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Hybrid Robust Model Predictive Based Controller for a Class of Multi-Agent Aerial dynamic Systems

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ABSTRACT: The decentralized control of a multi-agent system with leader-follower consensus is investigated. The system is formulated in graph theory, and a general configuration for L-F formation is proposed. The goal for the formation is defined to track the predefined trajectory in the presence of high-frequency noise. The controller for the system is proposed on the basis of a model predictive-based controller. Different scenarios for a multi-agent system are considered, which lead to the linearization of the plant. Meanwhile, external structured disturbances are considered in the system. The novelty of the present paper addresses the gap between optimal controllers and robust controllers. The robustness of optimal controllers is not verified in the optimality of MPC controllers. Thus, a tube MPC theory is proposed to increase the robustness of the interacting noise system. Consequently, the optimal controller maintains robust throughout the existence of external disturbances and high frequency noises. Meanwhile, the closed-loop multi-agent response is investigated in the presence of external bounded disturbances. Next, The hybrid controller is designed for the formation. The switches take place between MPC and Tube-MPC controllers for each agent. On the other hand, hard constraints on control input and its variations and soft constraint on graph structures and topology of the multi-agent system are submitted. At length, the stability proof is considered for the closed loop multi-agent system. Finally, the simulation results demonstrate the formation results and the proposed controller can also satisfactorily deal with the high-frequency noise with hard and soft constraints.

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1- Introduction

The multi-agent formation control architecture is considered a well-studied problem throughout the literature [1-3]. In this respect, the main formation architectures in pre-studied literature are mainly categorized as leader-follower and leaderless architectures. The multi-agent system is subject to input saturation hard constraints and task constraints, as well as topology constraints [3]. Meanwhile, the satisfaction of constraints in the existence of more than one synchronized agent in a single formation is investigated as a challenging distributed problem. Collision and obstacle avoidance are examples of the aforementioned physical and task constraints, covered successfully in the previous literature [4], which describes a squared, fully actuated class of dynamic systems with model predictive based control architecture in a multi-agent formation manner. The definition leads to a Distributed Model Predictive Controller (DMPC) design. The terminal set assumed positively invariant; thus, the constraints are satisfied due to obstacle and collision avoidance. It is discernible that the MPC architecture is considered a widely applicable optimal control strategy for multi-agent systems. The capability of handling constraints in MPC consensus is investigated broadly for multi-agent problems [5-7]. Nevertheless, most of the described approaches are applicable only for squared systems. The underactuated control systems are considered to be even more challenging in MPC architecture. Thus, a vast domain of modifications and noble combination of different control theories with MPC formulation is studied in the previous researches [8-11]. The main challenge in this problem with optimal manner is to keep the robustness of the system. Therefore, multi-layer MPC architectures are designed to overcome this problem. Generally, this proposed control architecture uses a supervisory approach consisting of an MPC in the upper layer with a robust controller in the bottom layer, [8]. A quadcopter formation control problem is investigated in [8]. As it is widely known, practical Unmanned Aerial Vehicles (UAV) are considered underactuated systems. Hence, feedback linearization is a must for the system. A robust feedback linearization is designed in the bottom layer of the aforementioned supervisory MPC architecture and the hybrid theory provides the switching between the robust and optimal controller in the flight scenario. The exact identification of system parameters is considered a challenge for the practical implementation of this control scheme. Therefore, an adaptive DMPC is represented [9]. The previous works of literature are concerned with the adaptive control for linear systems with unknown parameters. The identification in

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this manner is based on the DMPC method. As predicted, the adaptive DMPC architecture is affected by high computational costs due to the online manner for both formulations. Explicit MPC is considered as an offline MPC architecture [10]. The noted approach is widely applicable in mobile robots and embedded systems. The possibility of implementation of the noted approach on limited hardware resources and for quite complex and discontinuous control functions is proved [10]. Since simultaneous consideration of the actuator faults and model uncertainties is considered a challenging aspect in the late researches, the previous literature covered the subject acceptably as in [11], and an underactuated quadcopter dynamic model is investigated. An intelligent Fault-Tolerant Control architecture (FTC) is proposed, and the method is divided into passive FTC and active FTC with an adaptive intelligent inference learning engine. Next, the external disturbances are added to the previous research [12]. The intelligent trajectory tracking control approach is designed for an aircraft, and internal and external disturbances are presented in the model. The neural network-based model predictive control scheme is presented, and the prediction engine is updated in each iteration based on a proposed online sequential structure. The robustness of optimal MPC controller is assumed as an important sequence of controller design. A novel approach in the definition of tubes for the noted prediction-based controller is developed in [13], and the proposed tube MPC method is designed for a class of discrete linear systems. The noted approach is developed in order to design an optimal robust controller for a class of nonlinear dynamic systems with unmodelled dynamics.

The multi-agent system is considered in the present paper. The theory of multi-agent system is proposed by graph theory in the previous literatures. In [25], the formation of UAV is considered, and two types of communications are proposed. First, the information sharing strategy between two UAV in the time-slot is investigated. Second, adoption of a dynamic time slot allocation scenario is considered for the UAV. Afterwards, a formation problem for UAV is considered based on a novel back-stepping control theory. In [26], the Lyapunov method is investigated to further verify the effectiveness and stability of the multi-agent system. Additionally, a comparison between the noted controller and the MPC controller is proposed. The formation technique for the system is considered a decentralized controller. The theory of hybrid system is combined with formation control problem in the previous literatures [27]. The formation topology switching is considered in [27]. Consequently, a time-varying formation tracking problem is considered. The novel TVFT protocol is proposed for the formation, and the feasibility of the problem is investigated. A flying experiment for three air vehicles is considered to test the simulation results. A generalized approach to multi-agent formation control problem is proposed in [28]. The distributed sliding mode control is modified for the time-varying UAV formation problem. The configuration of multi-agent system is considered to be leader-follower, and the decentralized controller is proposed for the noted system. The described theory utilized the neighboring relative information. As an example of advanced formation problems, the formation of UAV swarm system with multiple implicit leaders is considered in [29]. In the noted approach, leaders are implicitly integrated into the swarm and can be influenced by formation and other information from neighboring followers. The existence of multiple leaders in formation is considered of importance in the previous literature. Decentralized and centralized control configurations are considered the basic form architecture of the formation controllers. Nevertheless, the distributed control architecture is considered a novel approach to the problem. In [30], a formation maintenance problem is proposed and the distributed controller is investigated. The UAV swarm system operates in the absence and existence of obstacles in the literature. The relative change of relative positions between the agents is the main error dynamics in this research, and the controller is designed in order to minimize the relative error between the agents.

In this paper, a class of dynamic models is considered as an underactuated, highly coupled, and unstable system. The class of systems mentioned above is considered a six degree of freedom rigid mass in the three-dimensional space. The system has applications in aerial vehicles and underwater moving unmanned mobile robots. Meanwhile, it is assumed that the desired command of actuation and the actual amount of actuations are not equal. The noted assumption leads to a fault-tolerant system. The control architecture for the system is defined in the definition of Model predictive-based control architecture. The controller is considered an optimal controller with a prediction horizon over the model behaviors in the following steps in a discrete manner. Meanwhile, the system is highly nonlinear and coupled. In order to drive a relatively simple dynamic model, a robust controller is needed for the system to overcome the unmodeled dynamics in the system. The high computational cost of deep online methods and lack of robustness in the noted previous researches forces interest in tube-MPC methods. MPC and robust MPC contain advantages and disadvantages; thus, a new control approach is needed to privilege the advantages and avoid the disadvantages of the control architectures. Consequently, a hybrid controller is proposed in this paper. The noted control approach switches between MPC and Tube-MPC knowingly based on a guard condition. In the present paper, a switching controller is initiated to optimize the efforts of the control system. Accordingly, the selected control scheme is separated into two fragments: MPC and tube-MPC. The MPC controller consists of an optimal control structure, and the tube-MPC is considered a robust optimal controller. In this paper, the results are extended to outline a hybrid robust model predictive-based controller for a multi-agent system to perform an admissible path tracking result in the presence of external disturbances or, more precisely, high-frequency noises, unmodelled dynamics, and faulty actuators.

