

# *Assessment of Weighting Functions Used in Oppermann Codes in Polyphase Pulse Compression Radars*

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## **ABSTRACT**

Polyphase is a common class of pulse compression waveforms in the radar systems. Oppermann code is one of the used codes with polyphone pattern. After compression, this code has little tolerant against Doppler shift in addition to its high side lobe level. This indicates that the use of Oppermann code is an unsuitable scheme to radars applications. This paper shows that the use of amplitude weighting functions improves properties of code and makes it an appropriate technique. Noticeable reduction in sidelobe and false alarm as well as the increase of the target detection ability and Doppler tolerant are the signature of amplitude weighting functions investigated and simulated in this study.

## **KEYWORDS**

Pulse compression, polyphase, sidelobe, Oppermann code, window weighting function, Hamming, Hanning, Nuttallwin, Resolution, Peak power, width pulse

## **1. INTRODUCTION**

In order to increase range resolution, pulse compression techniques is widely used in many radar systems. Pulse compression is a method which combines the high energy of a long pulse width with the high resolution of a short pulse. The transmitted pulse is modulated by using binary phase coding, polyphase coding, frequency modulation, and frequency stepping in order to get large time-bandwidth product.

Phase-coded pulse compression can be implemented by applying the digital Correlator as the matched filter. Output of the matched filter will be an extremely narrow pulse with a large peak value, thus the transmitted pulse is compressed in time domain [1]. Unfortunately, the autocorrelation function (ACF) of a real expanded impulse consists not only of a main peak which is used for target detection but also of range sidelobes which can cover main peaks caused by small targets [1], [2].

Binary Phase and polyphase are two methods for of phase coding. In binary form, a long pulse of duration  $T$  is divided into  $N$  sub pulses each of width  $\tau$ . The phase of each sub-pulse is chosen to be either  $0$  or  $\pi$  radians.

The binary choice of  $0$  or  $\pi$  phase for each sub-pulse may be made at random. However, some random selections may be better suited than others for radar application. One criterion for the selection of a good random phase coded waveform is that its autocorrelation

function should have equal time sidelobes. Barker codes have called perfect codes because the highest side lobe is only one code element amplitude high. However, the largest pulse compression ratio that can be obtained with barker code is only 13 [1].

If the phases of subpulses in phase coded pulse compression are other than the binary phases of  $0$  and  $\pi$ , then the phase codes are called polyphase codes. They have lower sidelobes than binary codes and are more Doppler tolerant if the Doppler frequencies are not too large. Frank proposed a polyphase code called as Frank code [1]. Lewis and Kreuschmer have presented the variants of Frank code. P1 code which is derived from step frequency, Bolter matrix derived P2 code and linear frequency derived P3 and P4 codes. The significant advantage of P1 and P2 codes over the Frank code and the P4 code over P3 is that they are tolerant to receiver band limitations [3].

Oppermann Codes polyphase pulse compression waveform discussed by the author [4], provide a class of phase coded waveform that can be sampled upon reception and processed digitally. These codes were originally introduced within the context of applications for code-division multiple-access (CDMA) systems. Given the length of the code, Oppermann codes are defined by three parameters which then correspond to a distinct family of codes. For particular values of these parameters, the autocorrelation magnitude of Oppermann

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codes is controlled by one parameter while a second parameter influences only the phase characteristics. Further, the autocorrelation magnitude is then the same for all Oppermann codes in the family. Thus, this makes it a candidate for the design of integrated radar and communication systems where more than one code is needed.

The compressed pulse of the polyphase coded waveforms has sidelobes which decrease the pulse compression ratio (PCR). For PCR equal to 100, the sidelobe peaks range from 26 to 30dB below the main peak signal response, depending on the particular code [3].

