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High Frequency Numerical Solution to Evaluate the Impedance of the Vertical Grounding Electrode Using an Accurate Mathematical Approach

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ABSTRACT: Considering the air/earth interface to compute the electromagnetic field at the desired point is the most important problem in transient analysis of grounding system buried to lossy media like earth. In order to consider this issue in the available proposed method and to obtain a solution for the problem, one needs to account an integral solution of the so-called Semmerfeld integral in the cylindrical coordinate system. Analytical solution for such integral is almost impossible which is due to the presence of the oscillating the zeroth-order Bessel function of the first kind, and singularities and branch-cuts in its integrand function. In this paper, we investigate the behavior of the vertical grounding electrode in a high-frequency electromagnetic transient state based on the near field theory using the method of moments (MoM). To compute the electromagnetic field at the desired point, the main problem is calculating the well-known Sommerfeld integral in the cylindrical coordinate system, which its integral kernel includes the zeroth-order Bessel function of the first kind along with some singularities and branch-cuts. Since the analytical solution of this integral is not available in literature, we propose a numerical method, as well as a strategy based on the exact solution for far and near filed calculations. A detailed analysis of the obtained results compared with other techniques are provided to confirm the accuracy and validity of the proposed method.

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1- INTRODUCTION

The most important problem in transient analysis of a grounding system buried to lossy media like the earth is the computation of the electromagnetic field at the desired point. In the last few decades, one of the most important issues in electromagnetic transient state of power systems is investigating the transient behavior grounding systems against transient currents caused by lightning discharge, switching and faults. Due to renewed interest in near field precision calculation, there recently has been a growing number of publications focusing on this topic, see [1-18] for example and references therein.

Predicting the proper performance of a HV substation grounding system in electromagnetic transient states can play a significant role to improve the safety of personnel and correct functioning of protective devices. In addition, the study of minimizing flashover during transients, the insulation coordination of power systems and electromagnetic compatibility in sensitive electronic equipment also provides important insights of the power system [1]. Yet, several methods have been proposed to evaluate the high frequency behavior of a grounding systems. In general, evaluating the electromagnetic transient behavior of a grounding system can be arranged in four main categories. Including:

- Approximate circuit model: In this approach, all conductors of the grounding system are replaced by an equivalent circuit including resistance, capacitance and inductance, and later evaluated by the nodal analysis method. In Ref. [2], an equivalent electric circuit model is proposed to simulate the nonlinear soil ionization phenomenon in the frequency domain for the high-frequency analysis of the grounding system. Ref. [3] presents an efficient solution to obtain transient characteristics of grounding systems by a mathematical approach based on a circuit model. This method is able to evaluate the performance of the grounding system considering the soil ionization effect. In Ref. [4], the issue of computing transient ground potential rise of the grounding grid has been evaluated. The proposed method uses finite element analysis to model the main components of the grounding system containing distributed inductance, capacitance, and leakage resistance to earth. In [5], a generalized nominal-pi circuit model has been addressed to study the grounding system under transient conditions. The proposed method in [6] is able to model a grounding electrode by circuit elements such as resistors, inductors, and capacitors in parallel and/or series (RLC) in the time domain.

It should be noted that the approximate circuit model contains some approximations which produce some limitations.

- Transmission Line Model (TLM): In this procedure, all conductors of the grounding system are replaced by TLM

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which is based on the travelling wave calculations. Ref. [7] states a simple analytical expression based on TLM to compute the impulse impedance of a buried grounding electrode into the earth. In Ref. [8], equations of electromagnetic waves propagation along a grounding electrode have been presented by applying Kirchoff's laws for each small segment as it was well-known in transmission line model theory. The proposed TLM in Ref. [9] is in the time domain and includes electromagnetic coupling effects between the segments of an electrode. Ref. [10] presents a transient model for ground conductors based on the transmission line approach and states its integration in the EMTP software. The proposed method in Ref. [11] investigates the transient behavior of a grounding electrode by state space based on the transmission line approach. In this study, the soil ionization and mutual coupling between the electrode segments have been considered. According to surveys, the mutual coupling between segments of conductors and the correct impact of the air-earth interface are not considered in most of these applications. Consequently, its accuracy and precision will not be desirable.

• Electromagnetic field (EMF) theory model: In Ref. [12], a method based on the EMF theory to assess the transient performance of the grounding systems is presented. In this approach, quasi-static image theory has been applied to calculate the green function which is an approximate solution for considering the air-earth interface. Refs. [13-14] present the half-space Pocklington integral equation to determine the current distribution along the grounding electrode buried in the lossy medium in the frequency domain. In these methods, the impact of the air-earth interface in the green function is considered by the reflection coefficient method which is a computationally more efficient solution for far-field calculation. In Ref. [15], a simple EMF-based method has been proposed to assess the high-frequency behavior of the grounding electrode. This approach uses a modified reflection coefficient for the influence of the air-earth interface which leads to a computation approximation in obtained results. Ref. [16] presents an antenna model of the grounding electrode to calculate transient impedance calculation. The numerical analytical solution is carried out by the boundary element method. Additionally, the impact of the air-earth interface is considered by the simplified reflection coefficient.

The basis of this approach is based on solving the Maxwell's equations with a minimum of assumptions that can solve related problems in time and frequency domains despite the complexity. This approach provides great accuracy on this matter.

• Hybrid method: This method is a combination of circuit methods and electromagnetic field theory which is easier to solve problems than the field theory method but it has a lower accuracy [17-21].

