matrices B and A is described by $B \otimes A$. $\|\cdot\|$ denotes standard norm of vectors or matrices, while

$$\|z_{(i)}\|_{T_f} = \left(\int_0^{T_f} (z_{(i)}^T(t)z_{(i)}(t)dt) \right)^{1/2}.$$

2- Preliminaries and Problem Formulation

Here, we introduce some basic notions of graph theory.

2- 1- Communication Topology

Consider a multi-agent network composed of Λ agents whose information communication is modeled by an undirected graph $\Xi(V,E,I)$ with $V=\{1,\dots,\Lambda\}$ representing the set of nodes, and $E \subseteq V \times V$ describing the set of arcs. For agent i , the set of all its neighbors is described by $N_i = \{j \mid (i,j) \in E\}$. The adjacency matrix of the graph $\Xi(V,E)$ is represented by \mathcal{J} , which is a $\Lambda \times \Lambda$ matrix.

3- Problem Formulation

We assumed that there is a multi-agent networked system consisting of Λ agents, where the dynamic of the i -th agent is governed by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) + Ff_i(t) + \omega_i(t), \tag{1}$$

$$D\omega_i(t), y_i(t) = Cx_i(t),$$

where $x_i(t) \in \mathbb{R}^n$ is the state vector which should be estimated; $u_i(t) \in \mathbb{R}^m, f_i(t) \in \mathbb{R}^f$ and $\omega_i(t) \in \mathbb{R}^w$ represent the control signal, the actuator fault, and the external disturbance, respectively. Additionally, $y_i(t) \in \mathbb{R}^p$ denotes the system output vector of the i -th agent. Here, we assumed that $\omega_i(t) \in L_2$, and fault signal differentiable and its derivative are also bounded. Matrices A, B, F, D , and C are all known constant matrices with appropriate dimensions. In system (1), we also assumed that $\text{rank}(C) = p$.

In this paper, we consider an output-based control input, as follows:

$$u_i(t) = K \sum_{j=1}^{\Lambda} a_{ij} (y_i(t) - y_j(t)), \tag{2}$$

$$i = 1, 2, \dots, \Lambda$$

here, K is a weighted constant matrix with an appropriate dimension.

In the problem of multi-agent systems, each agent usually transfers its measurement output to its neighbors through a communication network at each time step. This inevitably leads the networked system to encounter the wastage of communication bandwidth. Therefore, to save energy consumption and reduce the communication bandwidth, the event-trigger mechanism should be implemented in these systems. Each agent i sends $y_i(t_k)$ to the event-trigger scheduler; then, it is transmitted to its neighbors only when the following condition is satisfied:

$$t_{k+1} = \inf \{t > t_k | \delta y_i^T(t) \Omega_i y_i(t) - e_{y_i}^T(t) \Omega_i e_{y_i}(t) \leq 0\}, (3)$$

where $e_{y_i}(t) = y_i(t_k) - y_i(t), 0 < \delta < 1, \Omega_i > 0$ are event-triggering parameters.

Before getting the main results, the following Lemma is needed.

Lemma 1 (Schur Complements) [7]: For given matrices X, Y , and R with appropriate dimensions,

where $X = X^T, Y = Y^T$, the following inequalities are equivalent:

$$I - \begin{bmatrix} X & R \\ * & Y \end{bmatrix} < 0.$$

2- Each of the following holds:

(i) $Y < 0$, and $X - RY^{-1}R^T < 0$.

(ii) $X < 0$, and $Y - R^T X^{-1}R < 0$.

4- Main Results

Here, at first the designing procedure of the proposed observer for each agent i with consideration of its neighbors is formulated over a communication graph. Finally, sufficient conditions in the form of LMIs are obtained in order to guarantee the stability of the proposed distributed fault/state estimation observer with the event-triggered communication. To formulate the dynamic of each agent i based on its neighbor agents, system (1) is rewritten as follows:

$$\begin{aligned} \dot{x}^i(t) &= A^i x^i(t) + B^i u^i(t) + F^i f^i(t) + \\ D^i W^i(t), y^i(t) &= C^i x^i(t), \end{aligned} (4)$$

Here:

$$x^i(t) = [x_{i1}^T(t), x_{i2}^T(t), \dots, x_{i|\bar{N}_i|}^T(t)]^T,$$

$$u^i(t) = \begin{bmatrix} u_{i1}(t) \\ u_{i2}(t) \\ \vdots \\ u_{i|\bar{N}_i|}(t) \end{bmatrix}, f^i(t) = \begin{bmatrix} f_{i1}(t) \\ f_{i2}(t) \\ \vdots \\ f_{i|\bar{N}_i|}(t) \end{bmatrix},$$

$$W^i(t) = \begin{bmatrix} \omega_{i1}(t) \\ \omega_{i2}(t) \\ \vdots \\ \omega_{i|\bar{N}_i|}(t) \end{bmatrix}, y^i(t) = \begin{bmatrix} y_{i1}(t) \\ y_{i2}(t) \\ \vdots \\ y_{i|\bar{N}_i|}(t) \end{bmatrix},$$

And:

$$B^i = I_{|\bar{N}_i|} \otimes B,$$

$$F^i = I_{|\bar{N}_i|} \otimes F, D^i = I_{|\bar{N}_i|} \otimes D, C^i = I_{|\bar{N}_i|} \otimes C.$$

and according to the control input (2), and taking the concept of the event-triggering (3), one can have:

$$\begin{aligned} u^i(t) &= \left(\text{diag} \left(\left[\sum_{j=1}^{\Lambda} a_{i_1j}, \sum_{j=1}^{\Lambda} a_{i_2j}, \dots, \sum_{j=1}^{\Lambda} a_{i_{|\bar{N}_i|}j} \right] \right) \otimes K \right) \times \\ &\begin{pmatrix} y_{i_1}(t_k) - y_j(t_k) \\ y_{i_2}(t_k) - y_j(t_k) \\ \vdots \\ y_{i_{|\bar{N}_i|}}(t_k) - y_j(t_k) \end{pmatrix}, \end{aligned} (5)$$

where \bar{N}_i stands for neighbors of agent i . In other words, agent i and its neighbors are described as $i_1, i_2, \dots, i_{|\bar{N}_i|}$, wherein $i = i_1$.