The paper proceeds as follows. A dynamic model of a single agent system is designed in the next section. The MPC controller is implemented in the nominal system. The control system is generalized to tube MPC in the presence of unexpected noise. A switching signal is defined, and the hybrid controller is introduced in the last part. The simulation results illustrate the capability of this controller in the fourth section.

2- Dynamics

The linear dynamic systems in the form of $\dot{x} = Ax$ have a unique solution. The nonlinear dynamic system covers a wide variety of solutions and behaviors[14]. In this paper, a class of nonlinear dynamic systems is considered as formulated in (1).

$$\dot{x} = f(x) + g(x,u) \tag{1}$$

Where, $f(x) \in \mathbb{R}^n$, $g(x,u) \in \mathbb{R}^n$ and $\dot{x} \in \mathbb{R}^n$ is the state vector of the state space notation for a general nonlinear system.

In this paper, a general class of nonlinear dynamic systems of aerial vehicles are considered in formulation (1). Generally, the aerial vehicle is six degrees of freedom, and the state-space model for these dynamic robotic systems is considered in the vector space R^{12}. The distribution of input signals in the autonomous dynamic model above described the actuation class of the system. With a physical sight, the described model is mainly categorized as under or fully actuated. In this paper, it is assumed that the system solution exists and it is unique. Thus, the open-loop system is considered Lipschitz in definition below:

$$|f(x) - f(y)| \le K|x - y| \tag{2}$$

First, a single agent dynamic is considered next, and the multi-agent system based on graph theory is proposed afterwards. As a practical approach, Unmanned Aerial Vehicles are precise examples of the noted dynamic systems. The noted vehicles are considered six degrees of freedom rigid body in space, and the system is presented as an underactuated system [8]. In the quadcopter configuration, the control inputs to the system are the sum of thrusts and three aerodynamic torques.

2- 1- Single-agent UAV System Dynamics

The position of the air vehicle with respect to the inertial frame is denoted respectively by $X = [x, y, z]$, and attitude dynamic is denoted by $\Theta = [\varphi, \theta, \psi]$ which represent the roll, pitch, and yaw angles, respectively. The main equations of motion in planar and rotational movement is presented as follows:

$$m\ddot{\varkappa} = -mge_3 + RF_Te_3 \tag{3}$$

$$\dot{R} = RS(\Omega) \tag{4}$$

$$J\dot{\Omega} + S(\Omega)J\Omega = \tau_i \tag{5}$$

Furthermore, F_T is considered resultant of the input forces, and τ_i s are considered input aerodynamic torques to the rotational subsystem.

$$F_T = T_1 + T_2 + T_3 + T_4 \tag{6}$$

$$\tau_1 = -lT_2 + lT_4 \tag{7}$$

$$\tau_2 = lT_1 - lT_3 \tag{8}$$

$$\tau_3 = -CT_1 + CT_2 - CT_3 + CT_4 \tag{9}$$

where, $T_i = [T_1, T_2, T_3, T_4]^T$ represents the thrust force generated by the rotor at the end of the arms. l is the length of the arm, and C is the aerodynamic drag coefficient.

$\varkappa = [x, y, z]$ and $\Omega = [\dot{\varphi}, \dot{\theta}, \dot{\psi}]$. . . F_T and τ_i are the system control inputs. The localization of an aerial vehicle in a single coordinate is considered challenging. Thus, the rotational matrix R in (4) represents the conformal transportation matrix from initial coordinate to body coordination. Finally, the $S(.)$ represents the SO3 transformation in the noted space.

Matrix R is represented in (10), and the transformation $S(.)$ is represented in (11), respectively.

$$R = \begin{bmatrix} 1 & 0 & -\sin(\theta) \\ 0 & \cos(\varphi) & \cos(\theta)\sin(\varphi) \\ 0 & -\sin(\varphi) & \cos(\theta)\cos(\varphi) \end{bmatrix} \tag{10}$$

$$S(\Omega) = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \tag{11}$$

Consequently, the nonlinear equations of motion of the noted single agent can be formulated as a six second order ordinary differential equations as follows [8]:

$$\ddot{x} = (\cos\psi\sin\theta\cos\varphi + \sin\psi\sin\varphi)\frac{F_T}{m} \tag{12}$$

$$\ddot{y} = (\sin\psi\sin\theta\cos\varphi - \cos\psi\sin\varphi)\frac{F_T}{m} \tag{13}$$

$$\ddot{z} = -g + (cos\theta cos\varphi)\frac{F_T}{m} \tag{14}$$

$$\ddot{\varphi} = {}^1/_{I_x}\left(\tau_1 + (I_y - I_z)\dot{\theta}\dot{\psi}\right) + n_1 \tag{15}$$

$$\ddot{\theta} = {}^1/_{I_y}\left(\tau_2 + (I_z - I_x)\dot{\varphi}\dot{\psi}\right) + n_2 \tag{16}$$

$$\ddot{\psi} = {}^{1}/_{I_z}\,(\tau_3 + (I_x - I_y)\dot{\theta}\dot{\varphi}) + n_3 \tag{17}$$

The topology of the aerial vehicle is assumed symmetric; thus, the inertial moment matrix is presented by a diagonal matrix $J = diag\{I_x,\ I_y,\ I_z\}$. here I_x, I_y, and I_z represent the moments of inertia align the directions x,y,z, respectively. g, m are the gravity acceleration and air vehicle mass.

The noted dynamic system in (12-17) is considered as a highly nonlinear, coupled, and underactuated model, as it can be observed.

As predicted, this paper aims to control this class of dynamic systems to track a predefined path. Meanwhile, the stability of the closed-loop system has to be grantees. The assumptions below are considered to partition the general problem into different scenarios that lead to the linear model of the system.

Assumption 1: Hovering and slow maneuvers: the attitude angles in this scenario vary around zero with a variation of fewer than 5 degrees.

Assumption 2: Aggressive navigation: the attitude angles in the aggressive movements can be greater than the previous assumption, but vary less than 30 degrees.

With the assumptions above, the external structured uncertainties and noises to the attitude state variables are denoted by $n_i\ (i \in [1,2,3])$. The unmodeled dynamics and nonlinear terms are denoted structured uncertainties.

First, the resultant thrust force is designed as follows:

$$F_T = \frac{m(\ddot{z} + g)}{cos\varphi cos\theta} \tag{18}$$

Second, replacing F_T by (18) in (12-14), and the altitude state is decoupled due to feedback linearization. Nevertheless, the aforementioned approach is not considered as a robust control scheme for the noted system. As it can be seen, the system is considered an under-actuated system. Meanwhile, the decoupled attitude dynamic is fully actuated by three independent moments. Consequently, in order to design a position tracking controller, a practical approach to the system is to design an attitude controller for the system. The attitude dynamic is tuned based on the horizontal acceleration of the system. Therefore, the desired attitude angles in (19-20) are tracked perfectly. In order to track the predefine reference trajectory, the attitude dynamics should be faster than position dynamics.

$$\theta^d = \arctan\left(\frac{\ddot{x}^d cos\psi + \ddot{y}^d sin\psi}{\ddot{z} + g}\right) \tag{19}$$

$$\varphi^d = \arctan\left(\tan(\theta^d).\ \cos(\theta^d)\right) \tag{20}$$

2- 2- Multi-agent UAV Dynamics

In this paper, the formation problem with the leader-follower structure is considered. Generally, the noted problem can be broken down into flexible or rigid sub-formation structures. As a result, the general configuration is considered a rigid form of known quantities of agents. States of follower agents are completely specified once the leader states are investigated in (L-F) standard configuration. In order to formulate the problem, graph theory is introduced to describe the formation. The basic definition of formation is presented as follows:

Definition 1: The sensing range of an agent is the maximum sensing distance that an agent cannot obtain the states of its neighbors whose inter-distances are more than the sensing range.

Definition 2: The agent that can follow the reference formation trajectory directly is considered a leader agent.

Definition 3: If an agent is not a leader, it is a follower and cannot reach the reference trajectory directly.

Definition 4: An agent that has no neighbors in its sensing range is considered an isolated agent.