Ref. [17] proposes an approach to present the frequency-dependent impedance of the grounding system in the EMTP. In Ref. [18], a computational procedure has been proposed to determine the frequency performance of grounding systems considering a soil electrical model and the current propagation and attenuation in dissipative media. Ref. [19] introduces a hybrid approach based on a combination of circuit parameters and electromagnetic field theory for transient analysis of grounding systems. In Ref. [20], a numerical approach is presented to analyze transient behavior of the grounding grid which combines the method of the moment with circuit theory and Fourier transform. Ref. [21] uses the simplified image theory formulations to calculate the Sommerfeld integral in the Green function. Therefore, the obtained results have considerable approximation.

Electromagnetic field theory methods are the most accurate procedures in electromagnetic field calculations, but with more complexity in the computations due to existing Sommerfeld integral in the correction term of the Green function to solve the problem of earth/air interface. Several methods have been proposed to solve the Sommerfeld integral in the Green's function. Some researchers have proposed a mathematical methodology, and used the image theory with an approximate reflection coefficient to calculate these integrals, which is valid for remote fields [13,22,23,24]. In order to simplify the electromagnetic field calculation caused by the buried grounding electrode in lossy media, some procedures used the quasi-static equations without considering the effect of the earth/air interface and Sommerfeld integral [25-27]. Based on the electromagnetic field theory model, osther methods have been proposed to compute electromagnetic fields accurately with considering the effect of the earth/air interface which applied to solve the Sommerfeld half-space problem from more complicated methods [25] and [28-30].

The main issue addressed in this paper is the presentation of a simple and accurate numerical method based on the electromagnetic field theory to investigate the high frequency behavior of the vertical grounding electrodes to calculate far and near fields in any frequency. In this mathematical methodology, the electromagnetic fields at any point of the grounding electrode can be calculated by direct solution of Maxwell's equations, and then the voltage and current values can be computed to each point on the grounding electrode with the least approximation. This approach has a number of attractive features:

• Presenting closed-form relations instead of Sommerfeld integral.

• Providing a simple and detailed numerical method to calculate the near and far field.

• Accessing to results with higher accuracy than the existing methods in the literature.

• Possibility to use the proposed method in a wide range of frequencies.

This article has been organized as follows: In section 2, the mathematical framework of this analysis is presented which includes the calculation of the current distribution on the grounding electrode and calculation of the Green function. Section 3 is concerned with the mathematical methodology employed for this study. Section 4 represents the computation flowchart of the proposed method. In Section 5, we present two strategies related to the validity of our approach and the numerical simulations are illustrated. Finally, in section 6, we

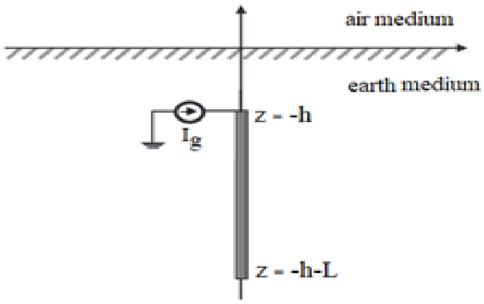


Fig. 1. The vertical grounding electrode buried to the lossy earth

give some concluding remarks.

2- THE MATHEMATICAL MODEL GOVERNING THE ISSUE

This section presents our mathematical framework for the analysis. First, we present the calculation of the current distribution on the grounding electrode. Later, we discuss the calculation of the Green function.

2.1. Calculation of Current Distribution on the Grounding Electrode

Fig. 1 shows a vertical grounding electrode with length of L and radius r, at depth h from the lossy media like the earth medium that is fed by a high frequency wave (Ig).

The basic relation for calculating the distribution of the longitudinal currents in different segments of the vertical earth electrode is written as Eq. (1):

$$\begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1N} \\ Z_{21} & Z_{22} & \dots & Z_{2N} \\ \vdots & \vdots & \dots & \vdots \\ Z_{N1} & Z_{N2} & \dots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = -\begin{bmatrix} Z_{s1} \\ Zs_2 \\ \vdots \\ Zs_N \end{bmatrix} I_g \tag{1}$$

The matrix $[Z]$ contains the self and mutual impedances between the segments of the grounding electrode and also the column vector $[I]$ including the longitudinal currents of the segments of the grounding electrode. In Eq. (1), the mutual impedance Z_{ij} equals the ratio of V_{ij} to I_j which is calculated by the following equation [1]:

$$Z_{ij} = \frac{V_{Lj}}{I_{Li}} = \frac{1}{4\pi} \times \left(\frac{1}{\sigma_2 + j\omega\varepsilon_2} + \mathrm{j}\omega\mu_2\right) \times \int_{l_i} f_i(l_i)\left[\int_{l_j} f_j(l_j) G(r_{ij}) dl_j\right] dl_i \tag{2}$$

where $f_i(l_i)$ and $f_j(l_j)$ represent the shape function and are defined as Eq. (4). r_{ij} is the distance between the i-th and j-th segments, for calculating the diagonal arrays of the mentioned matrices or self-impedance, radius of the grounding conductors is substitute to distance the source and observation point. In Eq. (2), ω is angular frequency. In

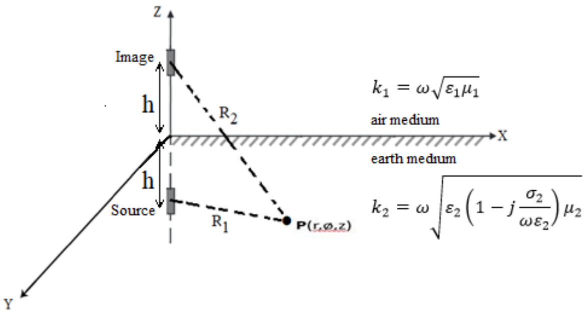


Fig. 2. Image theory for vertical grounding electrode buried to the earth.

addition, μ_2, ε_2 and σ_2 are the soil permeability, permittivity and conductivity, respectively. Therefore, the impedance between the source element and j-th segment can be calculated as follows:

$$Z_{sj} = \frac{V_{sj}}{I_f(j\omega)} = \frac{1}{4\pi}\left(\frac{1}{\sigma_e + j\omega\varepsilon_e} + j\omega\mu_e\right) \times \int_{lj} f_j(l_j) G(r_{sj}) dl_j \tag{3}$$

and

$$f_i = \frac{1}{x_{ie} - x_{is}}, \tag{4}$$

where x_{ie} and x_{is} are the starting and ending points of i-th segment. Additionally, in Eqs. (2) and (3), $G(r_{ij})$ is the Green function and will be described in the next section.