There are reduction techniques developed to reduce the sidelobe levels. Lewis proposed sliding window two-sample subtractor to reduce the sidelobes for the polyphase codes [5]. Weighting in frequency and time domain can generally be applied to reduce the sidelobes [6], [7]. This sidelobe reduction technique can be analysed twofold: as matched weighting (with weighting window at the transmitter and the receiver) and mismatched weighting, where amplitude weighting is performed only at receiver site. There is wide range of well-known window functions (Hanning, Hamming, and Nuttallwin) implemented in pulse compression technique.

This paper indicates that Oppermann code has an unsuitable sidelobes level and Doppler tolerant to radars applications. Also, this shows that the use of amplitude weighting functions improves properties of code and makes it as an appropriate technique.

The paper is organized as follows. Section II defines the measures used to facilitate a quantitative performance evaluation and comparison of the considered polyphase codes. Section III describes the class of Oppermann codes. On this basis, numerical results are given in Section IV. It also illustrates Effects of amplitude weighting window for Oppermann code in poly phase pulse compression. Finally, Section V concludes the paper.

## 2. PERFORMANCE MEASURES

Let  $N$  denote the length of each polyphase code  $u = [u(0), u(1), \dots, u(N-1)]$ . In the sequel, we provide the definitions of the measures [8], [9] used to assist with the performance comparisons of the examined classes of polyphase code.

### A. Aperiodic Autocorrelation

The aperiodic autocorrelation  $C(l)$  at discrete shift  $l$  between a polyphase code  $u$  and its shifted version, is defined as

$$C(l) = \begin{cases} \frac{1}{N} \sum_{i=0}^{N-1-l} u(i)u^*(i+l) & 0 \leq l \leq N-1 \\ \frac{1}{N} \sum_{i=0}^{N-1+l} u(i-l)u^*(i) & 1-N \leq l \leq 0 \\ 0 & |l| \geq N \end{cases} \quad (1)$$

where

$$u(i) = \exp[j \varphi(i)] \quad (2)$$

is the  $i$  th subpulse of length  $N$  of the poly phase code and  $u^*$  denotes the complex conjugate of the argument  $u$ . Also  $\varphi(i)$  denotes  $i$  the subpulse phase of length  $N$  of the polyphase code.

It is noted that the discrete shift  $l$  in the considered radar scenarios is associated with the delay by which a transmitted pulse code signal is received, which in turn translates to the range of a target.

### B. Figure of Merit

The figure of merit (FOM) of a code  $u$  of length  $N$  with aperiodic autocorrelation function  $c(l)$  measures the ratio of energy in the mainlobe to that in the sidelobes of the autocorrelation function. It is defined as

$$\text{FOM(dB)} = 10 \log_{10} \frac{|C(0)|^2}{2 \sum_{l=1}^{N-1} |C(l)|^2} \quad (3)$$

### C. Peak-to-Sidelobe Ratio

Similarly, the peak-to-sidelobe ratio (PSLR) of a code of length  $N$  with aperiodic autocorrelation function  $c(l)$  measures the ratio of the inphase value  $c(0)$  to the maximum sidelobe magnitude  $|c(l)|$  of the autocorrelation function. It is defined as

$$\text{PSLR} = 20 \log_{10} \frac{|C(0)|}{\max_{1 \leq l \leq N} |C(l)|} \quad (4)$$

## 3. OPPERMANN CODES

A family of polyphase codes that supports a wide range of correlation properties is proposed in [3]. The phase  $\varphi_k(i)$  of the  $i$  th element  $u(i)$  of Oppermann code of length  $N$  is defined as

$$\varphi_k(i) = \frac{\pi}{N} [K^m i^p + i^n + kiN] \quad i, k = 1, \dots, N \quad (5)$$

The parameters  $m$ ,  $n$  and  $p$  in (5) take real values and define a family of Oppermann codes. For a fixed



combination of these three parameters, all the codes have the same autocorrelation magnitude. If  $p=1$  this autocorrelation magnitude depends only on  $n$  and is given by [5]

$$|C_k(l)| = \left| \frac{1}{N} \sum_{i=0}^{N-1} \exp \left\{ \frac{j\pi}{N} \left[ i^n - (i+l)^n \right] \right\} \right| \quad (6)$$