Generally, in a multi-agent formation problem, a graph with sets of vertices and edges represents the interaction of agents. The adjacency, degree, and leader matrices are denoted respectively by G^A, G^D, G^L. Interacting topologies and agents are considered in the Adjacency matrix. The degree matrix represents the number of interacting agents in a neighbourhood, and the leader matrix obtains the leader agent. The Laplacian matrice G is introduced in (21).

$$G = G^D - G^A + G^L \tag{21}$$

For instance, consider a four-agent formation problem. The first agent is assumed as the formation leader. The graph contribution is described in (22-24).

$$G^A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} = w_{ij}^a \tag{22}$$

$$G^D = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} = w_{ij}^d \tag{23}$$

$$G^L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = w_{ij}^l \tag{24}$$

According to (22), the second agent is considered an interacting agent with third and fourth agents. According to

(23), the first agent has two neighbors, and the fourth agent has three. The configuration above is based on assumptions, as follows:

Assumption 3: The connection between two neighboring agents is a two-way connection; thus, the adjacency matrix is a symmetric squared matrix with zero diagonal components.

Assumption 4: The Laplacian matrix is a positive definite matrix, [22].

Eventually, a multi-agent formation error measurement relationship of the system is introduced in (25-26). The desired trajectory is assumed to be reachable only for the leader agent. Meanwhile, the follower agents are designed to follow the leader position states.

$$\frac{d}{dt}\left(Z_{ij}\right) = \frac{d}{dt}\left(\begin{bmatrix} x_i - x_j \\ \dot{x}_i - \dot{x}_j \end{bmatrix}\right) = \begin{bmatrix} \dot{x}_i - \dot{x}_j \\ \ddot{x}_i - \ddot{x}_j \end{bmatrix} \tag{25}$$

$$\frac{d}{dt}\left(Z_{io}\right) = \frac{d}{dt}\left(\begin{bmatrix} x_i - x_d \\ \dot{x}_i - \dot{x}_d \end{bmatrix}\right) = \begin{bmatrix} \dot{x}_i - \dot{x}_d \\ \ddot{x}_i - \ddot{x}_d \end{bmatrix} \tag{26}$$

Where Z_{ij} represents the overall error measurement for each couple of neighboring agents in formation, and indices $i, j \in [1,2\ldots,N]$ represent the number of agents. x_d represents the desired trajectory of the leader, and Z_{io} is the error measurement relation for the leader agent.

In order to represent (25-26), regardless of being a leader or a follower, the formation weighted error measurement is presented for the multi-agent system, as follows:

$$Z^f = \varepsilon_i^a \varepsilon_j^a z_{ij} + sgn\left(w_i^l\right) z_{io} \tag{27}$$

where Z^f is the weighted error measurement for the formation, ε_i^a is the permutation in Leibniz notation, and sgn represents the sign operation.

Consider the system in (1). The output for the system is represented for each agent in the formation as follows:

$$y = \mathrm{C}x \tag{28}$$

$$\mathrm{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\sin(\theta) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos(\varphi) & \cos(\theta)\sin(\varphi) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sin(\varphi) & \cos(\theta)\cos(\varphi) \end{bmatrix} \tag{29}$$

Consequently, the weighted error is rewritten in the output notation for the system:

$$\begin{aligned} &\varepsilon_i^a \varepsilon_j^a z_{ij} + sgn\left(w_i^l\right) z_{io} \\ &= \left[\left(G^D - G^A + G^L\right) \otimes I_6\right].\left[I_n \otimes C^t\right]x - \left[G^L \otimes I_6\right].\left[I_n \otimes \begin{bmatrix} R \\ \dot{R} \end{bmatrix}\right] \end{aligned} \tag{30}$$

3- Control

In this paper, a hybrid controller for a multi-agent system is considered. A linear decentralized MPC controller is proposed for the primary layer controller. Next, the structured disturbances and specific high-frequency noise are injected into the system. As mentioned earlier, the noted primary layer controller is not considered a robust approach. Therefore, a secondary control scheme is considered for the system. Followed by representing a Tube-MPC theory. The guard condition is defined for the known switches between MPC and Tube-MPC controllers. Afterwards, the hybrid architecture is proposed. The secondary controller is activated based on switching conditions. The guard conditions are based on error measurement for the formation. Reference tracking and topology stabilization are considered the final aim of the control theory in the presence of structured uncertainties and external noises.

3- 1- The Primary MPC Controller

MPC is an advanced and generalized approach. The noted approach can satisfactorily manage the nonlinear manner of the system, and same as other optimal control theories, it can deal with constraints as well. Generally, a constraint optimization problem is considered in the model predictive-based controller. In each iteration, an optimal control problem based on a cost function is solved. The future control input vector is determined in the previous input-output basis for the system.

As mentioned earlier, the system is underactuated. Thus, an attitude controller is considered as a practical approach to the system. The attitude dynamics in (15-17) are expected to follow the desired set point represented in (19-20). The attitude error dynamics in closed-loop yields:

$$e_\Theta = \Theta - \Theta^d \tag{31}$$

$$\ddot{e}_\Theta = \ddot{\Theta} - \ddot{\Theta}^d \tag{32}$$

The attitude error dynamic state space for each agent is defined as follows:

$$\dot{e}_\Theta = f_i(\varphi, \theta, \psi) + g_i(I_x, I_y, I_z, \tau_i)u \tag{33}$$

where f_i and g_i $i \in [1,2,3]$ represent the governing nonlinear state equations of the attitude dynamics system, and vector u represents the control input vector, respectively. Therefore, the path planning problem is now demonstrated by a regulation control problem. The error dynamics in (33) present the attitude dynamics error for the system. Based on the feedback law for under-actuated UAV dynamics system, the error in position dynamics is proportional to the error dynamics in attitude dynamics. Hence, the minimization of attitude error leads to the minimization of position error for the under-actuated UAV system.

The optimal cost function of the system is considered an error minimizer and control effort minimizer cost for a regulation problem. The main difference of the cost function in (34) and a conventional Linear Quadratic Regulator (LQR) cost function is the nonlinear manner of the equations in (31-32). A general form of the cost function is represented in (34).

$$J(k) = \sum_{i=1}^{N_p} \| e_\Theta(k + i|k) \|_q^2 + \sum_{i=0}^{N_u} \| \Delta u(k + i|k) \|_\xi^2 \quad (34)$$

Where N_p and N_u represent the prediction horizon and control horizon, respectively. In the noted approach, the cost function in (34) is reformulated in order to define a regulator control problem in the least squared optimization problem, as follows:

$$J(k) = \left(e_\Theta(kN_p)\right)^T Q \left(e_\Theta(kN_p)\right) + \left(\Delta u(kN_c)\right)^T \xi \left(\Delta u(kN_c)\right) \quad (35)$$

where Q and ξ are diagonal positive definite matrices.

The cost function in (34) is considered as a least-square optimization problem, and is reformulated as follows:

$$f(x) = \sum_{i=1}^{m} r_i^2(x) \quad (36)$$

where each r_i is a smooth functional from $\mathbb{R}^n$ to $\mathbb{R}$. Each r_i is referred to as a residual. The final aim is to minimize the residual in (36) in order to propose a tracking controller for each agent.

To minimize the noted cost function, the functional $J(k)$ derivatives can be expressed in terms of the jacobian matrix, an $m \times n$ matrix of the first partial derivatives of the residuals. Where m is the number of functionals and n is the number of variables. In the described aerial vehicle dynamics system, $m = 3 and\ n = 6$.

The first derivative of the residuals is expressed as follows:

$$\nabla f(x) = \sum_{i=1}^{6} r_i \nabla \mathrm{r}_i \quad (37)$$

The second-order optimality sufficient condition is represented as follows:

$$\nabla^2 f(x) = \sum_{i=1}^{6} \nabla r_i \nabla \mathrm{r}_j^T + \sum_{i=1}^{6} r_j \nabla^2 r_j \quad (38)$$

As mentioned earlier, the residuals in (36) are nonlinear. Thus, a nonlinear least squared problem is needed in order to find the optimal solution for the cost function. A nonlinear least square algorithm is proposed based on the Gauss-Newton method for the MPC controller. The nonlinear objective function that exploits the basis in the gradient ∇f and Hessian matrix $\nabla^2 f$ is described, and the Gauss-Newton approach is presented. The method can be viewed as a generalized algorithm of Newton with a line search strategy [23]. In each line search method iteration, the algorithm moves along a predefined direction, and the iteration is given by:

$$x_{k+1} = x_k + \alpha_k {P_k}^{GN} \quad (39)$$

In each iteration, the MPC controller performs a line search in the direction ${P_k}^{GN}$, and the step length α_k is required to satisfy the strong wolf condition in (40):

$$f\left(x_k + \alpha_k {P_k}^{GN}\right) \le f(x_k) + c_1 \nabla f^T {P_k}^{GN} \quad (40)$$

where the constant c_1 is chosen in the interval $c_1 \in (0,1)$.