2.2. Calculation of the Green Function

We start with the calculation of the Green function. Fig. 2 shows the image theory for the vertical grounding electrode buried to the earth. Where μ_1, ε_1 and σ_1 are the air medium permeability, permittivity and conductivity, respectively. According to this Figure, the Green function can be calculated by Eq. (5) based on the image theory [1].

$$G(r) = \int_0^\infty J_0(\lambda\rho) e^{-j\mu_2(z-h)} \frac{\lambda d\lambda}{j\mu_2} + \int_0^\infty \frac{\mu_2 - n^2\mu_1}{\mu_2 + n^2\mu_1} J_0(\lambda\rho) e^{-j\mu_2(z+h)} \frac{\lambda d\lambda}{j\mu_2} \tag{5}$$

where $\mu_2 = \sqrt{k_2^2 - \lambda^2}$ and $\mu_1 = \sqrt{k_1^2 - \lambda^2}$. The separation of the effects of the earth/air interface in the above equation, can be estimated as following:

$$G(r) = \int_0^\infty J_0(\lambda\rho) e^{-j\mu_2(z-h)} \frac{\lambda d\lambda}{j\mu_2} + \tag{6}$$

$$\int_0^\infty \left(\frac{\mu_2-n^2\mu_1}{\mu_2+n^2\mu_1}-1+1\right) J_0(\lambda\rho)e^{-j\mu_2(z+h)}\frac{\lambda d\lambda}{j\mu_2}$$

According to [31], the first term of Eq. (5) in cylindrical coordinates is equivalent to the Green's function from the source to the observation point, which is proved in spherical coordinates with the first term of Eq. (7). The second part of Eq. (5) is a combination of the Green's function from the image of the source to the observation point and the effect of the earth/air interface. Therefore, by removing the Green function related to the image of the source and replacing it by closed form relation in the spherical coordinates (second part of Eq. (7)), the remaining part of the relation represents the effect of two media of earth and air (correction term of the Green function). Due to the existence of the specific integrand function, it is not being able to present in spherical coordinates and need to be solved in the same cylindrical coordinates:

$$G(r)=\frac{e^{-jk_2R_1}}{R_1}+\frac{e^{-jk_2R_2}}{R_2}+\underbrace{\int_0^\infty \frac{\mu_2-n^2\mu_1}{\mu_2+n^2\mu_1}J_0(\lambda\rho)\,e^{-j\mu_2(z+h)}\frac{\lambda d\lambda}{j\mu_2}}_{\Delta G_e} \quad (7)$$

Where R_1 is distance between the observation point (P) and source, and R_2 is the distance between the observation point (P) and image. Additionally, h is the buried depth of the grounding electrode. The Correction term of the Green function in Eq. (7) can be written as Eq. (8).,

$$\Delta G_e=\int_0^\infty f_e(\lambda)J_0(\lambda\rho)\,e^{-j\mu_2(z+h)}d\lambda. \quad (8)$$

By simplifying the above relation and defining $n=k_2/k_1$, the relation for $f_e(\lambda)$ is derived as:

$$f_e(\lambda)=-\frac{2n^2\mu_1}{n^2\mu_1+\mu_2}\cdot\frac{\lambda}{j\mu_2}=\frac{-2k_2^2\mu_1}{k_2^2\mu_1+k_1^2\mu_2}\cdot\frac{\lambda}{j\mu_2} \quad (9)$$

Eq. (8) is the Sommerfeld integral while the analytical solution of this type of integrals is very difficult and time consuming due to the existence of singularity and branch points at $\lambda=k_1$ and $\lambda=k_2$ as well as the oscillatory behavior of the Bessel function. Due to the existence of decreasing exponential function depending on μ_2 in ΔG_e, the contribution of the point $\lambda=k_1$ in the integrand function is less than point of $\lambda=k_2$. By substituting $\mu_1\approx k_1$ and $\mu_1\approx k_1$ in Eq. (9) and Eq. (10) we get:

$$f_e(\lambda)=\frac{-2k_2^2\mu_1}{\frac{k_2^2}{k_1^2}\cdot\frac{k_1}{k_2}+1}\cdot\frac{\lambda}{jk_1^2\mu_2^2}\approx-\frac{2k_2^2(k_1)}{n^2\cdot\frac{1}{n}+1}\cdot\frac{\lambda}{jk_1^2\mu_2^2} \quad (10)$$

$$\Longrightarrow f_e(\lambda)=\frac{2jnk_2}{n+1}\cdot\frac{\lambda}{\mu_2^2}.$$

By simplifying these relations, Eq. (11) becomes:

$$\Delta G_e=\frac{2jnk_2}{n+1}\int_0^\infty J_0(\lambda\rho)e^{-j\mu_2(z+h)}\frac{\lambda}{k_2^2-\lambda^2}d\lambda=-\frac{2jnk_2}{n+1}\,I(\rho,R_2) \quad (11)$$

where in Eq. (11), $I(\rho,R_2)$ is as the following integral.

$$I(\rho,R_2)=\int_1^\infty \frac{e^{(-jk_2R_2t)}}{\sqrt{t^2-(\rho/R_2)^2}}dt \quad (12)$$

This integral will be solved by the proposed method described in section (3).