In this case, the optimal family in terms of FOM or PSLR as defined in (3) and (4), respectively, can be found by simple search over  $n$ . In the sequel, we will therefore concentrate on the case of  $p=1$ . Given  $p=1$  and the parameter  $n$  associated with the optimal family, the parameter  $m$  may be varied to produce favorable phase or crosscorrelation characteristics, for instance. With this parameter setting, the class of Oppermann codes provides us not only with a wide range of correlations but also flexibility to control the ambiguity function at scenarios other than those relating to the zero Doppler shift [10].

#### 4. RESULTS AND DISCUSSION

This section aims at illustrating major performance characteristics of the examined class of Oppermann code in polyphase pulse compression codes along with the related benefits and drawbacks. In the first step, performance assessment is based on PSLR, FOM and mainlobe width (-3dB) that reflect code characteristics in the absence of Doppler shifts. In the second step, the behaviour in non-zero Doppler shifts is evaluated using the ambiguity function.

Figure 1 shows Compressed pulse for Oppermann code at zero Doppler shift with length of  $N=100$  while  $p=1$ ,  $m=1$  and  $n=2$ . Note that for  $N=100$ , PSLR is equal to 26.32 dB. This level of sidelobes is unsuitable for applications radar.

It is seems that the nearby sidelobe to the mainlobe is more important than other sidelobes, because its increase, decreases the detection ability for small targets have been placed near a great target. The nearby sidelobe to the mainlobe hereinafter will be referred to as secondary peak for shortness.

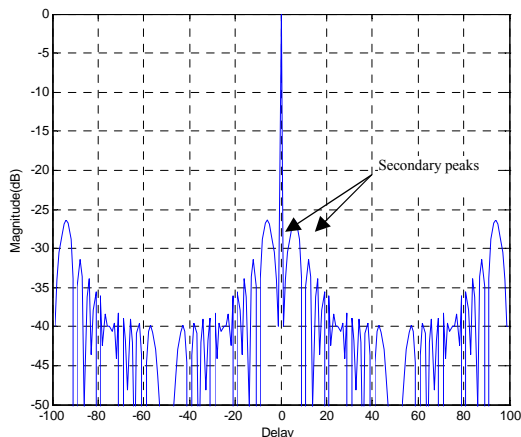


Figure 1: Autocorrelation function of Oppermann code at zero Doppler shift with length of  $N=100$  while  $p=1$ ,  $m=1$  and  $n=2$ .

Peak sidelobe level can be reduced by amplitude weighting window. In this paper Hamming, Hanning and Nuttallwin window are used that are shown in Figure 2. Hamming window is wider than Hanning, and Hanning is than Nuttallwin.

Figure 3 and Figure 4 show results of the amplitude weighting window on Oppermann code at zero Doppler shift of length  $N=100$ . The effects of amplitude weighting can be investigated as follows:

##### A. Peak -to-Sidelobe Level Ratio (PSLR)

Hamming, Hanning and Nuttallwin windows increase PSLR from 26.32 dB to 40.08, 39.91 and 35.76 dB, respectively. Increasing of PSLR causes the False Alarm probability to decrease and the small target detection ability to increase.

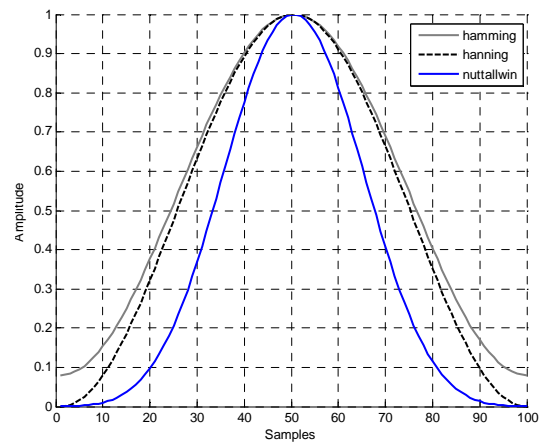


Figure 2: Hamming, Hanning and Nuttallwin windows. Nuttallwin is narrower than Hanning and Hamming.

