



## Closed-Loop Compensation of the Quadrature Error in MEMS Vibratory Gyroscopes

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**ABSTRACT:** In this paper, a simple but effective method for compensation of the quadrature error in MEMS vibratory gyroscope is provided. The proposed method does not require any change in the sensor structure, or additional circuit in the feedback path. The mathematical relations of the proposed feedback readout system were analyzed and the proposed solution assures good rejection capabilities. Based on the simulation results, the proposed method increases the dynamic range of the readout circuit by about **19dB** for the quadrature error with **10** times higher amplitude than the Coriolis signal. Furthermore, the feedback path reduces the effect of the **1** degree LO mixer phase error in the output path by about **95%**, which causes our system to be less sensitive to this error. In addition, the 2<sup>nd</sup> harmonic component at the output of the proposed feedback readout is much lower than that of the conventional readout. As a result, proposed feedback readout can relax the requirement of the output lowpass filter.

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## 1. INTRODUCTION

In the last decades, the micro-electro-mechanical systems (MEMS) are one of the most interesting subjects for researching and technical works. In addition to the high accuracy of MEMS gyroscope and its aerospace application, the ultra-low cost and small area are other advantages of the MEMS gyroscope. For these reasons, they have extensive applications in numerous fields, such as automotive, mechanical, military and electronics fields [1-2]. The operation of the Micromachined gyroscopes are based on the Coriolis force [3].

Most vibrating gyroscopes use angular velocity to measure the amplitude of vibration induced by the Coriolis force. Gyroscopes consist of two axes in the name of driving and sensing axis that are perpendicular to each other. Generally, the proof mass is stimulated alternately, and when it is affected by the angular velocity, a secondary vibration is generated by the Coriolis force. The secondary vibration amplitude is used to determine the speed. The perspective view of a MEMS gyroscope structure is shown in Fig 1.

Because of non-ideal factors in the gyroscope manufacturing process such as mechanical imbalances and misalignments, the oscillation amplitude of the driving axis is several times larger than the amplitude of the sensing axis [2, 5-7]. For this reason, these non-ideal factors give rise to cause a significant error called quadrature error. The amplitude of this error may be up to 10 times larger than the Coriolis signal and on the other hand, there is a 90 degree phase difference

between the Coriolis signal and the quadrature signal [2]. Because of the large quadrature error and its problems, there are several methods to eliminate the quadrature error. Fig 2 demonstrates some common ways to remove this error. One of these methods is the use of improved drive decoupled beam. In other words, improving the sensor causes to reduce this leak between two axes and improve the sensitivity of the suspension flexures [8].

Another method is the use of a 90 degree phase difference between the error signal and the Coriolis signal for separating these two signals. As be mentioned before, by using a synchronous demodulator, the quadrature signal and the Coriolis signal can be separated. In the following, by performing the necessary processing on the output signal the error will be eliminated. However, according to the large amplitude of the error signal, the most important problem in this case is the possibility of saturation of the readout circuit (capacitance to voltage converter). For this reason, the most attempting has been done for eliminating or minimizing this error before reaching to the readout circuit. In Fig. 2(a), the quadrature error is eliminated by applying differential DC potentials on the mechanical electrodes of the sensor. In Fig. 2(b), the gyroscope signal converted into the digital domain and the quadrature error control value applied on the compensations electrodes of the sensor [10, 12-13]. In Fig. 2(c), after converting the gyroscope signal to the digital domain, closed-loop control structure with force-feedback will be used. In Fig. 2(d), after separating the quadrature signal by a synchronous demodulator, the compensation

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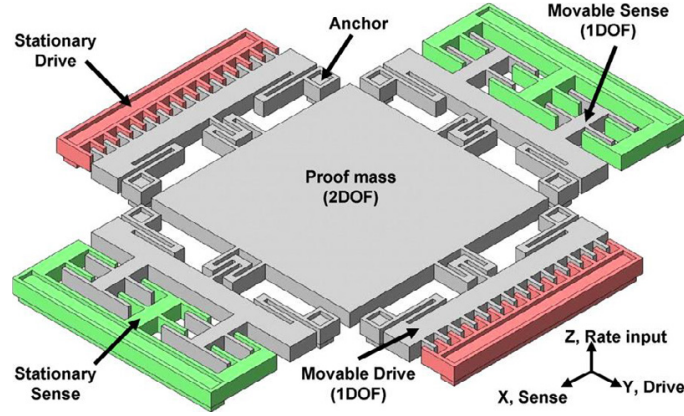


Fig 1. Perspective view of a MEMS gyroscope structure [4].