3- THE NUMERICAL TECHNIQUE OF THE PROPOSED APPROACH

The foregoing discussions on the necessity to develop an accurate method for considering the air-earth interface for far and near field computations in transient analysis of grounding electrodes, lead to propose the following mathematical formulation.

3.1. Mathematical Formulas

In this paper we deal with numerical approximation of the following improper integral:

$$I(\rho,R_2)=\int_1^\infty \frac{e^{(-jk_2R_2t)}}{\sqrt{t^2-(\rho/R_2)^2}}dt.,\quad k_2,R_2,\rho\in\mathbb{R},j^2=-1. \quad (13)$$

First, by applying the Euler's formula, Eq. (13) is divided into the real and imaginary parts, namely, $I_2=Re(I(\rho,R_2))$, $I_3=Im(I(\rho,R_2))$. Thus we have:

$$I(\rho,R_2)=\int_1^\infty \frac{e^{(-jk_2R_2t)}}{\sqrt{t^2-(\rho/R_2)^2}}dt=\underbrace{\int_1^\infty \frac{\cos(k_2R_2t)}{\sqrt{t^2-(\rho/R_2)^2}}dt}_{I_2}+\underbrace{i\int_1^\infty \frac{-\sin(k_2R_2t)}{\sqrt{t^2-(\rho/R_2)^2}}dt.}_{I_3} \quad (14)$$

Now, we approximate each part I_2 and I_3 separately. Assume that:

$$Q_2(t)=\frac{\cos(k_1R_2t)}{\sqrt{t^2-(\rho/R_2)^2}},\qquad Q_3(t)=\frac{-\sin(k_1R_2t)}{\sqrt{t^2-(\rho/R_2)^2}}.$$

It is worthy to mention that the numerical approaches for approximating the integral terms I_2,I_3 are generally threefold:

- Approximation based on the Laguerre polynomials: Since the Laguerre polynomials [32-36] form the basis functions of $L^2[0,\infty)$, one approximates the functions $Q_2(t)$ and $Q_3(t)$, using the elements of the Laguerre polynomials, and later the approximations of I_2 *and* I_3 will be calculated directly by integrating the linear combinations of the Laguerre polynomials as the approximations of $Q_2(t)$ and $Q_3(t)$), respectively. This procedure takes a large amount of computations. In addition. approximations over the unbounded domains often do not result in high accuracies.
- Approximation based on transforming the unbounded interval $[1,\infty)$ into a bounded interval:: By applying some transformations such as $t=\frac{1}{x}$ the approximation domain $t>1$ is transferred into the bounded domain $(0,1]$ and then by using the Jacobi polynomials as the basic functions, one can construct the approximations of $Q_2(t)$ and $Q_3(t)$. Thus, the approximations of I_2 *and* I_3 will be calculated directly by integrating the linear combinations of the Jacobi polynomials as the approximations of $Q_2(t)$ and $Q_3(t)$. After transforming the domain of the approximation into the interval $(0,1]$, the functions $Q_2(t)$ and $Q_3(t)$, become highly oscillatory. Hence, finding the stable approximations for such functions are quite difficult.
- Approximation based on cutting the domain of the problem [37-38]: Assume that the functions $Q_2(t)$ and $Q_3(t)$ are not increasing and tend to zero as $t\to+\infty$. Hence for a given $\varepsilon>0$, $\exists t_{max}>0$ such that $\forall t>t_{max}$, $\max_{t>t_{max}}\{|Q_2(t)|,|Q_3(t)|\}<\Gamma$.

By neglecting the value $\int_{t_{max}}^{\infty}Q_s(t)dt, s\in\{1,2\}$, it is obvious that:

$$I_s\approx\int_1^{t_{max}}Q_s(t)dt,\ s\in\{1,2\}. \tag{15}$$

To approximate the integral terms $\int_1^{t_{max}}Q_s(t)dt,\ \ s\in\{1,2\}$, consider the nodes t_i^* as the roots of the Legendre polynomial of degree $m+1$ and define.

$$t_i=\frac{t_{max}-1}{2}t_i^*+\frac{t_{max}+1}{2}\ \ ,i=\overline{0,m}, \tag{16}$$

then, the Lagrange polynomials corresponding to nodes $t_i, i=\overline{0,m}$ are defined as:

$$L_i(t)=\frac{(t-t_0)(t-t_1)\dots(t-t_{i-1})(t-t_{i+1})\dots(t-t_m)}{(t_i-t_0)(t_i-t_1)\dots\dots(t_i-t_m)},\quad i=\overline{0,m}. \tag{17}$$

We consider the approximations for $Q_2(t)$ and $Q_3(t)$ named by $y_2(t)$ and $y_3(t)$ as the following:

$$y_2(t)=\sum_{i=0}^{m}y_{2i}L_i(t)\ ,\ \ y_3(t)=\sum_{i=0}^{m}y_{3i}L_i(t), \tag{18}$$

where $y_{2i},y_{3i}, i=\overline{0,m}$ are the unknown coefficients. Since the function $L_i(t)$ satisfies the Kronecker property, it is straightforward to obtain:

$$y_{2i}=Q_2(t_i)\ ,y_{3i}=Q_3(t_i)\ ,\quad i=\overline{0,m}. \tag{19}$$

Now, the approximations of $Q_2(t)$ and $Q_3(t)$ are obtained without solving any system of equations. Finally, for calculating I_2, I_3 we use the Gauss-Legendre integration technique as following [38]:

$$I_2\simeq\int_1^{t_{max}}Q_2(t)dt\simeq\int_1^{t_{max}}y_2(t)dt=\int_1^{t_{max}}\sum_{i=0}^{m}y_{2i}L_i(t)dt=\sum_{i=0}^{m}y_{2i}\int_1^{t_{max}}L_i(t)dt,=\sum_{i=0}^{m}y_{2i}\left\{\frac{(t_{max}-1)}{2}\sum_{j=0}^{m}W_jL_i(t_j)\right\}=\frac{(t_{max}-1)}{2}\sum_{i=0}^{m}W_iy_{2i}\ , \tag{20}$$

where W_i's are the weights of the Gauss-Legendre integration technique defined by:

$$W_i=\frac{-2}{(m+2)P_{m+2}(t_i)P'_{m+1}(t_i)}\ ,\ i=0,1,\dots m, \tag{21}$$

and $P_m(t)$ is the Legendre polynomial of degree m. Also, by employing the same procedure we get

$$I_3\approx\frac{(t_{max}-1)}{2}\sum_{i=0}^{m}W_iy_{3i}.$$

3.2. Computational Algorithm

Computational algorithm to implement the proposed method is developed step by step as follows:

Step1: Choose the value $\Gamma=10^{-4}$ and select the value t_{max} for which $\max_{t>t_{max}}\{|Q_2(t)|,|Q_3(t)|\}<\Gamma$.

Step2: Choose the positive integer $m>0$ and calculate the roots of the Legendre polynomial of degree $m+1$, denoted by $P_{m+1}(t)$ and name those roots by $t_i^*, i=0,1\dots,m.$

Step3: Use the transformation $t_i=\frac{t_{max}-1}{2}t_i^*+\frac{t_{max}+1}{2}\ ,i=0,1,\dots,m\cdot$

Step4: Calculate

$y_{2i}=Q_2(t_i)\ ,\ y_{3i}=Q_3(t_i),\qquad i=0,1,\dots,m.$

Step5: Compute $P'_{m+1}(t_i)=\frac{d_{P_{m+1}(t)}}{dt}|_{t=t_i}$, $P_{m+2}(t):=\frac{(2m+3)}{m+2}tP_{m+1}(t)-\frac{(m+1)}{(m+2)}P_m(t)$, $W_i=\frac{-2}{(m+2)P_{m+2}(t_i)P'_{m+1}(t_i)}\ ,\ i=0,1,\dots m,$ and calculate $I_2^{(m)}:=\frac{(t_{max}-1)}{2}\sum_{i=0}^{m}W_iy_{2i}$, $I_3^{(m)}:=\frac{(t_{max}-1)}{2}\sum_{i=0}^{m}W_iy_{3i}$.

Step6: Go to Step2, set $m:=m+1$., repeat Steps 2-5 and compute $I_2^{(m+1)}$ and $I_3^{(m+1)}$.

Step7: If $\left|I_2^{(m)}-I_2^{(m+1)}\right|<10^{-2}$ and $\left|I_3^{(m)}-I_3^{(m+1)}\right|<10^{-2}$, Stop computing. Otherwise go to Step6 and repeat the computations