##### B. Figures of Merit (FOM)

Hamming, Hanning and Nuttallwin windows increase FOM from 12 dB to 20.47, 20.98 and 21.06 dB, respectively. Increasing of FOM denotes increase of the ratio of energy in the main lobe to the whole energy of the sidelobes. In fact, increasing of the FOM indicates increase of the target detection ability. Thus the detection ability of Hamming, Hanning and Nuttallwin windows increases, respectively, in comparison with the case in which window weighting is not used.

##### C. Mainlobe Width (-3dB)

Unfortunately, amplitude weighting increase the mainlobe width. This increase causes the range resolution loss. Range resolution is an ability of the receiver to detect nearby targets. Hamming, Hanning and Nuttallwin windows increase mainlobe width from 0.075 to 0.42, 0.51 and 0.88, respectively. That means decrease of resolution. Comparison of Figure 2 and Figure 4 indicates that window and mainlobe width are related

reversely. For example, Nuttallwin window is narrower than Hamming window, but the mainlobe of weighting Oppermann code with Hamming Window, is narrower than it with Nuttallwin window.

PSLR, FOM, and mainlobe width of weighting Oppermann code with used of Hamming, Hanning and Nuttallwin windows are presented in Table 1.

#### D. Secondary peak

Amplitude weighting with used of Hamming, Hanning and Nuttallwin windows have obvious effect on the secondary peak. Figure 3 is well shows that Hamming window decreases the secondary peak about 15dB. Hanning and Nuttallwin windows decrease it more than 23dB.

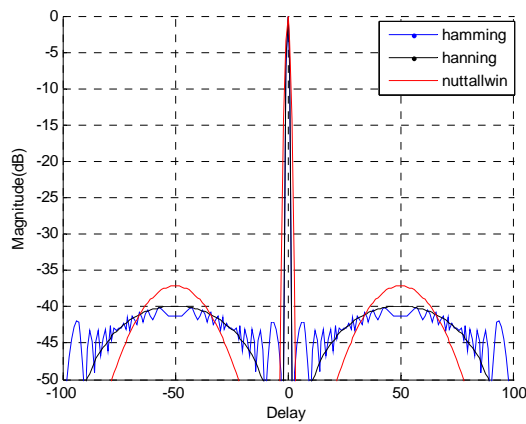


Figure 3: Autocorrelation function of weighting Oppermann code with used of Hamming, Hanning and Nuttallwin windows.

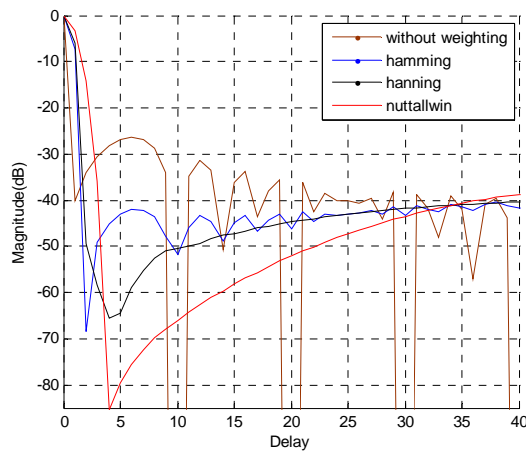


Figure 4: Effect of amplitude weighting window on mainlobe of Oppermann code with used of Hamming, Hanning and Nuttallwin windows.