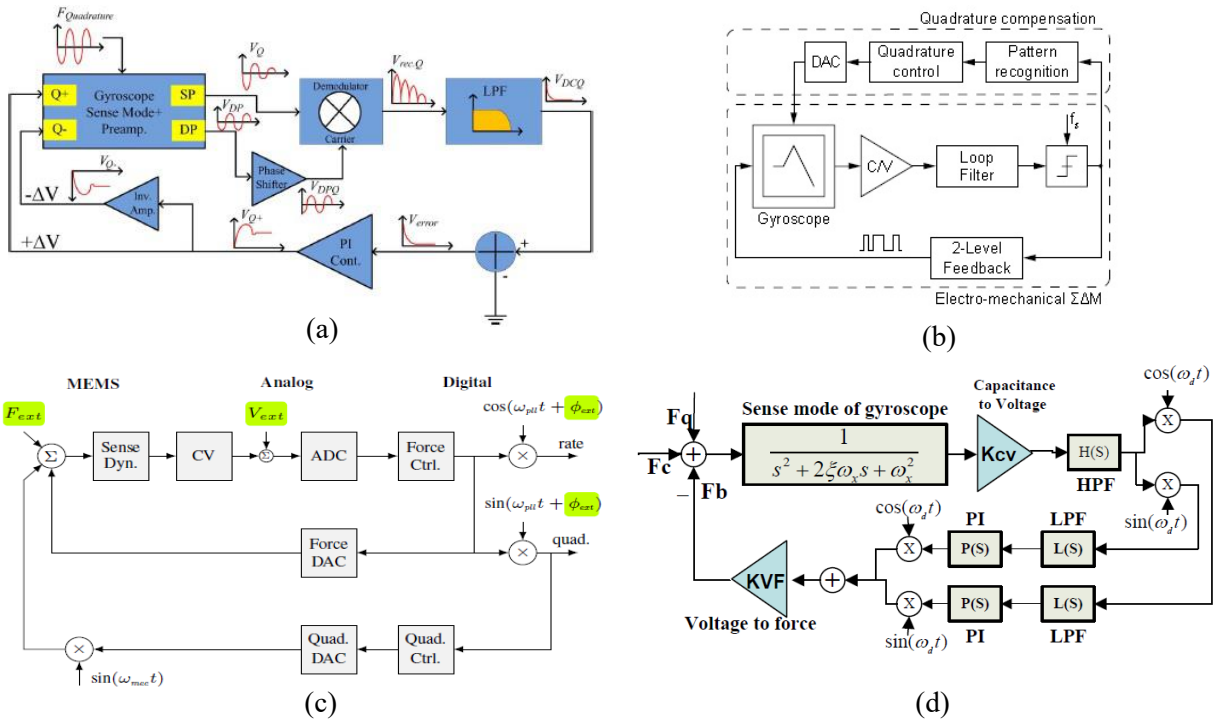


Fig 2. (a) Block diagram of the quadrature control electrodes [9]. (b) Electro-mechanical  $\Sigma\Delta$  modulator with the proposed quadrature compensation [10]. (c) Closed-loop control structure with force-feedback and quadrature compensation [11]. (d) Force rebalance control schematic for the quadrature error [12].

signal will be applied to the feedback electrodes and eventually the error signal before applying to the sensor readout circuit will be eliminated. In all of these methods, in addition to using the controller and digital converter in the feedback path, the sensor structure also becomes more complex and consequently, the cost of manufacturing the sensor increases.

In this paper, by using a connected feedback path to the input of readout circuit (instead of connecting to the sensor) the quadrature error is eliminated. Consequently, by using this method and needless to design a specific structure for the sensor, the dynamic range of the readout circuit will be increased. It should be noted that, unlike other past performed

works, the effects of various factors of the feedback loop is investigated by analyzing the mathematical relations of the system.