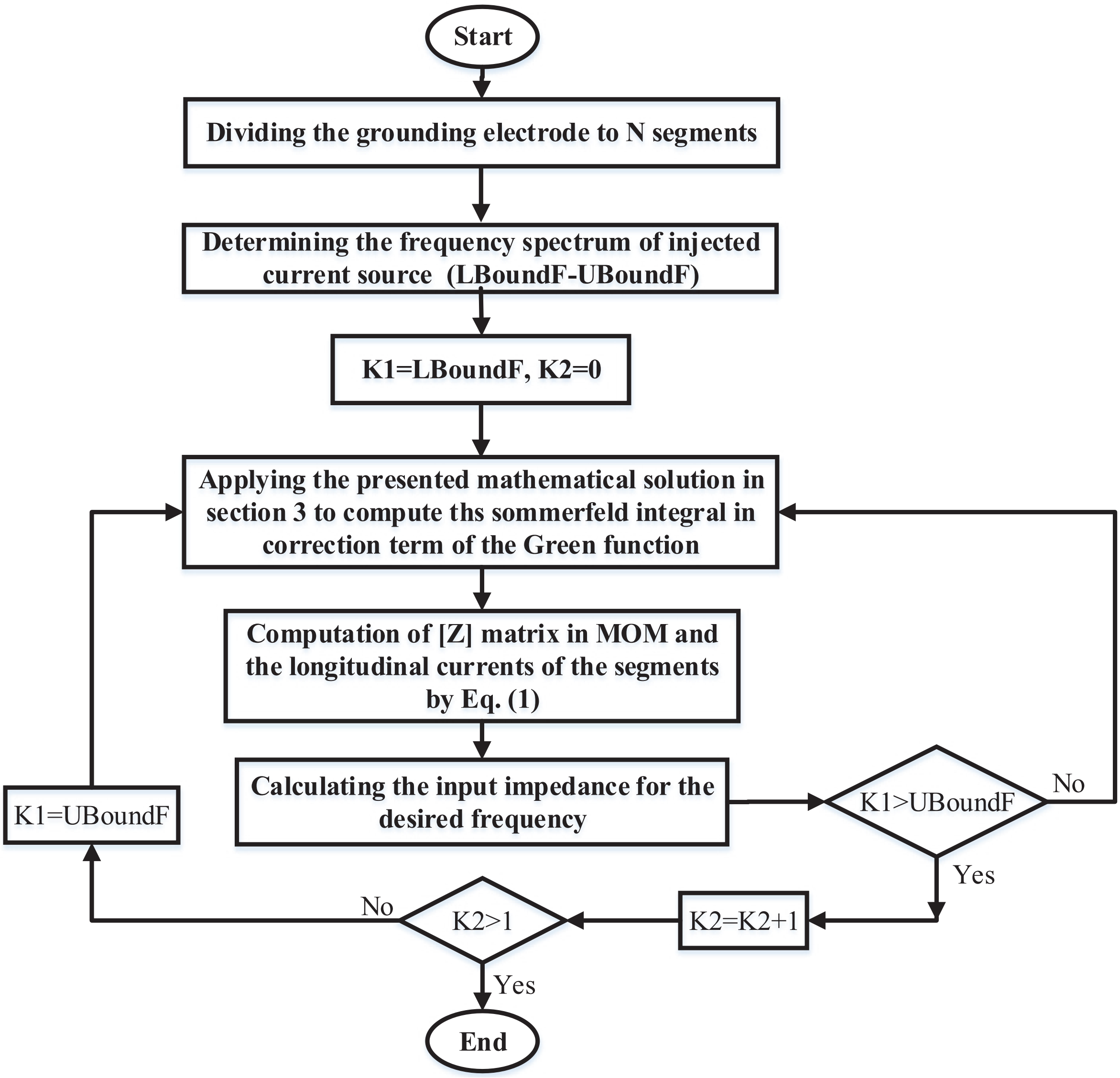


Fig. 3. Computation flowchart of the proposed approach

until for some integer $M > 0$ we get $\left|I_2^{(M)} - I_2^{(M+1)}\right| < 10^{-2}$ and $\left|I_3^{(M)} - I_3^{(M+1)}\right| < 10^{-2}$.

4- COMPUTATION FLOWCHART OF THE PROPOSED METHOD

Fig 3 shows the process of evaluating the high frequency behavior of a grounding electrode by the proposed method. In this flowchart, LBoundF and UBoundF parameters return the smallest and upset values of frequency in injected current to the grounding electrode.

5- RESULTS AND DISCUSSION

In this section, we present the main results of this analysis, namely the ΔG_e and frequency response of input impedance. We first discuss the scenario #1 which includes our results and a detailed comparison with other analyses in literature. Second, in scenario #2 we discuss the application of determined ΔG_e in the calculation of input impedance. Our findings are compared with the published results in literature. It should be noted that all simulations are implemented in the MATLAB software.

5.1. Validation of the Proposed Method

To evaluate the validity of the proposed method, two strategies are presented. This section is divided into two main sections to present and analyze the results. The first part of this section is related to compare the correction term of the Green function, i.e. ΔG_e , which was presented in Eq. (8). The second part of this section includes a detailed comparison of the obtained frequency response of the input impedance considering the proposed method and other methods in the

Table 1. Computing of ΔG_e by several other methods in different frequencies

F (MHz)	RCM [26] ×10⁴	HCA [23-24] ×10⁴	Refs. [23-24] ×10⁴	The proposed method ×10⁴	Exact result [23-24] ×10⁴
1.0	-10.0- j 5.45	-0.76- j 4.12	-0.90- j3.88	-0.89- j 3.93	-1.00- j 4.16
2.0	-15.2- j 3.81	-4.46- j 7.16	-4.55-j6.39	-4.59- j 6.50	-4.93- j 6.87
3.0	-18.8+ j 0.05	-9.48- j 7.77	-9.15- j6.46	-9.26- j 6.64	-9.95- j 6.92
6.0	-18.7+ j 16.6	-21.16+j 3.86	-18.20+j4.97	-18.74+ j 4.97	-19.56+j 5.82
9.0	-4.08+j 28.5	-15.93+ j 23.30	-11.75+ j20.97	-12.43+ j 21.63	-11.80+ j 22.90
12.0	16.3+j 26.7	5.00+ j 32.76	6.52+j 26.83	6.39+ j 28.16	8.09+ j 28.14

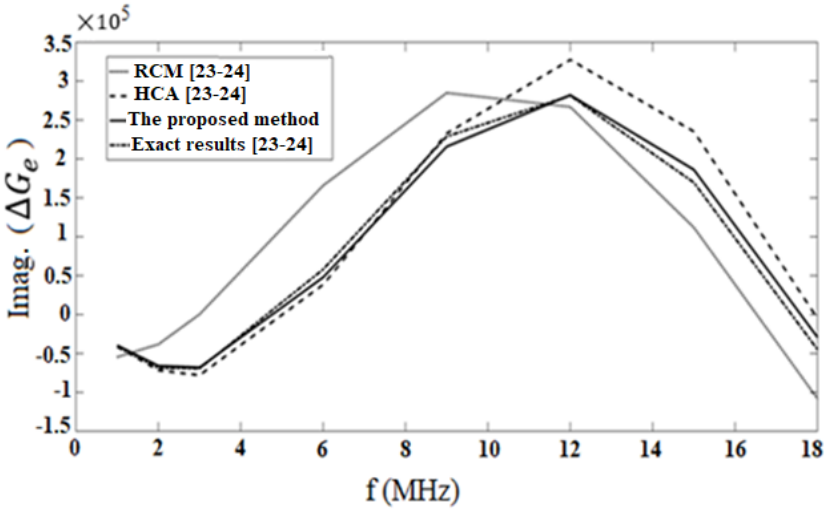


Fig. 4. The comparison between the imaginary term of ΔG_e . Our result (solid line). are compared with the results by RCM (dated line) and HCA (dashed line). The exact result also has been shown.

literature for the same vertical grounding electrode.