As mentioned earlier, the desired nonlinear least square optimization method is considered the Gauss-Newton method. Accordingly, the search direction ${P_k}^{GN}$ is obtained by solving the Newton equation below:

$$\nabla^2 f(x_k) {P_k}^{GN} = -\nabla f(x_k) \quad (41)$$

The noted optimization algorithm in (39-41) has two main disadvantages in computation. First, since the Hessian matrix may not always be positive definite, ${P_k}^{GN}$ may not always be a descent direction. Second, the computation of individual Hessians in (41) is considered a high-cost computation. Hence, with the definition of Jacobian matrix in (37) and approximation bellow, the noted approach is simplified.

$$\nabla^2 f(x_k) \approx \mathfrak{J}^T(x_k) \mathfrak{J}(x_k) \quad (42)$$

Table 1. Hybrid Subsystems

Number	Controller	Guard Condition
(1)	$u_i = v_i$	$S \le 0$
(2)	$u_i = -K_p(e_i) - K_d(\dot{e}_i) + v_i$	$S > 0$

Where $\Im(x_k) = \nabla f(x_k)$ is the Jacobian matrix for the nonlinear least-square optimization problem.

3- 1- 1- Convergence of the Gauss-Newton Optimization Method

The convergence of the Gauss-Newton optimization method is considered quadratic. Consequently, the optimal solution is evaluated for the system and the extremal points of tracking error minimization cost is obtained. Accordingly, the tracking of the reference trajectory is considered feasible [23].

3- 2- Tube-MPC Controller

In this section, the noise augmented system is proposed to be controlled. The nonlinear state in (12-17) is linearized due to assumptions (1,2) and extended to a noise augmented system, as follows [7]:

$$x^+ = Ax + Bu + w \tag{43}$$

Where $w \in \mathbb{R}^{12}$ represents the vector of external noises and structured uncertainties due to unmodelled dynamic and simplifications in assumptions (1,2) in (12-17). Accordingly, the control authority is separated into two individual systems: a portion that steers the nominal, noise-free system to origin with first layer primary MPC controller, and a portion that compensates for deviation from the system in the presence of noise and noted uncertainties. The new control signal for each agent in multi-agent formation is defined as follows:

$$u_i = K(x_i - z_i) + v_i \tag{44}$$

The nominal multi-agent system output is presented by z_i based on the first layer primary MPC controller with control signal v_i, . More precisely, a state feedback controller along with a primary MPC controller are augmented to define the tube-MPC architecture. The second layer of the noted controller is modified in order to eliminate the structured uncertainties and reject the augmented noise in the single-agent system on the basis of Proportional-Derivational (PD) controllers, and the noted approach is generalized for the multi-agent system in the sense of decentralized control architecture as follows:

$$u_i = -K_p(x_i - z_i) - K_d(\dot{x}_i - \dot{z}_i) + v_i \tag{45}$$

Where K_p and K_d are tunable positive definite matrices. The noted matrices are designed to derive the error dynamic into the origin.

3- 3- Hybrid Controller

The hybrid input-output automaton is defined as follows [10],[8]:

$$H = (Q, X, U, Y, Init) \tag{46}$$

Where the $(Q, X, U, Y, Init)$ represents a set of discrete variables, a set of continuous states, a set of continuous or discrete control inputs, a set of output variables, and a set of initial states for the noted hybrid system, respectively. In this approach, the hybrid automaton model is a composite actor containing the continuous and discrete dynamics. The guard condition and reset relation on a transition are defined as a function of output error, as illustrated in (47):

The system switches between two predefined subsystems in Table 1, if the predefined threshold is trespassed by guard condition in (47).

$$S \triangleq \| \Theta \|_\infty - \epsilon \tag{47}$$

4- Stability Proof

The stability of the closed-loop system is considered in two stray parts as follows: First, the stability of the MPC controller is considered, then the stability of multi-agent UAV system is considered.

As mentioned earlier, an optimization problem is considered in each iteration for MPC controller. Therefore, the controller is considered stable, if and only if the optimal solution for optimization problem provided in each iteration. Accordingly, based on the description in 3.1.1, the convergence of the Guass-Newton optimization method is the key to stability proof for the MPC controller [23]. Consequently, the assumptions (5,6) are proposed in order to utilize the convergence rate for the optimization problem.

Assumption 5: The bounded disturbances and noises are assumed to be fully estimated with predefined accuracy.

Assumption 6: The reference trajectory is twice differentiable for time interval without the necessity of an observer existence.

After the stability proof for the MPC controller, for single UAV system, the stability proof for the multi agent is remained intact. The equivalent linear system for a multi-agent system is presented as (48-49):

$$\dot{\tilde{\boldsymbol{x}}} = A\tilde{\boldsymbol{x}} + B\bar{\boldsymbol{u}} \tag{48}$$

$$\dot{\tilde{\boldsymbol{x}}} = A\tilde{\boldsymbol{x}} + B\bar{\boldsymbol{u}} \tag{49}$$

Where $\tilde{\boldsymbol{x}} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n, \dot{\boldsymbol{x}}_1, \dot{\boldsymbol{x}}_2, \ldots, \dot{\boldsymbol{x}}_n]$, $\boldsymbol{u} = [\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_n]$
The state-space form in (48) is represented as follows:

$$\dot{\tilde{\boldsymbol{x}}} = \begin{bmatrix} 0 & I_n \\ 0 & 0 \end{bmatrix} \tilde{\boldsymbol{x}} + \begin{bmatrix} 0_{n\times n} \\ I_n \end{bmatrix} \bar{\boldsymbol{u}} + \begin{bmatrix} 0_{n\times n} \\ I_n \end{bmatrix} \boldsymbol{\delta_1} \tag{50}$$

Where $\boldsymbol{\delta_1}$ presents the nonlinear terms for multi-agent system.

Error measurement signal in (31-32) is represented as follows for the leader-follower formation:

$$\boldsymbol{z_{ma}} = \left(I_2 \otimes G_{T_k}\right).\left(\tilde{\boldsymbol{x}} - [R_i^d, \dot{R}_i^d] \otimes 1_n\right) \tag{51}$$

Where $G_{T_k} = \left[G^D - G^A + G^L\right]$ and R are the reference trajectory. The positive definite interaction matrix is presented by G_{Tk}, in each time step $T_k = [t_k, t_{k+1})$. Unknown switching signal is defined over the control input for the system by $\sigma_c[\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_n] = [\sigma_c(\boldsymbol{u}_1), \sigma_c(\boldsymbol{u}_2), \ldots, \sigma_c(\boldsymbol{u}_n)]$ for n sample switches.

The $\boldsymbol{u_i}$ is defined as a proportional and derivational gain for tube-MPC control signal:

$$u_i = -\sigma_c\left(K\left(\sum_{j=1}^{n} W_{ij}^a\left(\begin{bmatrix} e_j \\ \dot{e}_j \end{bmatrix}\right) + W_i^l\left(\begin{bmatrix} e_i \\ \dot{e}_i \end{bmatrix}\right)\right)\right) \tag{52}$$

Consequently, the switched control input with an unknown switching signal for the multi-agent system is represented (53).

$$\overline{\boldsymbol{u_s}} = -\sigma_c((K \otimes I_n).(I_2 \otimes G_{Ts}).\bar{\boldsymbol{e}}) \tag{53}$$

Where $\bar{\boldsymbol{e}} = [e_1, e_2, \ldots, e_n, \dot{e}_1, \dot{e}_2, \ldots, \dot{e}_n]$ is considered as error collective vector for the formation.