In the following, a distributed observer is proposed which each agent i is able to estimate states and fault signals. The input signal of the Fault Estimation observer is rewritten by the event-triggered mechanism (i.e., $y^i(t) \rightarrow y^i(t_k), t \in [t_k, t_{k+1})$).

$$\begin{aligned} \hat{x}^i(t) &= A^i \hat{x}^i(t) + B^i u^i(t) + \\ F^i \hat{f}^i(t) + L_1^i \Delta_1^i(t) + L_2^i \Delta_2^i(t), \\ \hat{f}^i(t) &= E_1^i \Gamma_1^i (y^i(t_k) - \hat{y}^i(t)) + \\ E_2^i \Gamma_2^i (y^i(t_k) - \hat{y}^i(t)), \quad \hat{y}^i(t) &= C^i \hat{x}^i(t) \end{aligned} (6)$$

in which:

$$\Gamma_1^i = \left(\text{diag} \{ a_{i_1i}, a_{i_2i}, \dots, a_{i_{|\bar{N}_i|}i} \} \right) \otimes I_p, (7)$$

$$\Delta_1^i(t) = \Gamma_1^i (y^i(t_k) - C^i \hat{x}^i(t)),$$

$$\Gamma_2^i = \left(\text{diag} \left\{ \sum_{k=1}^{|\bar{N}_i|} a_{i_ki}, \dots \right\} \right) \otimes I_p, (8)$$

$$\Delta_2^i(t) = \Gamma_2^i (y^i(t_k) - C^i \hat{x}^i(t)),$$

Here, $\hat{x}^i(t)$ is the estimations of $x^i(t)$. L_1^i, L_2^i, E_1^i , and E_2^i are observer gain matrices which will be obtained later. For each agent i , Γ_1^i and Γ_2^i are devoted as the weight out-put estimation of neighbors and the agent itself, respectively.

Remark: There are considerable fault/state estimation approaches in the literature for multi-agent systems, especially for interconnect systems, where it is needed to construct the observer in each agent to estimate all possible faults in the whole system [13-15, 30-33]. Additionally, in the works [16-18], the main goal of these works is to study the Fault Estimation problem for multi-agent systems, where each agent is only capable of estimating its own possible fault. To monitor the possible faults in networked systems considered in these works, observers would have to be installed in each agent. To the best of our knowledge, the work in which the distributed fault/state estimation observer with an event-triggered communication mechanism in each agent could be able to estimate the states and faults simultaneously both within itself and in its neighbors for linear multi-agent systems was not found to mention. In Equation (6), two design parameters are embedded to consider only the neighbors of each i (i.e., L_1^i, E_1^i). It must be noted that the main contribution of this approach lies in the structure of the proposed observer for multi-agent systems. This observer has a straightforward structure and also is able to estimate the possible fault within itself and in its neighbors. However, a straightforward event-triggered mechanism has just been used to reduce the computing cost in exchanging data as much as possible.

Due to Eqs. (6) and (3), obtaining the gain matrices of the state, Fault Estimation, and the event-triggered mechanism is needed to construct the proposed observer. We define the following variables:

$$\begin{aligned} \xi^i(t) &= [(z_x^i(t))^T \quad (z_f^i(t))^T]^T, \\ z_x^i(t) &= x^i(t) - \hat{x}^i(t), z_f^i(t) = f^i(t) - \hat{f}^i(t), \end{aligned} \tag{9}$$

The following augmented system is obtained using the above variables:

$$\begin{aligned} \dot{z}_x^i(t) &= (A^i - L_1^i \Gamma_1^i C^i - L_2^i \Gamma_2^i C^i) z_x^i(t) + \\ & F^i z_f^i(t) + D^i W^i(t) + (-L_1^i \Gamma_1^i - L_2^i \Gamma_2^i) e_{y^i(t)}, \end{aligned} \tag{10}$$

$$\begin{aligned} \dot{z}_f^i(t) &= \hat{f}^i(t) - E_1^i \Gamma_1^i C^i z_x^i(t) - \\ & E_2^i \Gamma_2^i C^i z_x^i(t) + (-E_1^i \Gamma_1^i - E_2^i \Gamma_2^i) e_{y^i(t)}. \end{aligned} \tag{11}$$

According to (10) and (11), we have:

$$\begin{bmatrix} X \dot{z}_x^i(t) \\ \dot{z}_f^i(t) \end{bmatrix} = \begin{bmatrix} \Omega_1 & F^i \\ -E_1^i \Gamma_1^i C^i - E_2^i \Gamma_2^i C^i & 0 \end{bmatrix} \begin{bmatrix} z_x^i(t) \\ z_f^i(t) \end{bmatrix} + \tag{12}$$

$$\begin{bmatrix} D^i & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} W^i(t) \\ \hat{f}^i(t) \end{bmatrix} + \begin{bmatrix} -L_1^i \Gamma_1^i - L_2^i \Gamma_2^i \\ -E_1^i \Gamma_1^i - E_2^i \Gamma_2^i \end{bmatrix} e_{y^i(t)},$$

Where:

$$\Omega_1 = A^i - L_1^i \Gamma_1^i C^i - L_2^i \Gamma_2^i C^i.$$

Then we can redescribe equation (12) in a straightforward form:

$$\dot{\xi}^{(i)}(t) = A_1^{(i)} \xi^{(i)}(t) + D_1^{(i)} \bar{W}^{(i)}(t) + E_y^i e_{y^i(t)}, \tag{13}$$

Where:

$$\begin{aligned} A_1^{(i)} &= \begin{bmatrix} \Omega_1 & F^i \\ -E_1^i \Gamma_1^i C^i - E_2^i \Gamma_2^i C^i & 0 \end{bmatrix}, \\ E_y^i &= \begin{bmatrix} -L_1^i \Gamma_1^i - L_2^i \Gamma_2^i \\ -E_1^i \Gamma_1^i - E_2^i \Gamma_2^i \end{bmatrix}, D_1^{(i)} = \begin{bmatrix} D^i & 0 \\ 0 & I \end{bmatrix}, \\ \bar{W}^{(i)}(t) &= \begin{bmatrix} W^i(t) \\ \hat{f}^i(t) \end{bmatrix}. \end{aligned} \tag{14}$$

In the presented approach, we have derived a distributed Fault Estimation observer (6) with an event-triggered communication mechanism. In this way, a sufficient condition is assigned to guarantee the stability and H_∞ performance of the system (13). The provided results are presented in the following theorem.