I) Scenario #1

This subsection discusses the specific methods by which the research and analysis were conducted for the numerical solution for the calculation of the Green function. Among the presented methods to solve this problem by researchers, approximate methods have a very high application due to their simplicity. The methods of the high-contrast approximation (HCA) and the reflection coefficient (RCM) are convenient to apply in high frequency analysis of the grounding electrode but have not a high accuracy [23-24]. In addition, the exact Sommerfeld integration formula based on antenna theory is presented to obtain rather high accuracy in these references. Therefore, the methods addressed in two references are used to compare and evaluate the proposed mathematical method. The calculations were carried out for $\varepsilon_1 = \varepsilon_0$ and $\varepsilon_{21}/\varepsilon_1 = 10$, $\mu_2 = \mu_1 = \mu_{0=} 4\pi \times 10^{-7}$, $\sigma_2 = 0.01(\Omega.m)^{-1}$, $R_2 = 10m$, $\theta_2 = 10^o$ and $\varphi = 0$.

Table 1 and Figs. (4) and (5) show and compare the obtained results of the proposed method and several other analyses presented in different references for the calculation of the correction term of the Green function. The numerical values of the variables are the same as reported data in [23-24]. The obtained results have been presented for a vertical dipole on the lossy earth. By comparing the results, it can be illustrated that for a wide range of frequency f (MHz), our results for the real and imaginary part of the Green functions are in good agreements with the results by the exact solution [23-24]. For the case of imaginary part of the Green function, our results also agree with the findings of HCA [23-24] for the smaller values of frequency f (MHz). For the real part, our results confirm the previous findings by [23-24] for all frequency ranges. Comparison of these results show that the proposed method has higher accuracy compared to the other methods.

According to the results presented in Figs. (4) and (5), one can conclude that the proposed method is much more accurate in the calculation of the correction term of the Green

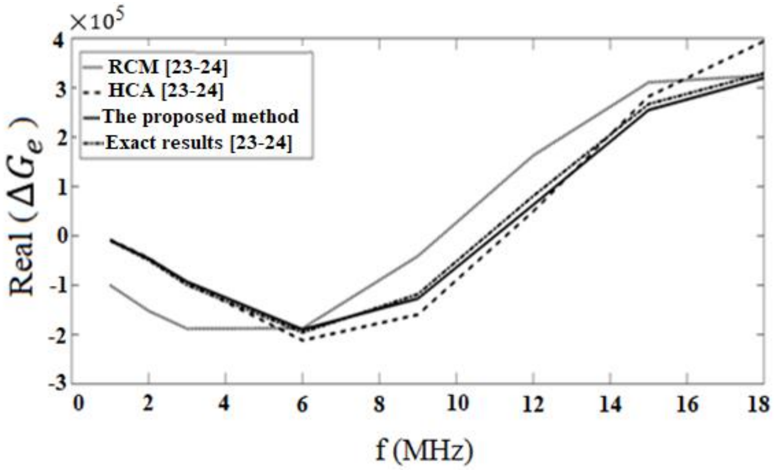
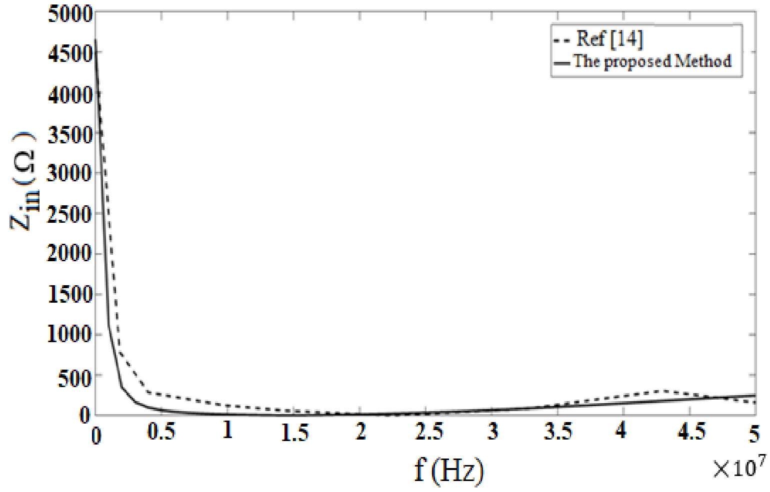
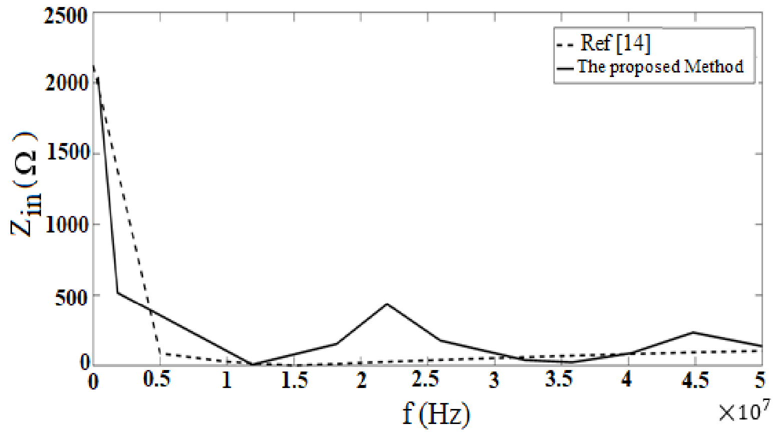


Fig. 5. comparison between the real term of ΔG_e . See the caption of Fig. 3 for further details.



(a)



(b)

Fig. 6. input impedance spectrum of the grounding electrodea. L=1m and b. L=2m. Our results (solid line) compared by the findings of [14] (dashed line).