The definition of the error dynamics system in (33) leads to the final result for the error open-loop dynamic model (54).

$$\dot{\bar{\boldsymbol{e}}} = \begin{bmatrix} 0 & I_n \\ 0 & 0 \end{bmatrix} \bar{\boldsymbol{e}} - \begin{bmatrix} 0 \\ I_n \end{bmatrix} \sigma_c([k_1 G_{Tk} \quad k_2 G_{Tk}]\bar{\boldsymbol{e}}) \tag{54}$$

Rewriting the (54) into the linear dynamic system for $\dot{\bar{\boldsymbol{e}}} = A_{HS}\bar{\boldsymbol{e}}$, we have:

$$A_{HS} = \begin{bmatrix} 0 & I_n \\ -k_1 D_n G_{Tk} & -k_2 D_n G_{Tk} \end{bmatrix} \tag{55}$$

The characteristic equation for (56) is represented by:

$$A_{HS}\underline{\boldsymbol{p}} = \lambda\underline{\boldsymbol{p}} \tag{56}$$

$$(\lambda^2 I_{\boldsymbol{n}} + \lambda k_2 D_n G_{TS} + I_n k_1 D_n G_{TS})\underline{\boldsymbol{p}} = 0 \tag{57}$$

The term $k_1 D_n G_{TS}$ is positive definite as the G_{TS} is positive definite. For the noted multi-agent system, the A_{HS} matrix is Hurwitz in the characteristic equation. $(A_{HS})^T P + P(A_{HS}) = -Q$ is considered as a trivial common Lyapunov function for the system.

5- Simulation Results

To examine the performance and stability of the controller, a circular reference trajectory is defined for the class of aerial vehicle dynamics system. As a practical example, a formation problem of quadrotors is proposed in the simulation scenarios. The mass of quadcopter is equal to 2kgs, and other numerical values of the system parameters are taken from [8]. The prediction horizon and control horizon are defined 20 steps and 15 steps, respectively. The time step for the simulation is defined 0.01s during all simulations. The optimization engine is defined with step size $\alpha = 0.2$.

The desired trajectory in the case of nominal flight condition is satisfactorily tracked in Fig.1.

As mentioned before, the controller is designed based on the attitude regulation for each UAV. Hence, the attitude error signal is manifested in Fig.2 in the case of nominal flight condition. It should be noted that the attitude dynamics is faster than translational dynamics, thus the convergence of attitude states is considered in Fig.2.