## 2. QUADRATURE ERROR EFFECT IN THE CONVENTIONAL READOUT

Fig. 3 shows the conventional block diagram of the gyroscope readout. As mentioned before, compared to Coriolis signal, mechanical imbalances and misalignments can generate large quadrature error signal at the input of the capacitance to voltage (C/V) convertor. Defining  $\Delta C_{is}$  and  $\Delta C_{iq}$  as the amplitude of the Coriolis and quadrature error

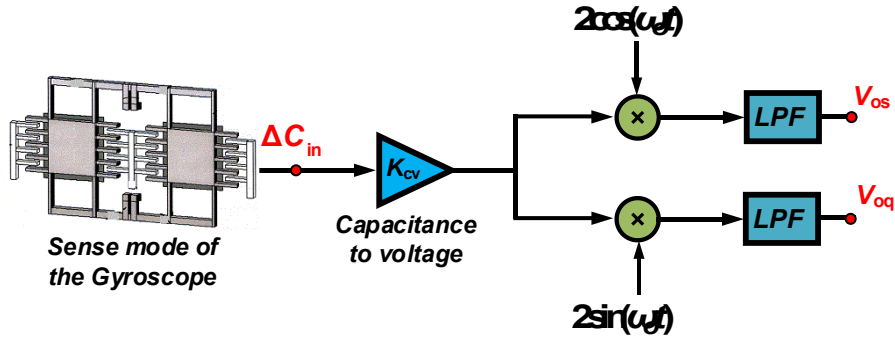


Fig 3. Conventional block diagram of the gyroscope readout.

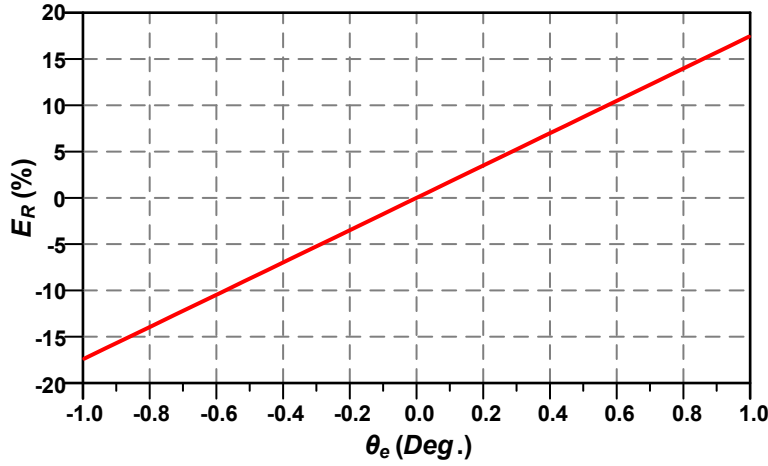


Fig 4. Normalized error ( $E_R$ ) versus LO phase error ( $\theta_e$ ) for  $\Delta C_{iq} / \Delta C_{is} = 10$

signals, the input voltage of the C/V convertor can be written as

$$\Delta C_{in} = \Delta C_{is} \cos(\omega_0 t) + \Delta C_{iq} \sin(\omega_0 t) \quad (1)$$

where,  $\omega_0$  is the operating frequency of the gyroscope.

According to (1), the voltage amplitude at the input of the C/V convertor can be calculated as

$$|\Delta C_{C/V}| = \sqrt{\Delta C_{is}^2 + \Delta C_{iq}^2} \quad (2)$$

which is much higher than that in the absence of the quadrature error signal. This large amplitude can limit the dynamic range of the readout circuits. For example, the quadrature error with 10 times higher amplitude (i.e.  $\Delta C_{iq} / \Delta C_{is} = 10$ ) can degrade the dynamic range by about 20 dB.