Theorem 1: Assume that $0 < \delta < 1$, the dynamic system (13) for each agent i is exponentially stable with an H_∞ level λ , that is, (i) for $\bar{W}^{(i)}(t) = 0$, the system (13) is asymptotically stable:

$$\lim_{t \rightarrow \infty} \xi^{(i)}(t) = 0. \tag{15}$$

(ii) for all non-zero $\bar{W}^{(i)}(t) \in L_2[0, \infty)$, the system (13) has an H_∞ level λ :

$$\|\xi^{(i)}(t)\|_{T_f} < \sqrt{\lambda} \|\bar{W}^{(i)}(t)\|_{T_f}, \lambda > 0, \tag{16}$$

if there are positive definite matrices P_i, Ω_i matrices R_1^i, R_2^i , and a positive scalar λ such that the following LMI is feasible:

$$\bar{\Pi} = \begin{bmatrix} \Pi_{11} & \Pi_{12} & P_i D_1^{(i)} \\ * & \Pi_{22} & 0 \\ * & * & -\lambda I \end{bmatrix} < 0, \tag{17}$$

Where:

$$\bar{A}_1 = \begin{bmatrix} A^i & F^i \\ 0 & 0 \end{bmatrix},$$

$$\Pi_{11} = [P_i \bar{A}_1 - R_1^i [\Gamma_1^i C^i \quad 0] - R_2^i [\Gamma_2^i C^i \quad 0]]_s.$$

$$\Pi_{12} = -R_1^i \Gamma_1^i - R_2^i \Gamma_2^i, \Pi_{22} = -\Omega_i I,$$

Thus, the Fault Estimation observer parameters gains are given by $\bar{L}_1 = P_i^{-1} R_1^i$, $\bar{L}_2 = P_i^{-1} R_2^i$.

Proof: According to the system (13), we consider the following Lyapunov function candidate [34] for i-th agent, as follows:

$$V^i(t) = \xi^{(i)T}(t) P_i \xi^{(i)}(t), \tag{18}$$

where P_i is a positive definite matrix.

The time derivative of (18) with the help of (13) is given by:

$$\begin{aligned} \dot{V}^i(t) &= \xi^{(i)T}(t) (P_i A_1^{(i)} + A_1^{(i)T} P_i^T) \xi^{(i)}(t) + \\ & \cdot 2\xi^{(i)T}(t) P_i \times D_1^{(i)} \bar{W}^{(i)}(t) + \\ & 2\xi^{(i)T}(t) P_i E_y^i e_{y^i(t)}. \end{aligned} \tag{19}$$

Considering the case of $\bar{W}^{(i)} = 0$, from the event-triggered (3), we can add the term $\delta y_i^T(t) \Omega_i y_i - e_{y_i}^T(t) \Omega_i e_{y_i}(t) \leq 0$ to (19):

$$\begin{aligned} \dot{V}^i(t) &= \xi^{(i)T}(t) (P_i A_1^{(i)} + A_1^{(i)T} P_i^T) \xi^{(i)}(t) + \\ & 2\xi^{(i)T}(t) P_i E_y^i e_{y^i(t)} + \delta y_i^T(t) \Omega_i y_i - \\ & e_{y_i}^T(t) \Omega_i e_{y_i}(t). \end{aligned} \tag{20}$$

Using Lemma 2.2, from (20) we can have the following LMI:

$$\Phi_i = \begin{bmatrix} P_i A_1^{(i)} + A_1^{(i)T} P_i^T + \delta \tilde{C}_i^T \Omega_i \tilde{C}_i & P_i E_y^i \\ 0 & -\Omega_i I \end{bmatrix}, \tag{21}$$

with $\tilde{C}_i = [C^i, 0]$, when $\Phi_i < 0$ in (21), one has $\dot{V}^i(t) < 0$, and specifying the error dynamics is asymptotically stable. In other words, it implies that (15) is satisfied.

Now, we assume the case that $\bar{W}^{(i)} \neq 0$:

$$\bar{Y}_i = \int_0^{T_f} (\xi^{(i)T}(t) \xi^{(i)}(t) - \lambda (\bar{W}^{(i)}(t))^T \bar{W}^{(i)}(t)) dt, \tag{22}$$

By utilizing (19) and (22), we have:

$$\begin{aligned} \bar{Y}_i &= \int_0^{T_f} ((\xi^{(i)}(t))^T \xi^{(i)}(t) - \lambda \bar{W}^{(i)}(t) \bar{W}^{(i)}(t) + \\ & \dot{V}^i(t)) dt - \int_0^{T_f} \dot{V}^i(t) dt = \\ & \int_0^{T_f} \xi^{(i)T}(t) \xi^{(i)}(t) + \xi^{(i)T}(t) (P_i A_1^{(i)} + \\ & A_1^{(i)T} P_i^T) \xi^{(i)}(t) + 2\xi^{(i)T}(t) P_i D_1^{(i)} \bar{W}^{(i)}(t) + \\ & 2\xi^{(i)T}(t) P_i E_y^i e_{y^i(t)} - \lambda \bar{W}^{(i)}(t) \bar{W}^{(i)}(t) dt - \\ & \int_0^{T_f} \dot{V}^i(t) dt. \end{aligned} \tag{23}$$

Now using (20), (23) can be rewritten as the following form:

$$\bar{Y}_i = \int_0^{T_f} [(\xi^{(i)}(t))^T \quad (\bar{W}^{(i)}(t))^T] \bar{\Xi} \begin{bmatrix} \xi^{(i)}(t) \\ \bar{W}^{(i)}(t) \end{bmatrix} dt - \int_0^{T_f} \dot{V}^i(t) dt \tag{24}$$

Where:

$$\bar{\Xi} = \begin{bmatrix} P_i A_1^{(i)} + (A_1^{(i)})^T P_i^T + \delta \tilde{C}_i^T \Omega_i \tilde{C}_i + I & P_i E_y^i & P_i D_1^{(i)} \\ * & -\Omega_i I & 0 \\ * & * & -\lambda I \end{bmatrix}, \tag{25}$$

Also, under zero initial condition $\xi^{(i)}(0) = 0$, we have:

$$\begin{aligned} \int_0^{T_f} \dot{V}^i(t) dt &= (\xi^{(i)}(T_f))^T P_i \xi^{(i)}(T_f) - \\ & (\xi^{(i)}(0))^T P_i \xi^{(i)}(0) = V^i(T_f) > 0. \end{aligned} \tag{26}$$