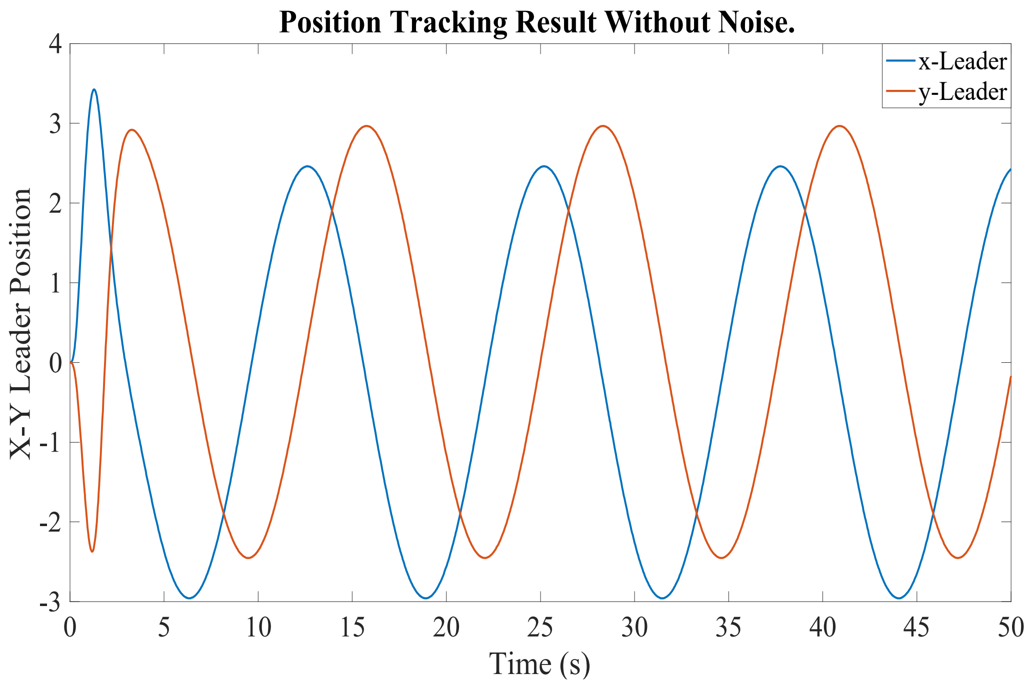

Fig. 1. Nominal position tracking of the system

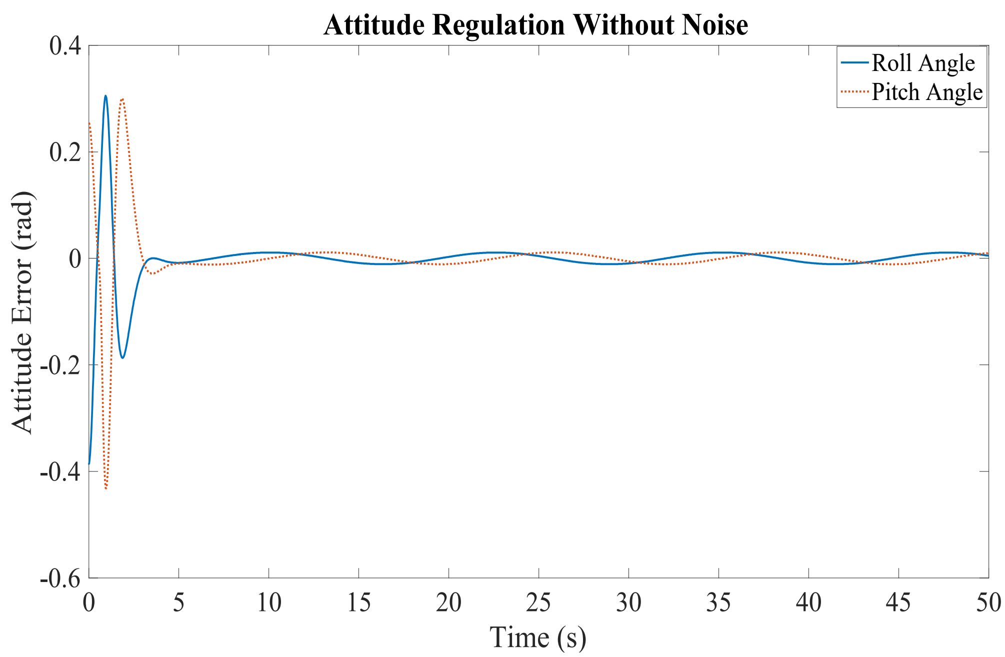

Fig. 2. Attitude Error Signal

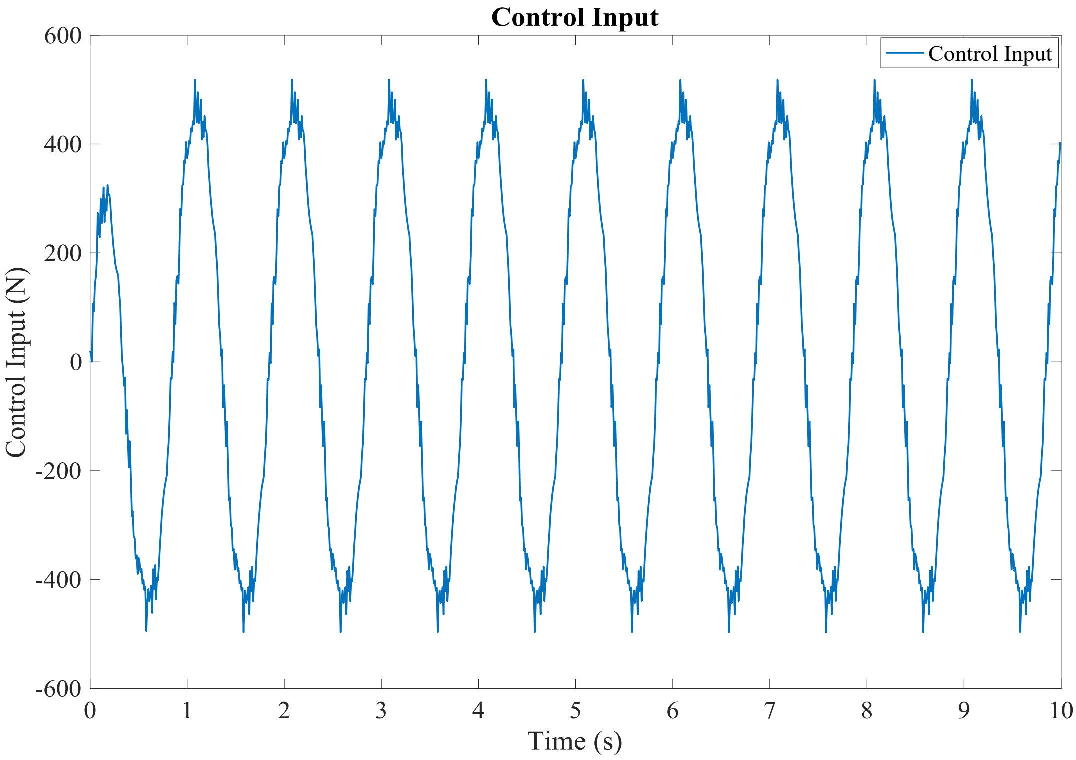

Fig.3. Constraint control signal

The control effort signal in the aforementioned scenario, is considered for a single agent system as Fig. 3. The control signal is bounded by hard constraints as it shows in the following figure.

Fig.4 represent the optimization conclusion in each iteration during the trajectory tracking.

Afterwards, the Fig.5 represents the multi-agent nominal system. The desired circular trajectory is tracked by each UAV as the figure illustrates.

The desired trajectory in the presence of single white noise in each attitude state and external constant disturbances in transitional dynamics are considered next. The simulation result illustrates that the primary MPC is not able to eliminate the attitude error utterly in the interrupted system. As anticipated, based on the guard condition, system switches in the 8^{th} second of the simulation and Fig.6 describes the effectiveness of tube-MPC in order to reduce the noise effect in an acceptable manner.

Robot formation position tracking result is considered in Fig.7.

Subsequently, Fig.8 illustrates the phase portrait of switching multi-agent systems in the case of noise presence.

Eventually, the comparison between the results of the primary MPC controller and the modified method mentioned in the paper is contemplated in Fig.9. As expected, the capabilities of the linear MPC controller are affected significantly in the presence of external disturbances, namely the high-frequency noises.

Fig.10 demonstrates a better comparison between the noted controllers. The attitude error is compared in the figure as mentioned earlier.

Finally, the comparison between the proposed hybrid robust controller and the conventional PID controller is considered in fig.11.

Obviously, the PID controller is not able to track the predefine trajectory precisely in the presence of external disturbances. Meanwhile, the robust proposed controller tracks the predefine reference trajectory in an acceptable manner.

