Dynamic Harmonic Modeling and Analysis of VSC-HVDC Systems

E. Karami1*, M. Madrigal2, G. B. Gharehpetian1

1 Department of Electrical Engineering, Faculty of Electrical Engineering, Amirkabir University of Technology, Tehran, Iran
2 Department of Electrical Engineering, Instituto Tecnológico de Morelia, Michoacán, Mexico

ABSTRACT: Harmonic analysis has become an important issue in modern power systems. The widespread penetration of non-linear loads into the emergence of power systems has turned power quality analysis into an important operation issue under both steady state and transient conditions. This paper employs a Dynamic Harmonic Domain (DHD) based framework for the dynamic harmonic analysis of VSC-HVDC systems. These systems are widely used in modern power systems in both distribution and transmission levels in order to provide voltage profile improvement, power flow control, and power loss reduction. In this paper, appropriate modeling of VSC-HVDC systems for harmonic propagation is performed by means of switching function which provides a connection between DC and AC sides. Also in this paper, dynamics related to DC side capacitor are taken into account which can greatly affect the transient response. In order to validate the results, the proposed method has been successfully tested on a test system and the obtained results are compared to those of a time-domain software, followed by a discussion on results.

1- Introduction
Harmonic analysis and power quality assessment have become important issues in modern power systems [1]. Nowadays, power systems include many power electronic devices, such as Voltage Source Converter (VSC) which is the main building block of Flexible AC Transmission System (FACTS) devices that are widely used in modern power system in order to improve voltage profile, transient stability, loadability and power flow control [2]. VSC uses multilevel arrays and/or Pulse Width Modulation (PWM) to control power, voltage and current [3]. However, these devices are one of the main harmonic sources in the modern power systems due to their switching nature [4].

Harmonic analysis can be conducted in both time and frequency domains based on the available tools. Harmonic analysis in the time-domain would need an additional processing procedure, such as Windowed Fast Fourier Transform (WFFT) which allows the calculation of the harmonic content by sliding a Fast Fourier Transform (FFT) window [5]. It is worth noting that numerical errors, such as Gibbs oscillation and the picket-fence effect are considerable especially for fast transient studies [6]. A great approach for steady-state harmonic analysis is the harmonic domain (HD) which models the coupling of harmonics in the nonlinear systems very accurately [7]. This methodology has been effectively applied to power electronic systems and FACTS devices [8-18]. It should be noted that an alternative hybrid time-frequency domain method in order to compute the steady-state response of an electrical system is presented in [19].

The HD approach has been further extended to include the dynamic analysis of harmonics during transient states. The method is called Extended Harmonic Domain (EHD) or Dynamic Harmonic Domain (DHD). As shown in [20], the DHD is a powerful method which contributes to the accurate assessment of power quality. This approach provides the calculation of harmonic content step-by-step. One of the main salient features of this method is its straightforward initialization in comparison with the time domain. Combining this method with companion circuit modeling leads to a powerful analytical technique called dynamic companion circuit modeling [13]. The DHD has been used in order to exact harmonic analysis of FACTS devices, synchronous machines, transmission lines and transformers [13], [20-26]. In [26], physical meaning of transient harmonics is put forward by using DHD methodology. A novel approach for steady and dynamic states harmonic analysis of power systems is presented in [27]. It employs a decomposition framework so that harmonic producing devices are considered as separate subsystems which are solved via the extended harmonic domain technique. Application of DHD approach for investigating the effect of the source phase angle on harmonic content and time domain response during both transient and steady states is presented in [28]. It is shown that shifting all the sources does not affect the harmonics magnitude and only harmonics phase angles are linearly shifted according to their harmonic order.

In this paper, the DHD methodology along with switching function concept is used for the dynamic harmonic analysis of VSC-HVDC systems by obtaining state-space model. Also, a general procedure for the calculation of switching function is proposed which can be implemented without any complexity and takes into account different switching types. The proposed method has been successfully tested on a test system and the obtained results are compared to those of a time-domain software, followed by a discussion on results.
2- Dynamic Harmonic Domain

The main idea behind DHD is that periodic function $x(t)$ can be approximated by time dependent Fourier series as shown in Eq. (1):

$$x(t) = \sum_{n=\infty}^{\infty} X_n(t)e^{j\omega n t}$$

The complex Fourier coefficient $X_n(t)$ is time varying. By considering only first $h$ harmonics, Eq. (1) can be rewritten in matrix form as follows:

$$x(t) = G^T(t)X(t)$$

where

$$G(t) = \begin{bmatrix}
e^{-j\omega_0 t} & X_{-h}(t) \\
e^{-j\omega_1 t} & X_{-h+1}(t) \\
\vdots & \vdots \\
e^{j\omega_h t} & X_{-1}(t) \\
1 & X_0(t) \\
\vdots & \vdots \\
e^{j\omega_{-h} t} & X_{-1}(t) \\
e^{-j\omega_{-h+1} t} & X_{-h}(t)
\end{bmatrix}, X(t) = \begin{bmatrix}
X_{-h}(t) \\
X_{-h+1}(t) \\
\vdots \\
X_{-1}(t) \\
X_0(t) \\
\vdots \\
X_{-h}(t)
\end{bmatrix}$$