Moreover, due to the large amplitude of the quadrature error signal, the phase error in the LO (local oscillator) signal of the mixer can cause a significant error at the output voltage. In the vibratory gyroscopes, the cosine signal for the output path modulator is usually generated from the sinusoidal signal in the gyroscope drive section by phase shifter (or a quadrature circuit). Assuming the phase error of  $\theta_e$  in the cosine signal,

the output voltage ( $V_{os,DC}$ ) and the normalized error ( $E_R$ ) can be calculated as

$$V_{os,DC} = k_{CV} \Delta C_{is} \cos(\theta_e) - \Delta C_{iq} \sin(\theta_e) \quad (3a)$$

$$E_R \triangleq \frac{k_{CV} \Delta C_{is} - V_{os,DC}}{k_{CV} \Delta C_{is}} \approx \frac{\Delta C_{iq}}{\Delta C_{is}} \theta_e \quad (3b)$$

where,  $k_{CV}$  is the gain of the C/V convertor circuit.

Fig. 4 shows the normalized error versus  $\theta_e$  for  $\Delta C_{iq} / \Delta C_{is} = 10$ . As it can be seen, a one degree phase variation in the LO signal of the mixer can cause about 17% error in the output voltage.

The quadrature error signal can also affect the undesired 2<sup>nd</sup> harmonic component at the output voltage. In the presence of the quadrature signal, the amplitude of the 2<sup>nd</sup> harmonic at the output ( $V_{os,2}$ ) can be calculated as

$$V_{os,2} = \frac{k_{CV}}{\alpha} \sqrt{\Delta C_{is}^2 + \Delta C_{iq}^2} \quad (4)$$

where,  $\alpha$  is the attenuation coefficient of the 2<sup>nd</sup> harmonic by lowpass filter. Due to the large amplitude of quadrature error signal, this harmonic component maybe cannot be

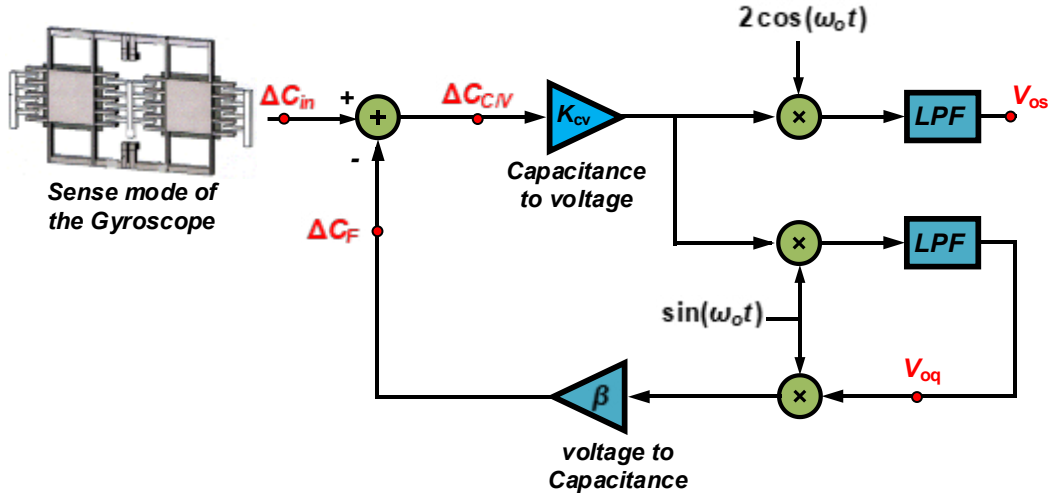


Fig 5. Proposed closed-loop readout structure.

sufficiently eliminated by a simple lowpass filter.

### 3- PROPOSED CLOSED LOOP READOUT

As shown in Fig. 3, the amplitude of the quadrature error signal can easily be extracted by a mixer. Therefore, reconstructing the quadrature signal by using its amplitude and subtracting it from  $\Delta C_{in}$  at the input of the C/V can eliminate this undesired signal. Based on this idea, Fig. 5 shows the proposed closed loop readout structure.