Recalling the form of $A_1^{(i)}$ in (13), the following LMI can be obtained based on (25):

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & P_i D_1^{(i)} \\ * & -\Omega_i I & 0 \\ * & * & -\lambda I \end{bmatrix}, \tag{27}$$

Where:

$$\begin{aligned} \Phi_{11} &= \left[P_i \bar{A}_1 - P_i \begin{bmatrix} L_1^i \\ E_1^i \end{bmatrix} [\Gamma_1^i C^i \quad 0] - \right. \\ & P_i \begin{bmatrix} L_2^i \\ E_2^i \end{bmatrix} [\Gamma_2^i C^i \quad 0] \left. \right]_s + \delta \tilde{C}_i^T \Omega_i \tilde{C}_i + I, \Phi_{12} = \\ & -P_i \begin{bmatrix} L_1^i \\ E_1^i \end{bmatrix} \Gamma_1^i - P_i \begin{bmatrix} L_2^i \\ E_2^i \end{bmatrix} \Gamma_2^i. \end{aligned} \quad (28)$$

Now, using the variables such as $\bar{L}_1^i = \begin{bmatrix} L_1^i \\ E_1^i \end{bmatrix}$, $\bar{L}_2^i = \begin{bmatrix} L_2^i \\ E_2^i \end{bmatrix}$ and $R_1^i = P_i \bar{L}_1^i$, $R_2^i = P_i \bar{L}_2^i$, LMI (17) is obtained. Π in term of (17), and from (24) and (26), one has $\bar{\gamma}_i < 0$, which indicates $\|e_{(i)}\|_{T_f} \leq \sqrt{\lambda} \|v_{(i)}\|_{T_f}$. It completes the proof. So far the observer and controller gains have been obtained in a straightforward way that satisfies the requirements (i) and (ii) in Theorem 1. Due to the ultimate results which are in the form of LMIs, the consequences of Theorem 1 offer the following optimization problem that is a routine problem to control engineers. The optimal H_∞ control problem, where λ is a bound of the norm l_2 , i for all nonzero $\bar{W}^{(i)}(t) \in l_2[0, \infty)$:

$$\min_{P_i > 0, \Omega_i > 0, R_1^i, R_2^i} \lambda$$

subject to (16)

4- 1- Numerical Simulation

In this section, a practical example is employed to verify the validity and practicality of the introduced approach. In this example, the network of the F-404 aircraft engine system is considered, where the model of each aircraft engine system is borrowed from [24]. Afterwards, we have the following parameters for each agent:

$$A = \begin{bmatrix} -1.46 & 0 & 2.428 \\ 0.1643 & -0.4 & -0.378 \\ 0.3107 & 0 & -2.231 \end{bmatrix}, B = \begin{bmatrix} 0.11 & 0 \\ 0.14 & -0.4 \\ 0.1 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -1.46 & 0 & 2.428 \\ 0.1643 & -0.4 & -0.378 \\ 0.3107 & 0 & -2.231 \end{bmatrix}, B = \begin{bmatrix} 0.11 & 0 \\ 0.14 & -0.4 \\ 0.1 & 0 \end{bmatrix}$$

These matrices stand for constructing system (1). Agents exchange data with one another according to the considered communication topology in Fig. 1.

According to the communication topology, the associated adjacency matrix can be described by:

$$\mathfrak{S} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

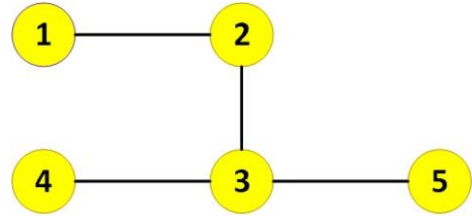


Fig. 1. The communication topology among five agents.

We assume that the faults occur in the considered network for each agent as follows:

$$\begin{aligned} f_1(t) &= \begin{cases} 0.5 & 0s \leq t \leq 10s; \\ -1 & 10s < t \leq 30s; \end{cases} \\ f_2(t) &= \begin{cases} 0 & 0s \leq t \leq 5s; \\ 1.5 & 5s < t \leq 10s; \\ -1.5 & 10s < t \leq 15s; \\ 1.5 & 15s < t \leq 30s; \end{cases}, f_3(t) = 0, \\ f_4(t) &= \begin{cases} 0 & 0s \leq t < 9s; \\ 1.5 & 9s \leq t < 15s; \\ -0.5 & 15s \leq t \leq 30s; \end{cases} \\ f_5(t) &= \begin{cases} 0 & 0s \leq t \leq 5s; \\ -1 & 5s < t \leq 30s; \end{cases} \end{aligned}$$

In (29), we assumed that the fault in agent 3 is zero in order to examine the effectiveness of the proposed method to consider the different cases in which the fault may or may not occur. Due to space limitations, the required parameters to construct an observer in agent 1 are only provided here. To solve this problem numerically, one of the best tools is the MOSEK solver in the MATLAB TOOLBOX YALMIP. By applying Theorem 1, the corresponding event-triggered mechanism and observer gains for agent 1 can be extracted as:

$$L_1^1 = \begin{bmatrix} -0.0009 & 0.0288 & 0.2836 & 0.0001 & -0.0000 & -0.0023 \\ 0.0579 & 0.0671 & 0.0576 & -0.0001 & -0.0011 & -0.0168 \\ 0.0209 & 0.0048 & 0.0463 & -0.0000 & -0.0003 & -0.0036 \\ -0.0008 & 0.0048 & 0.0001 & -0.1117 & 2.8295 & 28.3055 \\ -0.0320 & 0.0141 & 0.0044 & -1.4366 & -3.8310 & 6.1072 \\ -0.0008 & 0.0004 & 0.0001 & 2.0870 & 0.4629 & 4.6026 \end{bmatrix}$$

$$E_1^1 = \begin{bmatrix} -0.7635 & -0.92961 & 0.262 & 0.0021 & 0.0130 & 0.182 \\ 0.3662 & -0.1611 & -0.0506 & 6.3614 & 23.9162 & -30.1539 \end{bmatrix}$$

$$L_2^1 = \begin{bmatrix} -0.1122 & 2.8354 & 28.3771 & -0.0001 & -0.0006 & -0.0000 \\ -1.4294 & -3.7906 & 6.5614 & 0.0050 & -0.0110 & -0.0020 \\ 2.0841 & 0.4618 & 4.6342 & -0.0001 & -0.0006 & -0.0000 \\ -0.0002 & -0.0003 & -0.0033 & -0.0027 & 0.0170 & 0.2833 \\ 0.0002 & -0.0013 & -0.0155 & -0.0904 & -0.5034 & 0.0552 \\ -0.0003 & -0.0000 & -0.0008 & 0.0189 & -0.0073 & 0.0461 \end{bmatrix}$$

$$E_2^1 = \begin{bmatrix} 6.2805 & 23.4285 & -35.4077 & -0.0567 & 0.1259 & 0.0226 \\ -0.0070 & 0.0144 & 0.1790 & 0.9328 & 5.5496 & -0.2361 \end{bmatrix}$$