Considering the following state-space equation:

$$\dot{x}(t) = a(t)x(t) + b(t)u(t)$$

where, $x(t)$, $a(t)$, $b(t)$ and $u(t)$ are periodic functions. Using the general form of (2) in (3) it is not difficult to show that:

$$\dot{x}(t) = G^T(t)D(j\omega_0)X(t) + G^T(t)X(t)$$

where, $D(j\omega_0)$ is given by:

$$D(j\omega_0) = j\omega_0
\begin{bmatrix}
-h & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & 1
\end{bmatrix}$$

Also

$$a(t)x(t) = G^T(t)AX(t)$$

In Eq. (5), $A$ has Toeplitz structure given by:

$$A = \begin{bmatrix}
A_0 & A_{-1} & \cdots & A_{-h} \\
A_1 & \cdots & \cdots & \cdots \\
\vdots & \cdots & \cdots & \cdots \\
A_h & A_1 & \cdots & A_0 \\
A_{h-1} & A_0 & \cdots & A_{-1} \\
\vdots & \cdots & \cdots & \cdots \\
A_{-h} & \cdots & A_1 & A_0
\end{bmatrix}$$

where, the entries in $A$ are the harmonic coefficients of $a(t)$. In the same manner, we can obtain that:

$$b(t)u(t) = G^T(t)BU$$

Using (4), (5) and (6) in (3):

$$G^T(t)D(j\omega_0)X(t) + G^T(t)X(t) + G^T(t)AX(t) + G^T(t)BU$$

Dropping $G^T(t)$ from both sides of (7) and solving for the state variable results in:

$$\dot{X}(t) = \{A - D(j\omega_0)\}X(t) + BU$$

Eq. (8) is the transformation of (3) into the DHD, where the state variable in (3) is $x(t)$ and in (8) is the harmonics of $x(t)$. By comparing (3) and (8), one can observe that DHD transforms linear time periodic (LTP) system to linear time invariant (LTI) system. A particular case of (8) is the steady state which is reduced to a set of algebraic equations. Solving Eq. (8) needs a numerical integration method. In this paper, ode45 solver in Matlab® with a time step of $10^{-5}$ s is used.

2- Steady State Response

Steady state response can be easily reached by DHD method. In the steady state condition, a derivative of the state variables in Eq. (8) is equal to zero which means $0 = (A - D)X + BU$. This equation can be rewritten as follows:

$$0 = (A - D)X + BU$$

Eq. (9) shows one of the main salient features of DHD method against time domain simulations. In the steady state all the above matrices are constant and therefore the solution for state variables is straightforward. However, in the presence of non-linear components, an iterative process is required. It should be noted that, Eq. (9) can be used as initial condition for Eq. (8) for dynamic analysis purposes. This equation is used in following sections.

3- Voltage Source Converter State Space Model

The basic VSC scheme is shown in Fig. 1. The operation of VSC based devices depends on the conduction of semiconductor valves such as IGBTs, and it interconnects the AC system with the DC system. This conduction interval of the valves can be described by means of switching function. In this study, the switching function is used in order to provide a meaningful connection between AC and DC sides.

The voltage equations that relate the AC side with the DC side are given by:

$$v_{a1} = s_a v_{DC}, v_{b1} = s_b v_{DC}, v_{c1} = s_c v_{DC}$$

where, $s_a$, $s_b$, and $s_c$ are switching functions that represent the operation of VSC. If the VSC is assumed to be lossless, instantaneous power at AC side is equal to instantaneous power at DC side as shown in Eq. (11):

$$i_a v_{a1} + i_b v_{b1} + i_c v_{c1} = i_{DC} v_{DC}$$

If the inverter is not ideal, losses can be modeled through adding a resistance to the DC side [22]. Moreover, switching losses of the converter can be included by adding a current dependent resistance to the DC side as described in [3].

Employing (10) in (11) yields an expression for $i_{DC}$. 

32
I at the AC sides and associated switching functions are
\[ i_{a} + s_{b}i_{b} + s_{c}i_{c} = i_{DC} \]  
(12)

Keeping in mind that DC side current may be written in terms of the dynamic equation of the capacitor.
\[ i_{DC} = C \frac{dv_{DC}}{dt} \]  
(13)

The voltage drop across the three phase impedance of VSC circuit in Fig. 1 is as follows:
\[ v_{a} - v_{a0} = R_{a}i_{a} + L_{a} \frac{di_{a}}{dt} \]
\[ v_{b} - v_{b0} = R_{b}i_{b} + L_{b} \frac{di_{b}}{dt} \]
\[ v_{c} - v_{c0} = R_{c}i_{c} + L_{c} \frac{di_{c}}{dt} \]  
(14)