In order to evaluate the desired output voltage ( $V_{os}$ ), assuming the input voltage as explained in (1), let us consider the quadrature output ( $V_{oq}$ ) as

$$V_{oq} = V_{oq,DC} + V_{oq,2} \sin(2\omega_0 t + \theta_{oq,2}) \quad (5)$$

where,  $V_{oq,DC}$  is the DC component of the quadrature output.  $V_{oq,2}$  and  $\theta_{oq,2}$  are the amplitude and the phase of the 2<sup>nd</sup> harmonic component of the quadrature output, respectively.

Using (5), the feedback signal ( $\Delta C_F$ ) can be calculated as

$$\Delta C_F = \left[ \beta V_{oq,DC} \sin(\omega_0 t) + \frac{\beta V_{oq,2}}{2} \cos(\omega_0 t + \theta_{oq,2}) \right] - \frac{\beta V_{oq,2}}{2} \cos(3\omega_0 t + \theta_{oq,2}) \quad (6)$$

Subtracting (6) from the input signal yields

$$\Delta C_{C/V} = \left[ \Delta C_{is} \cos(\omega_0 t) + (\Delta C_{iq} - \beta V_{oq,DC}) \sin(\omega_0 t) - \frac{\beta V_{oq,2}}{2} \cos(\omega_0 t + \theta_{oq,2}) \right] + \frac{\beta V_{oq,2}}{2} \cos(3\omega_0 t + \theta_{oq,2}) \quad (7)$$

where,  $V_{C/V}$  is the voltage at the input of C/V circuit.

According to (7) and assuming the lowpass filter attenuates the 2<sup>nd</sup> harmonic by a factor of  $\alpha$ , the quadrature output can be rewritten as

$$V_{oq} = k_{CV} \left( \frac{\Delta C_{iq}}{2} - \frac{\beta V_{oq,DC}}{2} + \frac{\beta V_{oq,2}}{4} \sin(\theta_{oq,2}) \right) + k_{CV} \left( \left( \frac{\beta V_{oq,DC}}{2\alpha} - \frac{\Delta C_{iq}}{2\alpha} \right) \cos(2\omega_0 t) - \frac{\beta V_{oq,2}}{2\alpha} \sin(2\omega_0 t + \theta_{oq,2}) + \frac{\Delta C_{is}}{2\alpha} \sin(2\omega_0 t) \right) \quad (8)$$

Equating (5) and (8), the DC component and the amplitude and the phase of 2<sup>nd</sup> harmonic of the quadrature output can be calculated as

$$V_{oq,DC} = \left( \frac{k_{CV} (4\alpha + k_{CV} \beta)}{4\alpha (k_{CV} \beta + 2) + k_{CV} \beta (k_{CV} \beta + 4)} \right) \Delta C_{iq} \quad (9a)$$

$$V_{oq,2} = \left( \frac{-4k_{CV} / \sin(\theta_{oq,2})}{4\alpha (k_{CV} \beta + 2) + k_{CV} \beta (k_{CV} \beta + 4)} \right) \Delta C_{iq} \quad (9b)$$

$$\theta_{oq,2} = \tan^{-1} \left( \frac{-4(2\alpha + k_{CV} \beta)}{4\alpha (k_{CV} \beta + 2) + k_{CV} \beta (k_{CV} \beta + 4)} \frac{\Delta C_{iq}}{\Delta C_{is}} \right) \quad (9c)$$

Now, by substituting (9) into (7) and neglecting 3<sup>rd</sup> harmonic component, the input voltage of the C/V convertor can be estimated as

$$\Delta C_{C/V} \simeq \Delta C_{is} \left( \frac{4\alpha + k_{CV} \beta}{4\alpha + 2k_{CV} \beta} \right) \cos(\omega_0 t) + \Delta C_{iq} \left( \frac{8\alpha + 2k_{CV} \beta}{4\alpha (k_{CV} \beta + 2) + k_{CV} \beta (k_{CV} \beta + 4)} \right) \sin(\omega_0 t) \quad (10)$$

as can be seen, both coefficients  $\beta$  and  $k_{CV}$  are multiplied together, therefore, to simplify the feedback path,