TABLE 1  
PERFORMANCE FOR 100 ELEMENT OPPERMANN CODE

Window name	PSLR (dB)	FOM (dB)	Mainlobe Width (-3dB)
Rectangular	26.32	12	0.075
Hamming	40.08	20.47	0.42
Hanning	39.91	20.98	0.51
Nuttallwin	35.76	21.06	0.88

#### E. Doppler properties

Figure 5 shows the ambiguity diagram of compressed Oppermann code which is a function of normalized delay and normalized Doppler shift.  $\tau$  denotes the delay between the transmitted signal and the returned signal from a target and  $f_d$  denotes the Doppler frequency induced by a moving target.  $T_c$  denotes the duration of a subpulse and  $N$  length of code, in other words the period  $T = NT_c$ . The ratios  $\tau/T$  and  $f_d/B$  ( $B$  represents bandwidth) are then called normalized delay and normalized Doppler, respectively, which hereinafter will be referred to as delay and Doppler for shortness. Oppermann code has two smaller ridges relatively far from the diagonal ridge, in the corners of the second and fourth quadrant of the delay-Doppler plane.

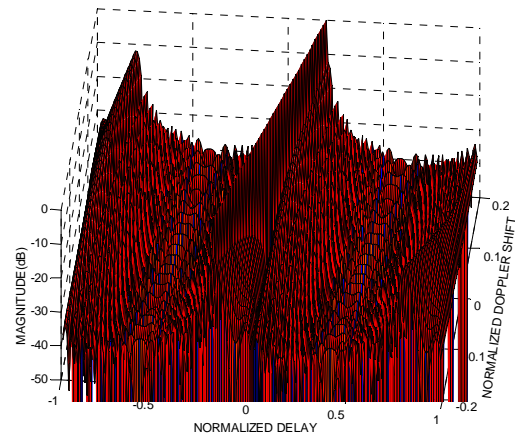


Figure 5: Ambiguity diagram of Oppermann code with length of  $N=100$  while  $p=1$ ,  $m=1$  and  $n=2$ .

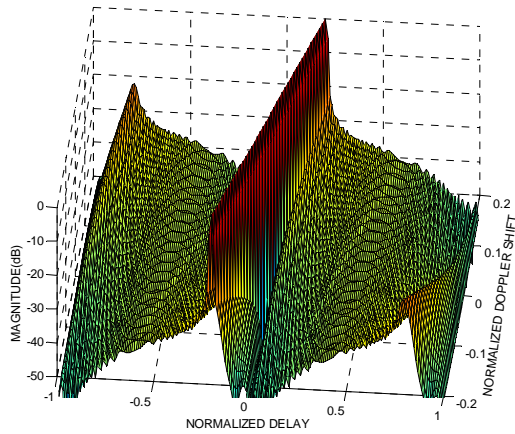
Effect of amplitude weighting with Hamming, Hanning and Nuttallwin windows functions for Doppler shift are investigated and presented. Use of the weighting window functions on Oppermann code causes many evolutions in the ambiguity diagram of it. Figure 6 shows the effects of windowing functions Hamming, Hanning and Nuttallwin windows on the Oppermann code.

Figure 6.a indicates that use of the Hamming window causes extreme reduction of the ridge in the corner of the

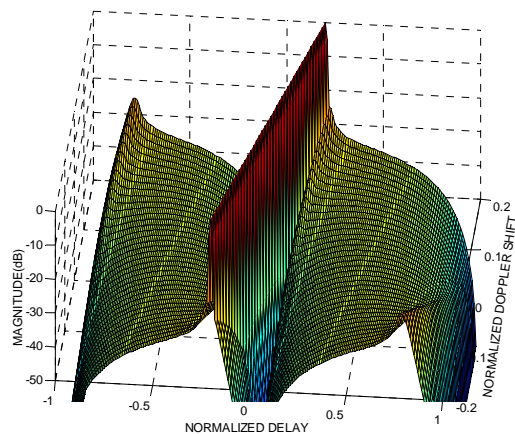


second and fourth quadrant of the delay-Doppler plane. Figure 6.b and Figure 6.c represents that Hanning and Nuttallwin windows are more effective on the reduction magnitude of ridges in comparison with Hamming window.