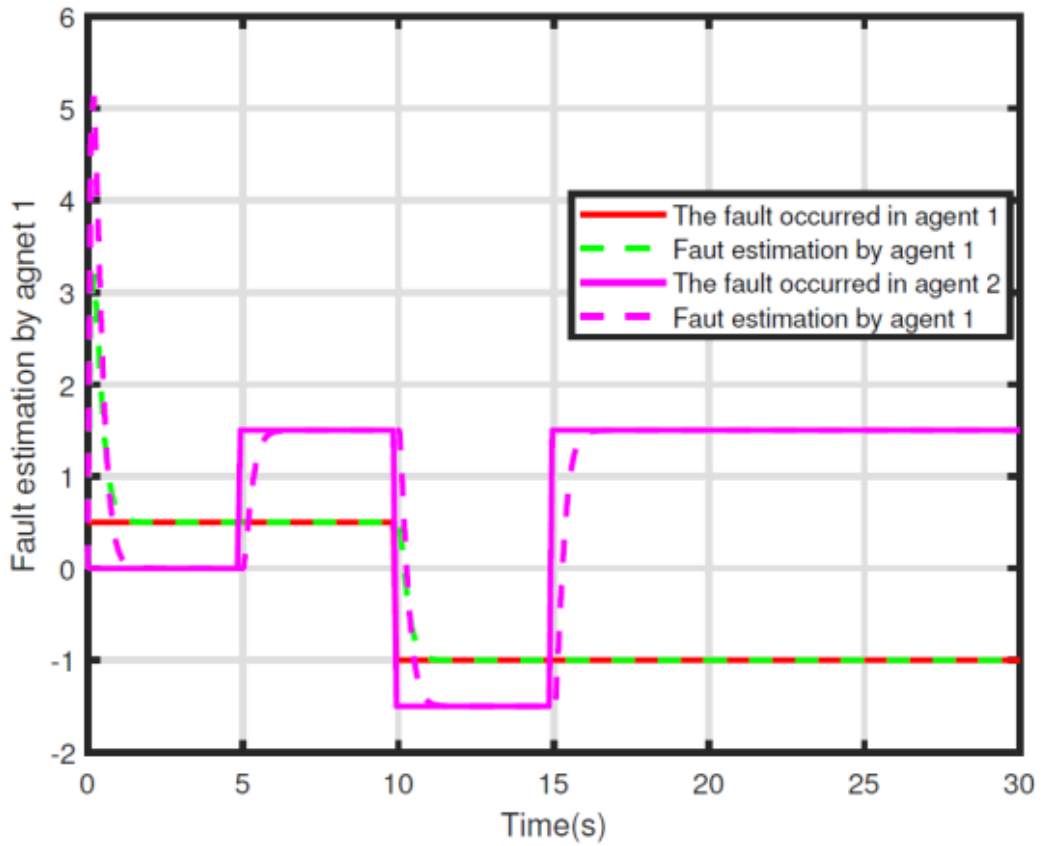


Fig. 2. The performance of agent 1 in estimating its fault and its neighbors

$$\Omega_1 = 1.0e + 02 * \begin{bmatrix} 1.0007 & 0.0866 & 0.1028 & 0 & 0 & 0 \\ 0.0866 & 0.0088 & 0.0108 & 0 & 0 & 0 \\ 0.1028 & 0.0108 & 0.0144 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0007 & 0.0866 & 0.1028 \\ 0 & 0 & 0 & 0.0866 & 0.0088 & 0.0108 \\ 0 & 0 & 0 & 0.1028 & 0.0108 & 0.0144 \end{bmatrix}$$

$$\lambda = 1.5098.$$

We take the external disturbance $\omega_i(t) = 0.2\sin(4\pi t)e^{-3t}$ for $t \in [0, 30]$. Here, to demonstrate the performance of the above observer gains, the simulation results of the distributed Fault Estimation and the event-triggered mechanism are illustrated in the following Figures. The simulation results are established over a sample time $0.1s$. For agent 1, the simulation results are demonstrated in Fig. 2.

It can be seen from figure 1 that agent 1 estimates the occurred fault within itself and in its neighbors. The simulation results to investigate the efficiency of the proposed approach for agent 2 can be seen in Fig. 3.

Fig. 4 depicts the effects of the event-trigger mechanism, the results of event-triggered times in this agent show that the executions of date are needed to exchange.

Based on the discussion in section 4, each agent should be able to estimate the possible fault within itself and its neighbors by considering the event-triggered communication. According to the communication graph in Fig. 1, agent 2 should estimate the possible fault in this agent and its neighbors (i.e., agent 2 and agent 3). This goal is met in Fig. 3. The simulation results for agents 3, 4, and 5 can be seen in Figs. 5-8.

Note that in the introduction, we discussed the case in which it is necessary to install the observer in only some agents instead of all agents to reduce the cost of implementing observers or controllers. Therefore, to validate the proposed approach in this direction, based on the communication graph in Fig. 1, the observers constructed in agents 1 and 3 should be able to estimate the occurred faults in agents 1, 2, 3, 4, and 5. According to Figs. 2 and 5, we can achieve the mentioned goal by implementing the proposed approach. According to (29), where the fault in agent 3 is 0 , the Fault Estimation of agent 3 by agent 5 is approximate to 0 , which Fig. 8 proves. To investigate the effects of the proposed event-triggered strategy on the system performance, interevent intervals of agents 2, 3, and 5 were illustrated in Figs. 4, 6, and 9.