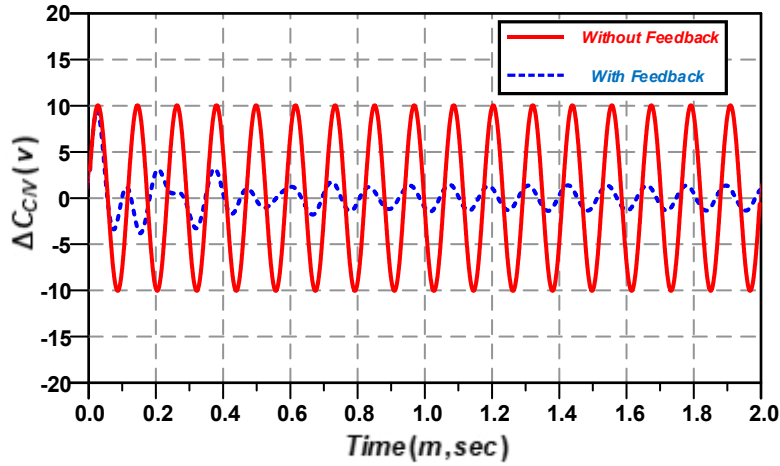


Fig 6. Input voltage of the C/V convertor with and without the feedback path ( $\Delta C_{iq} / \Delta C_{is} = 10, k_{CV} = 40, \alpha = 25dB$ )

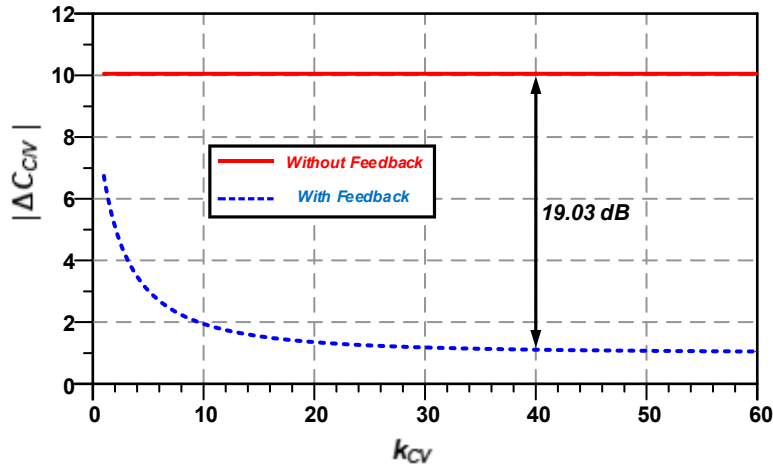


Fig 7. The amplitude of the input voltage of C/V convertor versus  $k_{CV}$ .

we can assume that  $\beta = 1$ , and assuming that the attenuation of 2<sup>nd</sup> harmonic by lowpass filter is much higher than amplification in the capacitance to voltage converter (i.e.  $\alpha \gg k_{CV}$ ), (10) can be rewritten as

$$\Delta C_{CV} \simeq \Delta C_{is} \cos(\omega_o t) + \Delta C_{iq} \left( \frac{1}{1 + k_{CV}/2} \right) \sin(\omega_o t) \quad (11)$$

Comparing (11) and (1) indicates that the proposed feedback loop does not affect the desired Coriolis signal while the undesired quadrature error signal is attenuated by a factor of  $(1 + 0.5k_{CV})$ . Assuming  $\Delta C_{iq} / \Delta C_{is} = 10$  and  $k_{CV} = 40$ , the input voltage of the C/V convertor with and without the feedback path are shown in Fig. 6. As it can be seen, due to the feedback path, the undesired signal at the input is sufficiently attenuated.

Fig. 7 compares the signal amplitude at the input of C/V circuit for the proposed and conventional readout for different values of  $k_{CV}$ . As it can be seen, the increment of the capacitance to voltage converter gain will reduce the

amplitude of  $\Delta C_{CV}$ ; and consequently, will result in dynamic range improvement. For example, choosing  $k_{CV} = 40$  will improve the dynamic range about **19dB**. It is worth to note that the value of  $\alpha$  has negligible effect on the dynamic range increment.