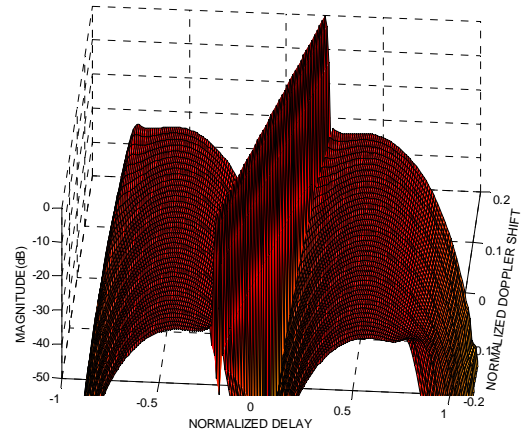
Usually the ambiguity diagram analysis is difficult, so it is showed in two dimensions. Autocorrelation function of Oppermann code with length  $N=100$  for various Doppler shift is shown in the Figure 7. Increase of Doppler frequency makes the peak of signal to shift right and increases the sidelobes level. The increase of sidelobe level causes the false alarm probability to increase. It seems that the secondary peak has more important than other sidelobes, because its growth with the increase of Doppler shift, is quicker than other sidelobes. Shift of peak signal creates error in calculating the distance. The more the peak is shifted, the more the error of calculated distance will be. So, Oppermann code has little tolerant against Doppler frequency.



(a)



(b)



(c)

Figure 6: Ambiguity diagram of weighting oppermann code with a) Hamming window, b) Hanning window, c) Nuttallwin window.

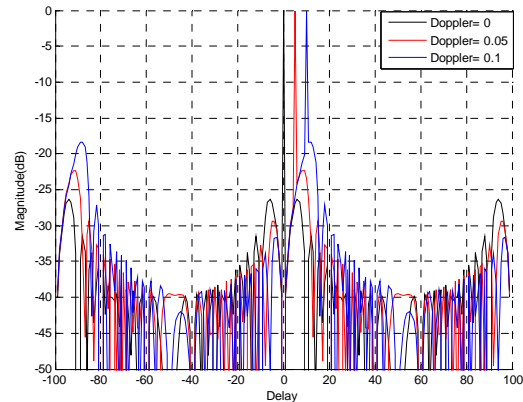


Figure 7: Autocorrelation function of Oppermann code with length  $N=100$  for Doppler shift=0, 0.05, 0.1.

Figure 8 presents Autocorrelation diagrams for various Doppler shifts of weighted Oppermann code with Hamming, Hanning and Nuttallwin windows. Comparison of Figure 7 and Figure 8 shows that using the weighting window in the Oppermann code does not affect the shifted main peaks, but is effective in the reduction of frequency Doppler role in the growth of sidelobe severely, in other words, increases Oppermann code tolerant against Doppler frequency. This result is more visible in weighting with the use of the Nuttallwin window (Figure 8.c).

In amplitude weighting with used of Hamming window for Doppler shift=0.05 and 0.1, the secondary peak is about -38dB and -32dB, respectively. This growth increases with the increase of Doppler shift. Also, Hanning and Nuttallwin window have a similar effect on the secondary peak. Note that, in weighting with Nuttallwin window, the secondary peak appears in Doppler shift=0.15.

Figure 9 shows Oppermann code without weighting

and with weighting using windows Hamming, Hanning and Nuttallwin windows for Doppler shift=0.2. In this figure the inefficiency of Oppermann code without weighting against Doppler shift which is relatively large, has been displayed well. Width of mainlobe can be extracted from this figure. It is indicated in the Figure 3 that regardless of Doppler shift, amplitude weighting increases the mainlobe width and this increase depends on the width of the used windows. But, Figure 9 represents that with regard to the Doppler shift, the width of the mainlobe in non-weighting Oppermann code, increases more intensively in comparison with the weighting Oppermann code and even becomes more than it.