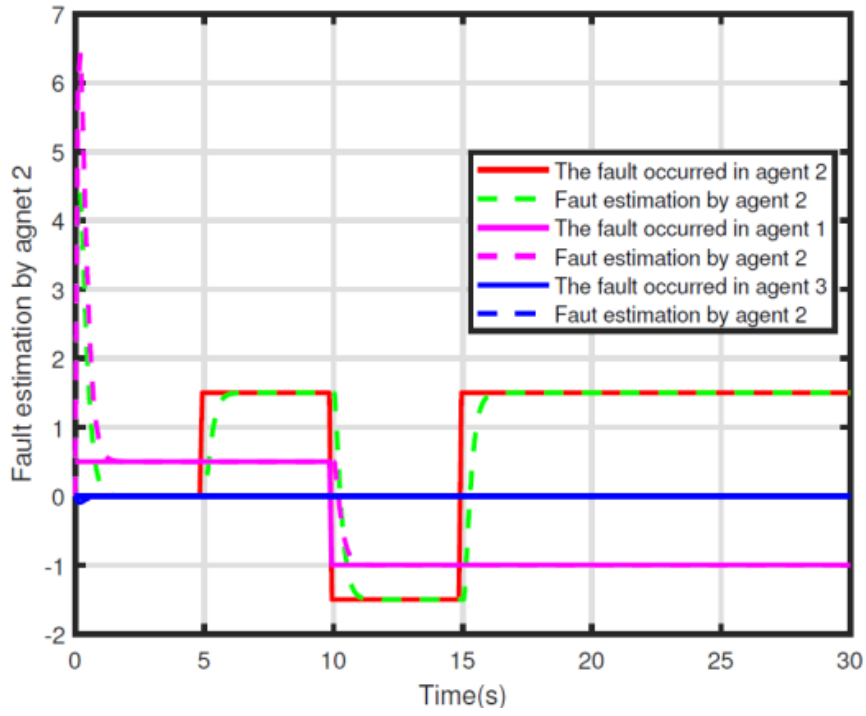


Fig. 3. The performance of agent 2 in estimating its fault and its neighbors.