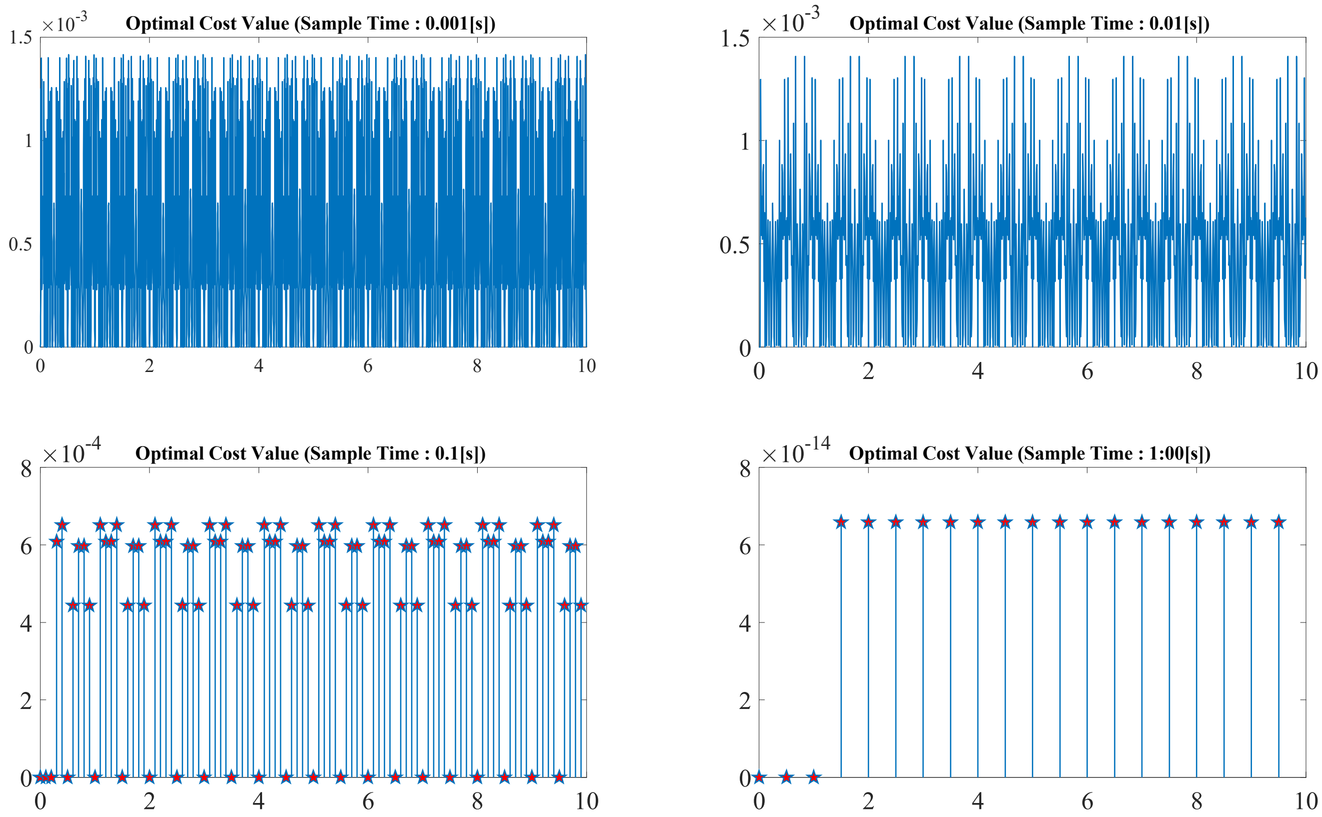

Fig.4. Optimal position calculation result

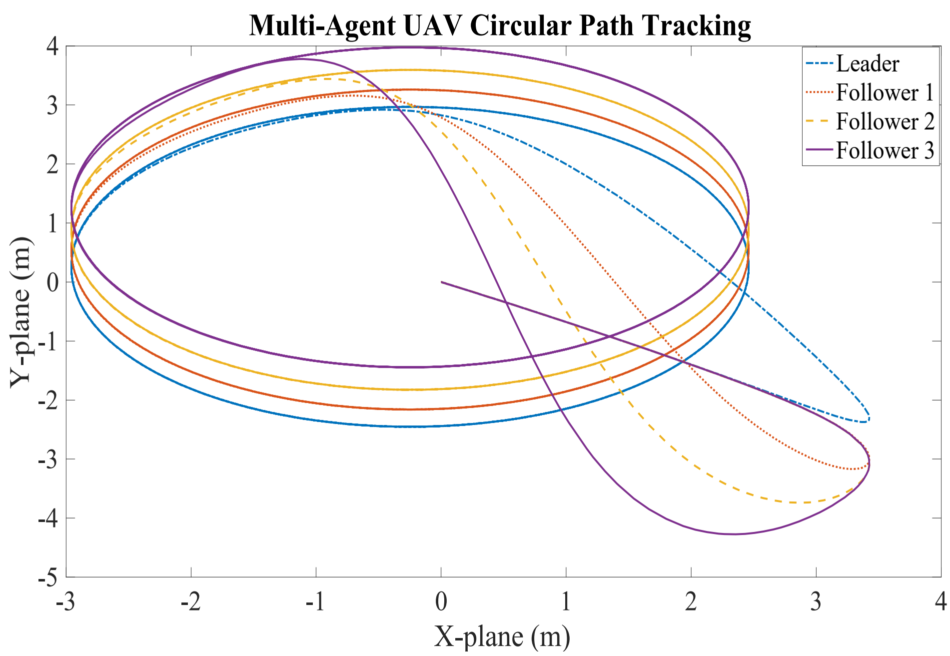

Fig.5. Multi-Agent nominal system

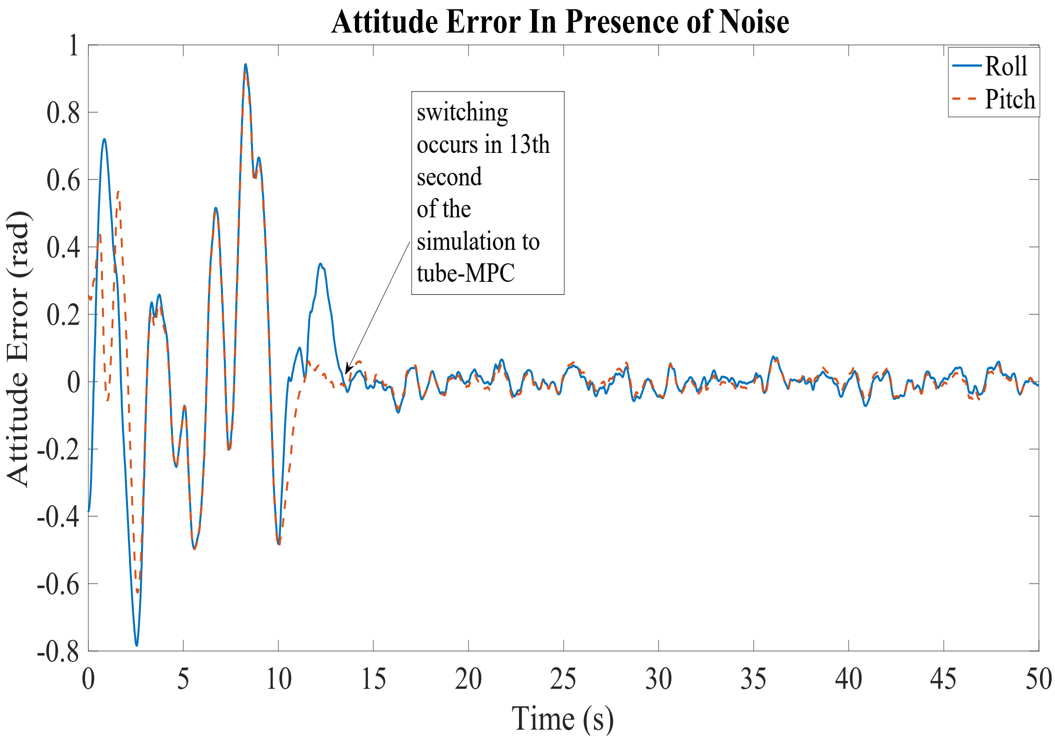

Fig. 6. Interrupted system attitude error

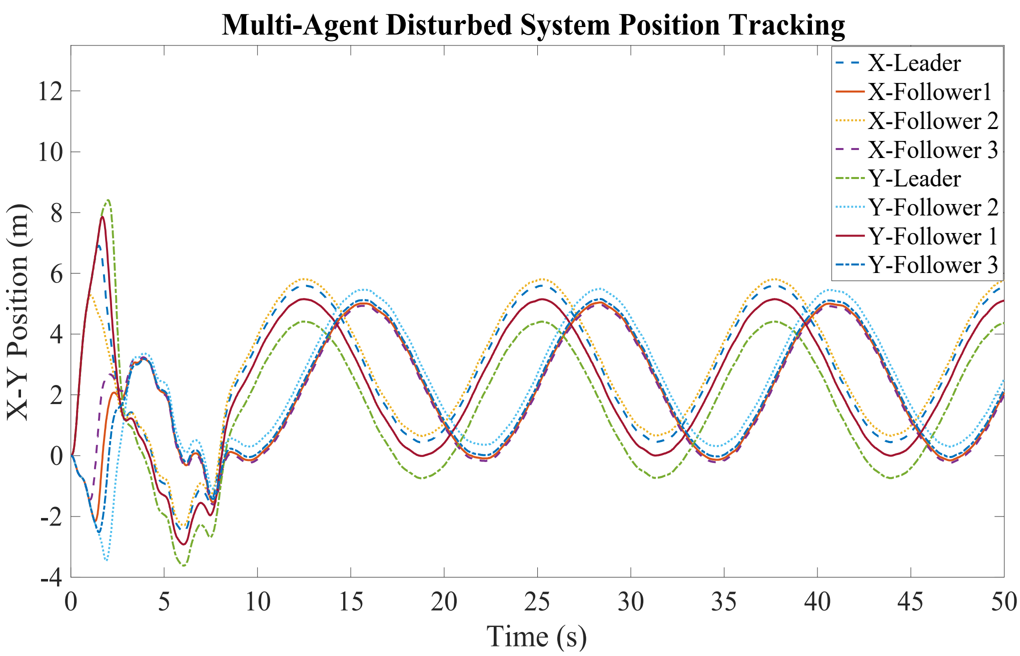

Fig. 7. Formation position tracking of interrupted system

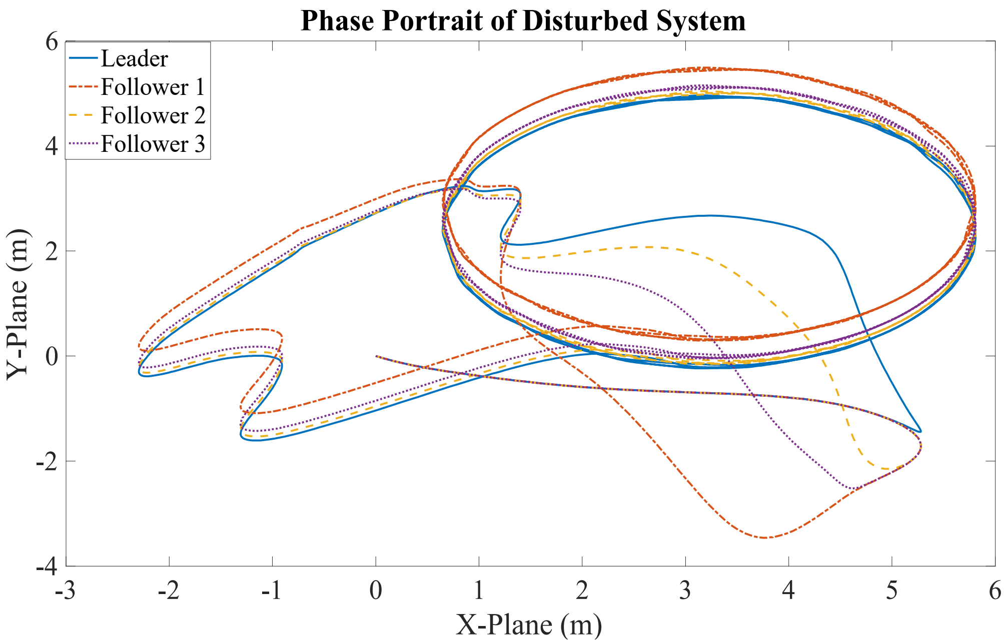

Fig. 8. Multi-UAV interrupted system phase portrait

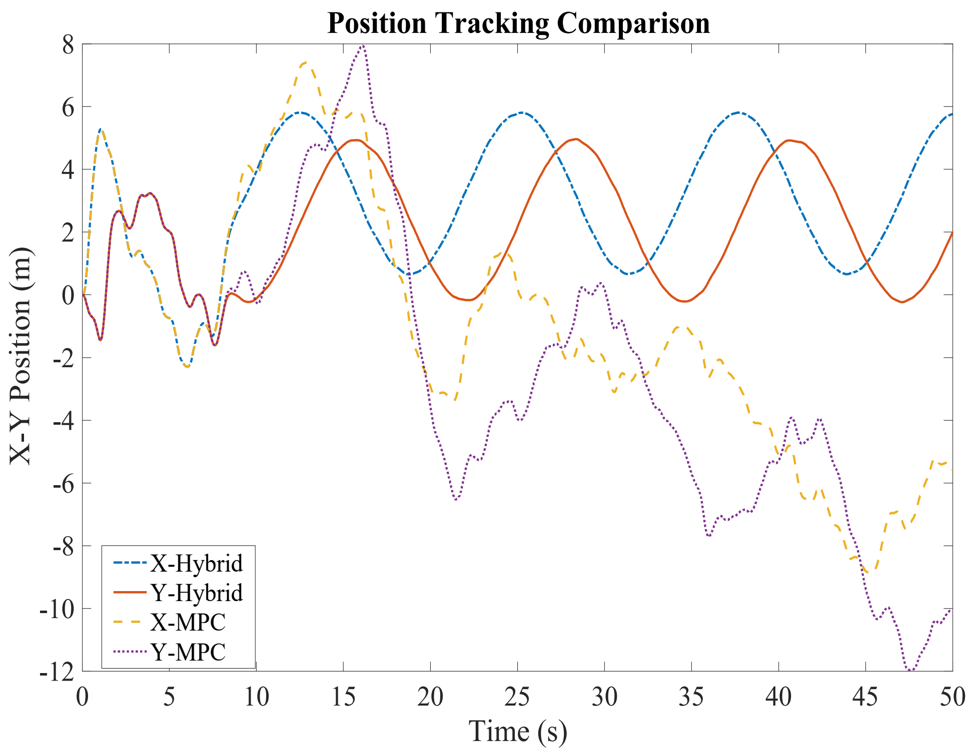

Fig. 9. Comparison between MPC and hybrid controller

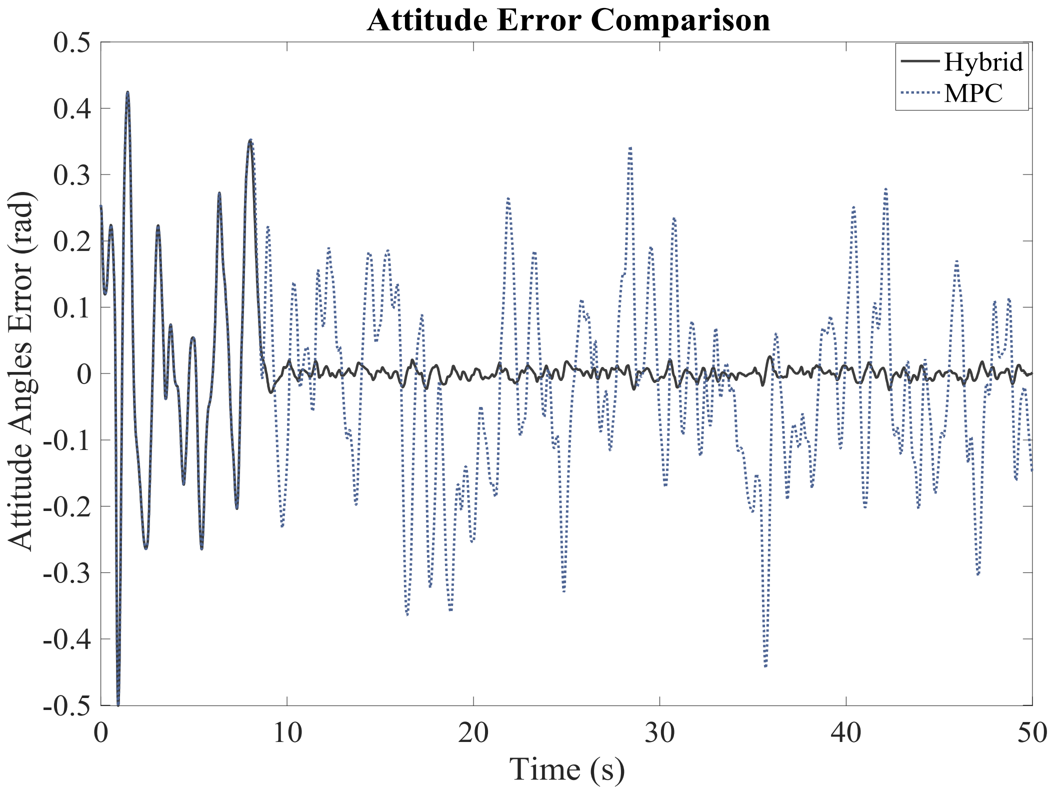

Fig. 10. Error reduction comparison

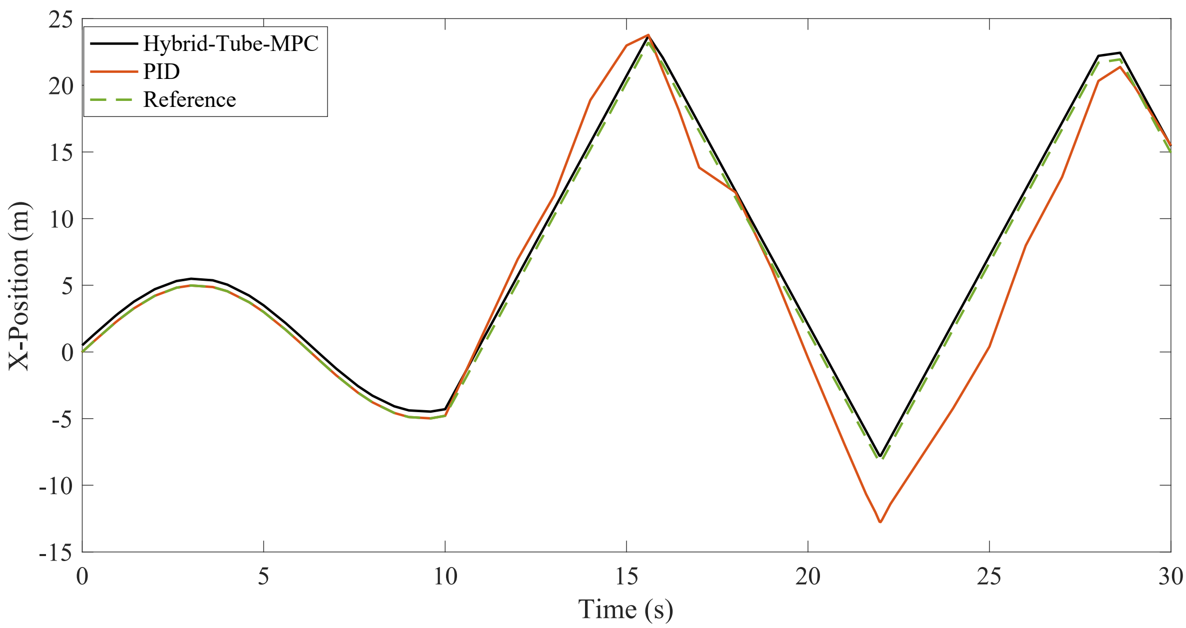

Fig.11. Comparison of trajectory tracking

6- Conclusion

A novel switching controller theory based on the tube-MPC and conventional MPC theories was proposed in this research. A primary MPC controller was introduced to control the nominal system in the absence of external disturbances. The simulation results illustrated the acceptable tracking result of the closed-loop nominal system. Afterwards, the interrupted system in the case of external disturbances presence was mentioned and the tube MPC was applied to the noted system. The stability of the controller is provided in two subsections. The stability of MPC optimization algorithm is proposed by the convergence rate and wolf condition criteria. Meanwhile, the stability and safety of the formation is considered by a common Lyapunov function for the swarm system. The desired trajectory was tracked perfectly after the switches took place in the system. The results are verified for the multi-agent system. Inner distance between the agents is kept satisfactorily uniform during the flight scenario. Hence, the noted approach improved the robustness capability of the regular MPC controller simultaneously the optimality of the solution has remained intact. Finally, the comparison between the proposed controller and a conventional PID controller, illustrates the claims above and the advantages of the hybrid controller.

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