According to (10), the output Coriolis voltage of the proposed readout can also be calculated as

$$V_{os,DC} = \left( \frac{4\alpha + k_{CV}}{4\alpha + 2k_{CV}} \right) k_{CV} \Delta C_{is} \quad (12)$$

which reveals that for  $\alpha \gg k_{CV}$ , the desired output Coriolis voltage does not affected by the proposed feedback loop. Fig. 8 shows the normalized gain of the proposed feedback readout ( $A_N = V_{os,DC} / (k_{CV} \Delta C_{is})$ ) in terms of capacitance to voltage converter gain ( $k_{CV}$ ) for different values of 2<sup>nd</sup> harmonic attenuation of the lowpass filter ( $\alpha$ ). As it can be seen, the normalized gain of the proposed feedback readout has negligible change for  $\alpha \geq 20dB$  and  $k_{CV}$ .

Due to quadrature error reduction, it can be expected that

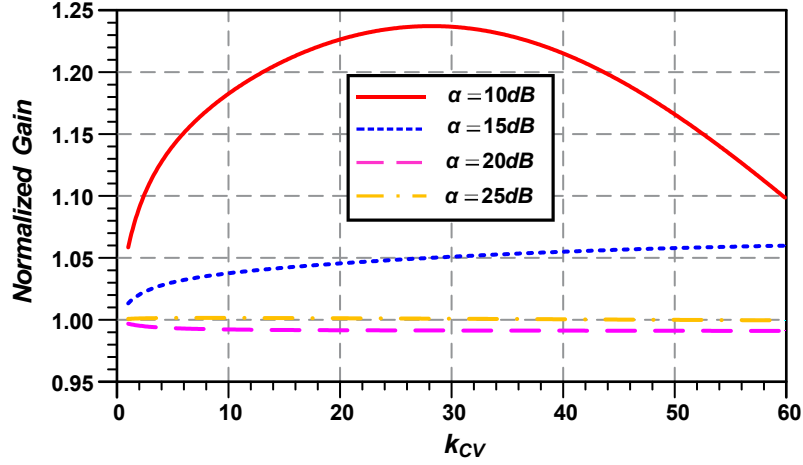


Fig 8. Normalized gain of the proposed feedback readout in terms of  $k_{CV}$  for different values of  $\alpha$ .

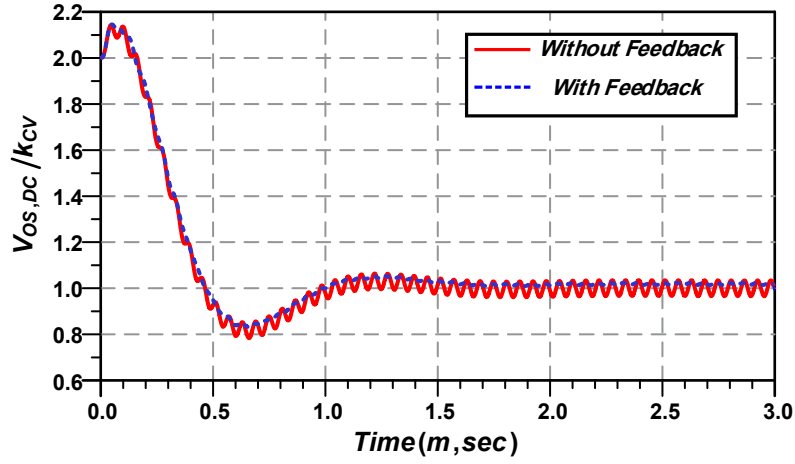


Fig 9. The effect of the 2<sup>nd</sup> harmonic component at the output voltage of the proposed and conventional readout circuit. ( $\Delta C_{iq} / \Delta C_{is} = 10$ ,  $k_{CV} = 40$ ,  $\alpha = 25dB$ )

the 2<sup>nd</sup> harmonic component at the output voltage should be reduced. Using (7), this component can be expressed as

$$V_{os,2} = k_{CV} \left( \Delta C_{is} \cos(2\omega_0 t) + \Delta C_{iq} \left( \frac{4(2\alpha + k_{CV})}{4\alpha(k_{CV} + 2) + k_{CV}(k_{CV} + 4)} \right) \cos(2\omega_0 t) \right) \quad (13)$$

and assuming  $\alpha \gg k_{CV}$ , the magnitude of the 2<sup>nd</sup> harmonic can be simplified as

$$|V_{os,2}| \simeq \frac{k_{CV}}{\alpha} \sqrt{\Delta C_{is}^2 + \Delta C_{iq}^2 \left( \frac{1}{1 + 0.5k_{CV}} \right)^2} \quad (14)$$