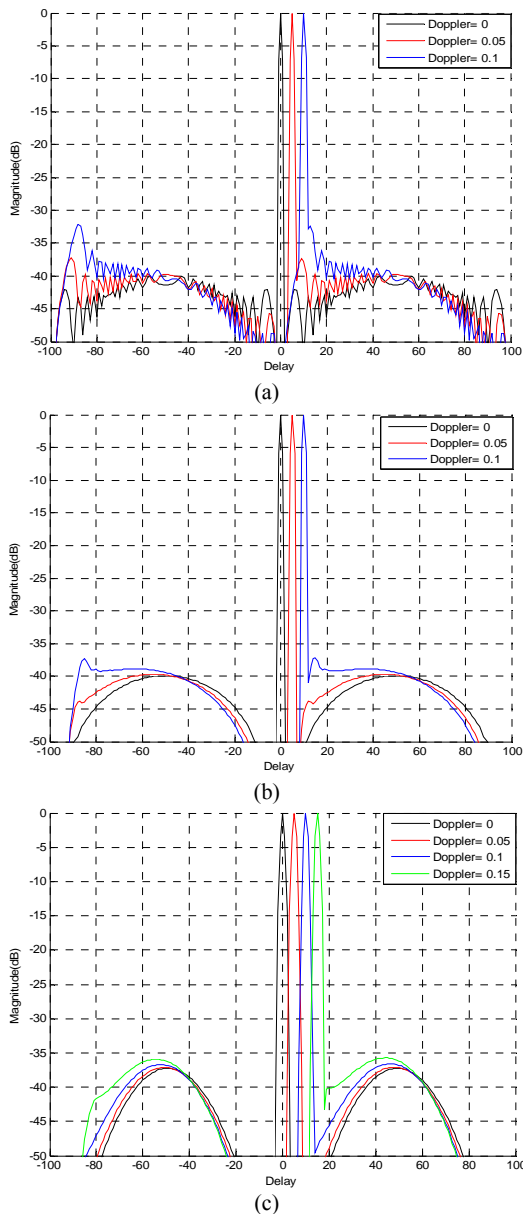


Figure 8: Autocorrelation function of weighting Oppermann code with a) Hamming window, b) Hanning window, c) Nuttallwin window, for Doppler shift=0, 0.05, 0.1.

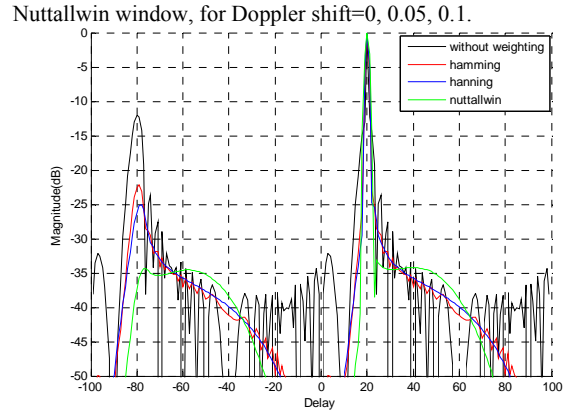


Figure 9: Autocorrelation function for Oppermann code without weighting and with weighting using Hamming, Hanning and Nuttallwin windows for Doppler shift= 0.2.

## 5. CONCLUSION

This paper investigated the properties of Oppermann code and indicated that this code has a high sidelobe level and lack of tolerant in the Doppler shift. Also, this instability occurs with an increase in the sidelobe level and the main peaks width and the shift of the main peaks with an increase of frequency Doppler. However, it is represented in this paper that the properties of this code can be improved by the use of the amplitude weighting window as Hamming, Hanning and Nuttallwin. Applying these windows reduces PSLR and increases FOM of compressed pulse, but increases the mainlobe width in time domain. In other words, this function causes the increase of target detection ability, decrease of false alarm probability and some inreduction the resolution range. It is also indicated that using the window weighting technique does not influence shifting of the main peak due to the Doppler frequency, but reduces the role of Doppler frequency on the growth of sidelobe and increases the main peak width, intensively.

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