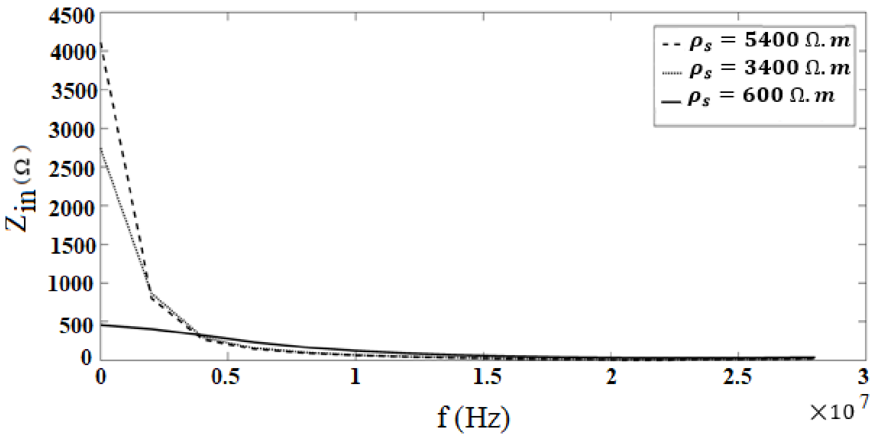


Fig. 7. Variation of input impedance with frequency and soil resistivity ($\tilde{n}_s$)

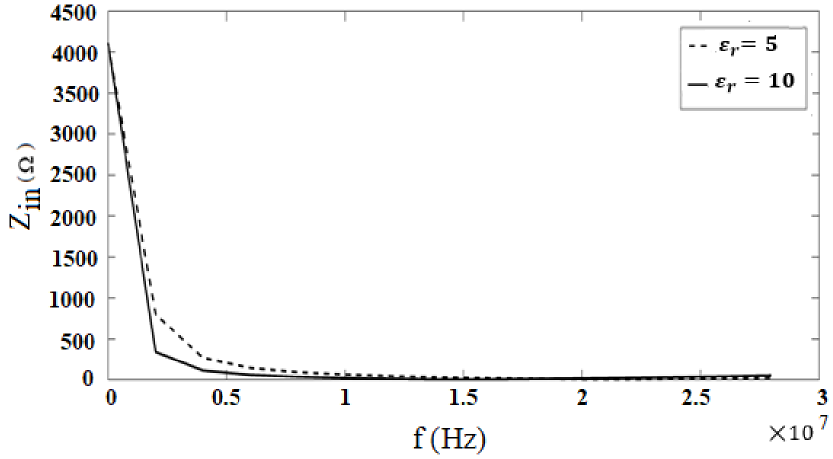


Fig. 8. Variation of input impedance with frequency and soil permittivity ε_r

function in comparison with the approximate methods used to far fields, and it's also very close to the results in [23-24], which are presented for the near field.

II) Scenario #2

Fig. 6 shows the calculated frequency response of the input impedance using the proposed method compared with [14] for a vertical grounding electrode buried at depth of 0.5 m with different lengths 1m and 2 m. and also its radii of electrode is 5 mm. The earth media is characterized with $\varepsilon_e = 8.85 \times 10^{-11}$ and $\mu_e = 4\pi \times 10^{-7}$. Additionally, earth resistivity is 5400 Ω.m. The current generator $I_f = 1.1043 \times \left(e^{-0.07924 \times 10^6 t} + e^{-4.0011 \times 10^6 t} \right)$ is excited to the top of the electrode. As depicted in Fig. 5, the obtained results by two procedure show a good agreement.

Furthermore, the input impedance of an electrode excited by high frequency current source is reduced with increasing the length of the electrode.

5.2. Influence of Soil Resistivity ($\tilde{n}_s$) on Input Impedance

In transient states, a power system involves higher frequencies up to kiloHertz and megahertz (for example, the frequency spectrum in lightning impulse is 10kHz to 3MHz), and restrikes on disconnectors and faults in GIS 100kHz-50 MHz. The grounding electrode impedance exhibited a constant value approximately equal to the DC resistance for a much higher frequency (e.g. antenna frequency), while it exhibits a decrease in its value for frequencies between 10 KHz to 10 MHz as shown in Fig. 7. The presented results in this Figure show the input impedance spectrum of the vertical grounding versus the different the soil resistivity for the three considered cases of 600, 3400 and 5400 Ω.m. Obtained results show that the input impedance of the vertical grounding electrode is significantly influenced by the variation of soil resistivity in frequencies less than 10 MHz

As shown in the obtained results, input impedance increases by increasing the soil resistivity and the input impedance tends to zero by increasing the frequency which is quite reasonable.

5.3. Influence of Soil Dielectric Permittivity (ε_r) on Input Impedance

To quantify the impact of the soil dielectric permittivity on the performance of the grounding electrode, the frequency response of a vertical grounding electrode was studied under two values of soil relative permittivity, $\varepsilon_r = 5$ and $\varepsilon_r = 10$, over frequencies DC to 30 MHz. The input impedance shows a considerable reduction by increasing soil permittivity at high frequency less than 1 MHz, while this variation is negligible in much higher frequency (e.g. higher than 1 MHz). As shown in Fig. 8, the input impedance reduces by increasing the soil dielectric permittivity. As is depicted in the results, this is more evident at frequencies below 10 MHz.

6- CONCLUSION

This study provides a novel approach to analyze the high-frequency behavior of the vertical grounding electrode. In this approach, for evaluating the impact of the earth/air interface, one needs to solve the rigorous Sommerfeld integral that appears in the Green function. We presented a simple and accurate mathematical method to investigate the behavior of the vertical grounding electrode in a high-frequency electromagnetic transient state based on the near field theory using the method of moments (MoM). The achievements in the calculation of the electromagnetic field that should be highlighted are as follows. As the first improvement, we use a simple closed-form relation instead of the Sommerfeld integral which is valid for a wide range of frequencies. Second, we show that the approach used in the paper is able to calculate the far and near electromagnetic fields. Third, by employing the appropriate numerical approach, more accurate results are obtained in comparison with other methods in the literature. The findings presented in this study are in good agreement with the previous findings.

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