Comparing (4) and (13) reveals that the proposed feedback path can relax the requirement of the output lowpass filter. Fig. 9 compares the output voltage of the proposed and conventional readout for  $k_{CV} = 40$ . As it can be seen, the 2<sup>nd</sup> harmonic component at the output of the proposed

feedback readout is much lower than that of the conventional readout. The simulation results shows a **17.5 dB** harmonic attenuation.

As mentioned before, due to the large quadrature error, a small phase error in the LO signal of the mixer can cause considerable change in the output Coriolis voltage. It can be expected that the attenuation of the quadrature error, due to the feedback path, will reduce this error. Assuming the phase error of  $\theta_e$ , the output Coriolis voltage in conventional readout and the normalized error can be calculated as

$$V_{os,DC} \simeq k_{CV} \Delta C_{is} \left( \frac{4\alpha + k_{CV}}{4\alpha + 2k_{CV}} \right) \cos(\theta_e) - k_{CV} \Delta C_{iq} \left( \frac{8\alpha + 2k_{CV}}{4\alpha(k_{CV} + 2) + k_{CV}(k_{CV} + 4)} \right) \sin(\theta_e) \quad (15a)$$

$$E_R \triangleq \frac{k_{CV} \Delta C_{is} - V_{os,DC}}{k_{CV} \Delta C_{is}} \simeq \frac{\Delta C_{iq}}{\Delta C_{is}} \frac{\theta_e}{1 + 0.5k_{CV}} \quad (15b)$$



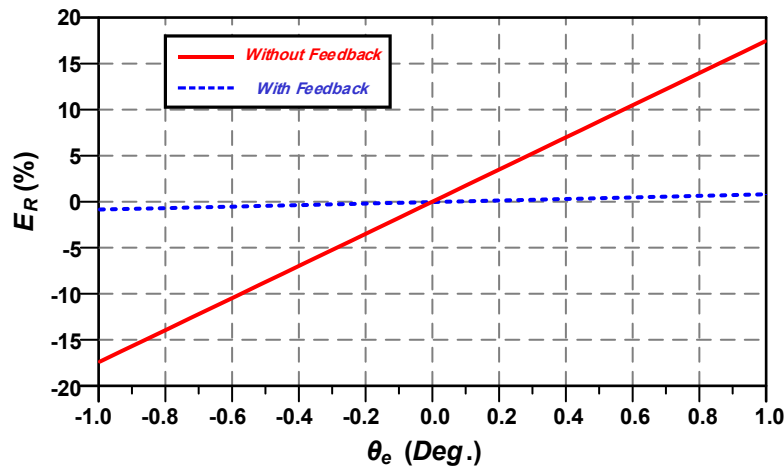


Fig 10. Comparison of the normalized error (due to LO phase error) for the proposed and conventional readout. ( $\Delta C_{iq} / \Delta C_{is} = 10$ ,  $k_{CV} = 40$ ,  $\alpha = 25dB$ )

Equation (15b) indicates that the normalized error will be attenuated by a factor of  $(1 + 0.5k_{CV})$ . The comparison of the normalized error for the proposed and conventional readout is shown in Fig. 10 for  $k_{CV} = 40$ . As it can be seen, the feedback path reduce this error by about **95%** for 1 degree phase error.

### 3. CONCLUSION

In this paper, the closed-loop system for rejecting the quadrature error in the MEMS gyroscope was presented and the mathematical relations were evaluated. With the help of this feedback path, needless to use an additional controller or extra circuitry, the quadrature error was nearly eliminated and as a result, the dynamic range of the readout circuit will be increased. Also, this added feedback path cause to reduce the sensitivity of the LO mixer in the output path to the phase error, and moreover, due to the reduction of the amplitude of 2<sup>nd</sup> harmonic component at the output, the required specifications for lowpass filter structure became simpler.

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