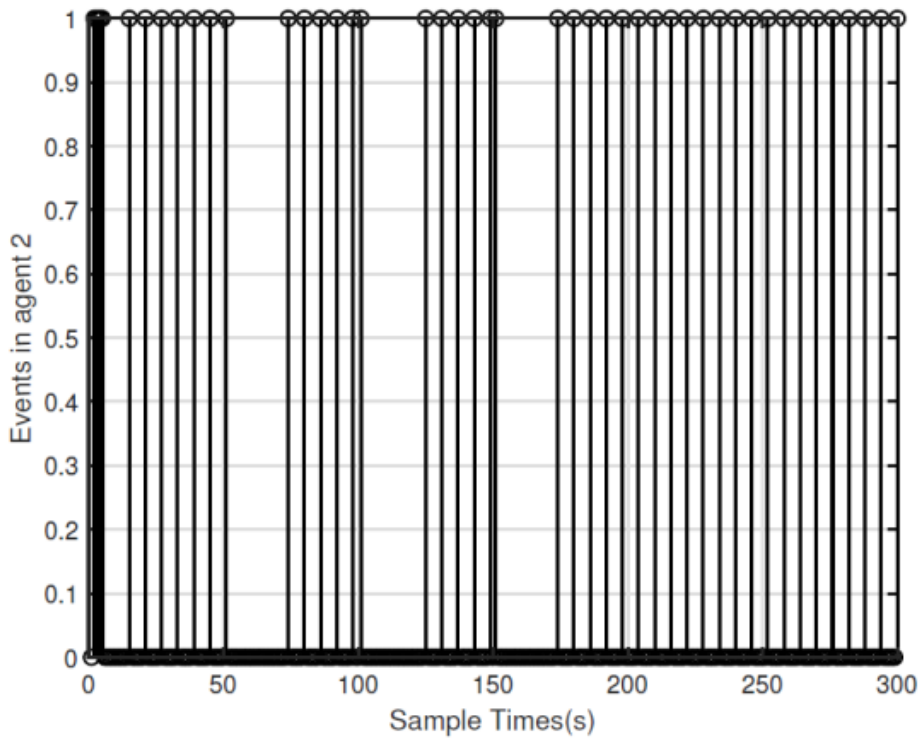


Fig. 4. Interevent intervals of agent 2

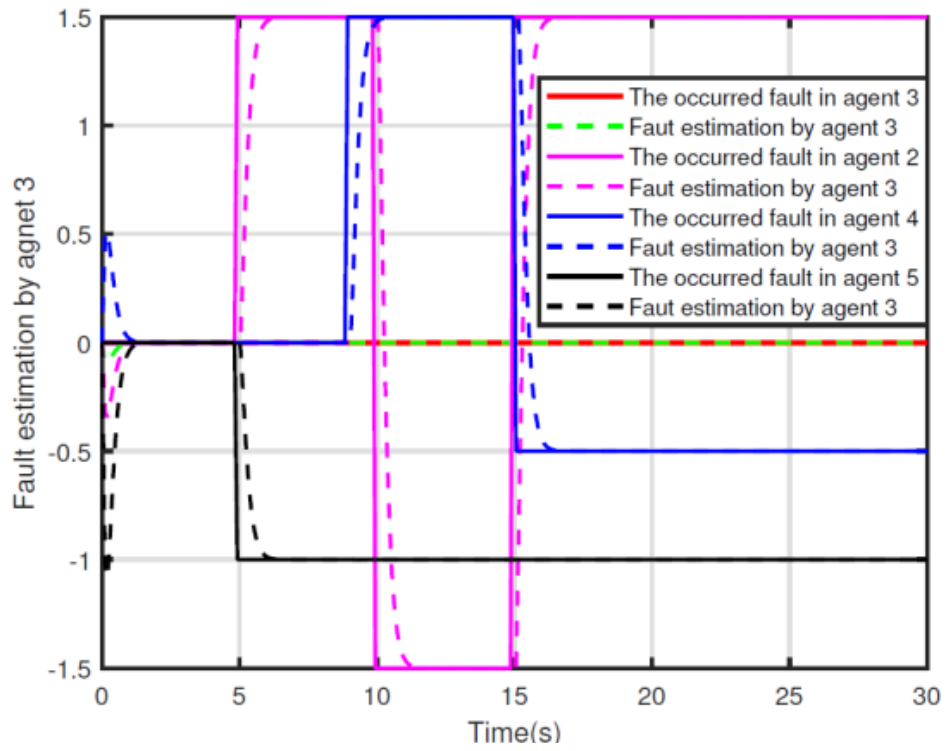


Fig. 5. The performance of agent 3 in estimating its fault and its neighbors

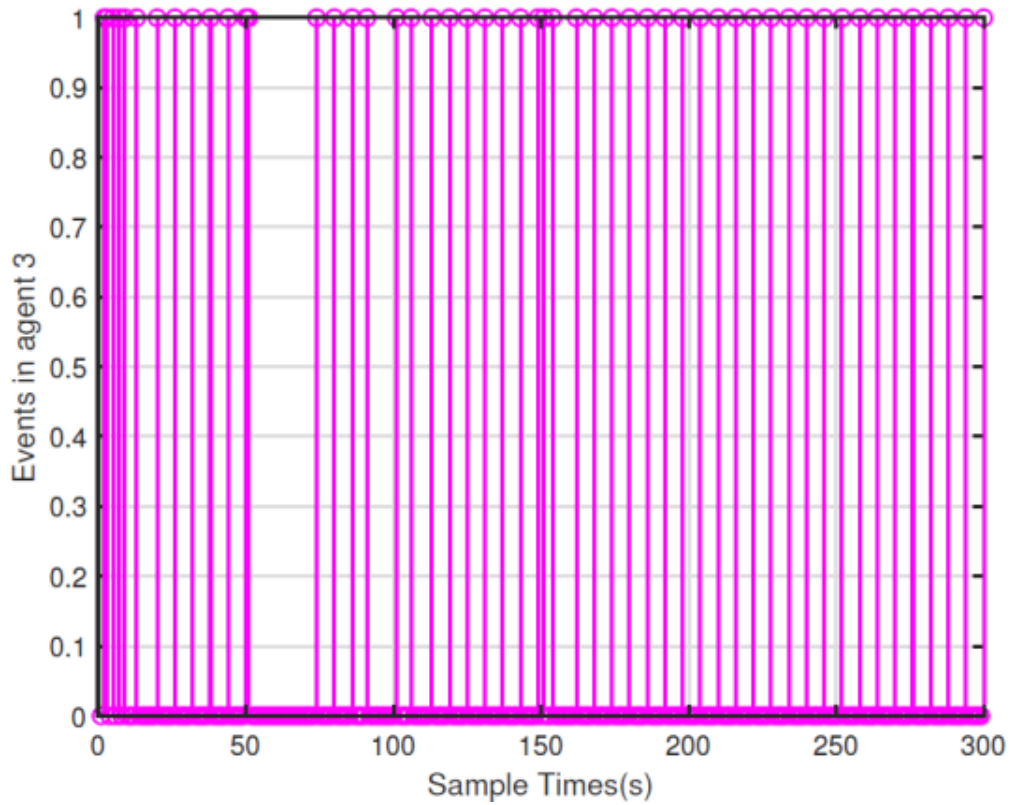


Fig. 6. Interevent intervals of agent 3

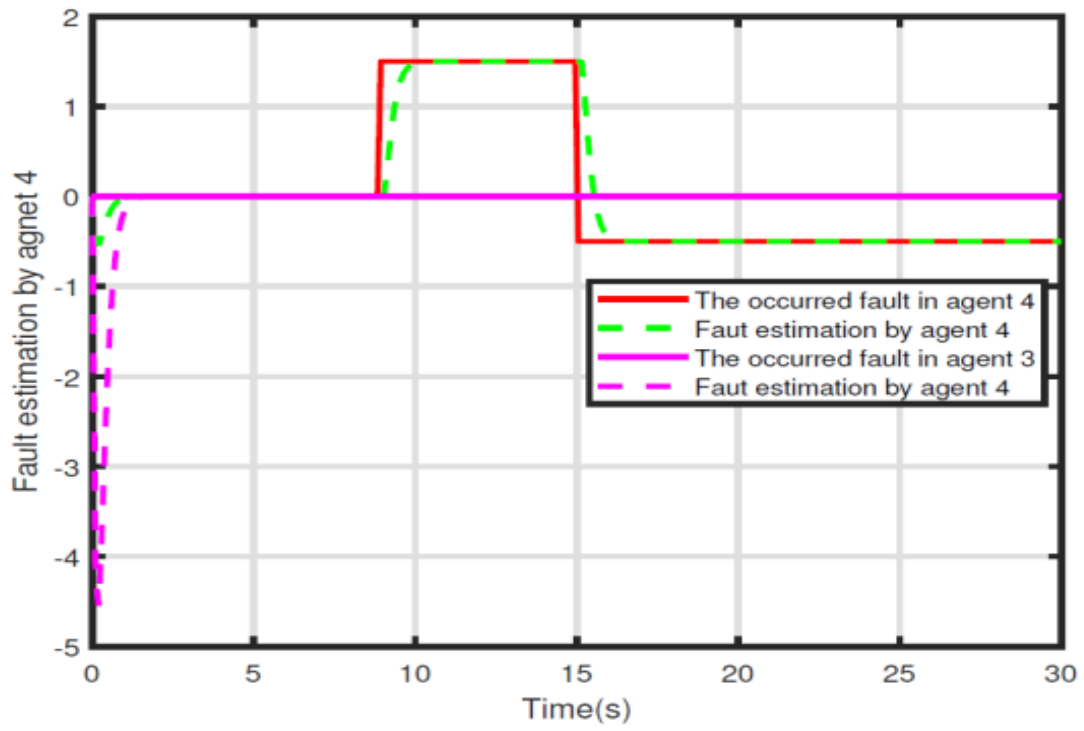


Fig. 7. The performance of agent 4 in estimating its fault and its neighbors

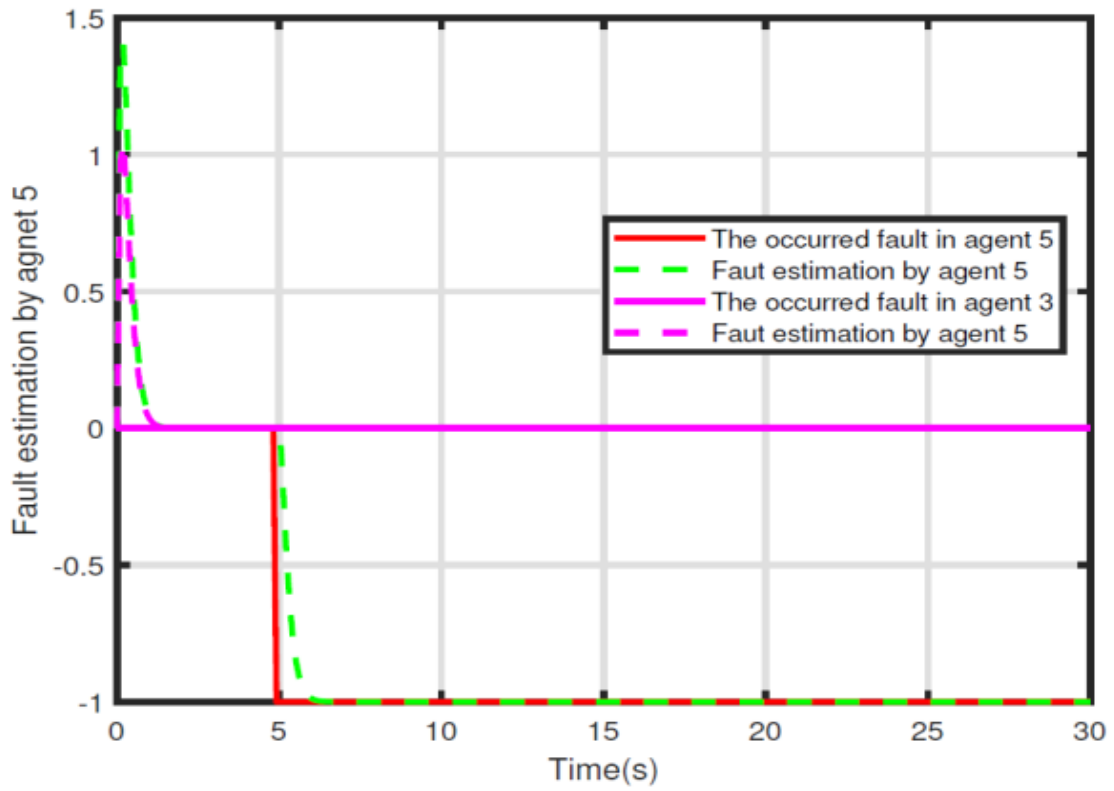


Fig. 8. The performance of agent 5 in estimating its fault and its neighbors

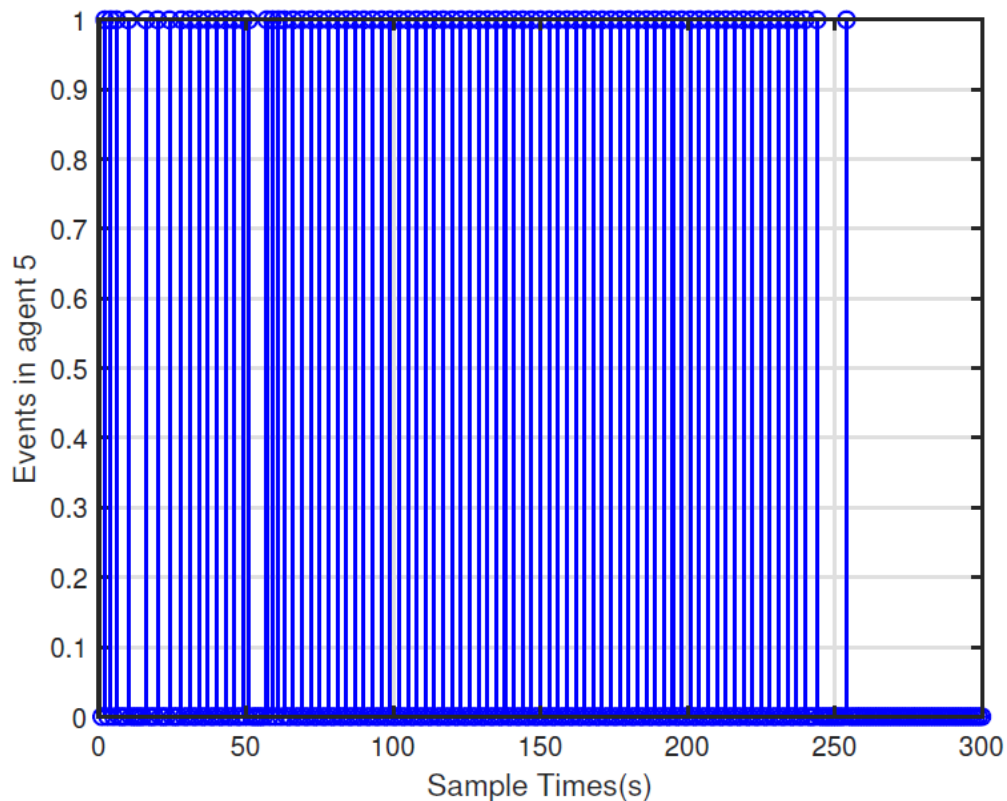


Fig. 9. Interevent intervals of agent 5

5- Conclusion

In this paper, a distributed Fault Estimation with an event-triggered strategy for the networked systems was presented. In the presented approach, using H_∞ the performance index, the influence of the external disturbances are alleviated as much as possible. Each agent can estimate the occurred fault in this agent and in its neighbors according to a given communication topology. These characteristics are evaluated by a numerical example in the simulation results section. The main results of this paper can be an extension to the following issues in order to improve the proposed method for the practical systems in future works: (1) Multi-agent systems with quantization effects [35, 36]; (2) Communication delays and communication link faults [37]; (3) For systems with uncertainty and mode reality mismatch. Considering these challenges can improve the performance of the proposed approach for further practical applications.

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HOW TO CITE THIS ARTICLE

Farshad Rahimi, Hero Shahi, Neighborhood-Based Event-Triggered Distributed Fault Estimation Observer for Multi-Agent Systems, *AUT J. Elec. Eng.*, 54(2) (2022) 281-294.

DOI: [10.22060/ej.2022.21368.5470](https://doi.org/10.22060/ej.2022.21368.5470)