Combining equations (10)-(14) yields the state space model for VSC as follows:
\[
\begin{bmatrix}
\frac{di_{a}}{dt} \\
\frac{di_{b}}{dt} \\
\frac{di_{c}}{dt} \\
\frac{dv_{DC}}{dt}
\end{bmatrix} =
\begin{bmatrix}
-R & 0 & 0 & -s_{a} \\
0 & -R & 0 & -s_{b} \\
0 & 0 & -R & -s_{c} \\
\frac{s_{a}}{C} & \frac{s_{b}}{C} & \frac{s_{c}}{C} & 0
\end{bmatrix}
\begin{bmatrix}
i_{a} \\
i_{b} \\
i_{c} \\
v_{DC}
\end{bmatrix} +
\begin{bmatrix}
\frac{1}{L} \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
v_{a} \\
v_{b} \\
v_{c} \\
v_{DC}
\end{bmatrix} \tag{15}
\]

This equation can be easily transformed into the DHD. According to Eq. (8), Eq. (15) in the DHD is given by:
\[
\begin{bmatrix}
I_{a} \\
I_{b} \\
I_{c} \\
V_{a}
\end{bmatrix} =
\begin{bmatrix}
\frac{R}{L} & -D & 0 & 0 & -\frac{1}{L}S_{a} \\
0 & \frac{R}{L} & -D & 0 & -\frac{1}{L}S_{b} \\
0 & 0 & \frac{R}{L} & -D & -\frac{1}{L}S_{c} \\
\frac{1}{C}S_{a} & \frac{1}{C}S_{b} & \frac{1}{C}S_{c} & -D
\end{bmatrix}
\begin{bmatrix}
I_{a} \\
I_{b} \\
I_{c} \\
V_{a}
\end{bmatrix} +
\begin{bmatrix}
\frac{1}{L} \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
V_{a} \\
V_{b} \\
V_{c} \\
V_{DC}
\end{bmatrix} \tag{16}
\]

where $U_{i}$ is identity matrix and 0 is matrix of zeroes. It should be noted that both $U_{i}$ and 0 have dimension of $(2h+1) \times (2h+1)$. These equations are the basics for modeling of VSC-HVDC systems.

VSC-HVDC system is modeled by following the same procedure and considering that DC side current and voltage are affected by two VSCs.

3-1- DC Side Voltage Control
In the VSC, it is desired that the voltage in the DC side be constant all the time. Hence, a PI controller as shown in Fig. 2 can be employed to maintain DC side voltage at the constant value by appropriate shifting of switching function [11].

4- Switching Function
There are different switching strategies for power electronic converters, such as harmonic elimination and PWM. In some cases, there is a straightforward manner to calculate switching function according to Fourier coefficients. For instance, in harmonic elimination technique switching pulses are applied so that predetermined harmonics are eliminated at the AC side. It can be shown that in order to eliminate $n$ harmonics at the AC side, $n$ pulses are needed to be applied to semiconductor valves which are obtained through an iterative process [1]. For example, in order to eliminate the fifth, seventh, 11th, 13th and 17th harmonics, the calculated switching angles are 11.35°, 17.27°, 23.81°, 34.88° and 37.27°, respectively. The harmonic content of switching function is shown in Fig. 3. According to this figure, associated harmonics are effectively eliminated at the AC side. Furthermore, this figure emphasizes on the significant magnitude of higher order harmonics. In some cases, extra mathematical computations are required in order to obtain the switching function. For instance, PWM switching is based on the comparison of two signals in which different parameters like modulation index and switching frequency affect the output. In order to overcome the mentioned complexities with both calculation and computer implementation, a general procedure as shown in Fig. 4 is proposed to calculate switching function of every switching strategy.

5- Simulation Results
In order to access the effectiveness and precision of DHD approach in analyzing VSC-HVDC, test system shown in Fig. 5 is used. Series resistance and inductance are 0.051Ω and 0.027mH, respectively. DC side capacitor has a value of 4950μF. Both sending and receiving end voltages are balanced with the magnitude of 1p.u. and phase shift of 120° between phases. Harmonic elimination technique is employed in both converters in order to eliminate fifth, seventh, 11th, 13th and 17th at the AC sides and associated switching functions are obtained by following the proposed method in Fig. 3. In order to control the power flow switching shift in sending and receiving ends are set to 10° and −10°, respectively. It should be noted that all results are shown in p.u.

5-1- Steady State Response
As mentioned in the previous sections, one of the main advantages of DHD method is the straightforward solution for steady state response by solving Eq. (8). Steady state harmonic content and time domain responses for receiving end currents are shown in Figs 6 and 7, respectively. Also, DC side voltage is shown in Fig. 8. By analyzing the waveforms, it can be seen that the steady state initial condition was exact since no transient at the beginning of the simulation was identified and simulation starts from steady state.
\[ \frac{V_{DC-ref}}{K/(1+TS) + V_{DC-measured}} \]

**Fig. 2.** PI controller block diagram

<table>
<thead>
<tr>
<th>Read the Input Data:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switching Method and</td>
</tr>
<tr>
<td>Associated Parameters</td>
</tr>
<tr>
<td>Calculation of Switching</td>
</tr>
<tr>
<td>Function in Time Domain</td>
</tr>
<tr>
<td>Using FFT to Extract Harmonic</td>
</tr>
<tr>
<td>Content of Switching Function</td>
</tr>
<tr>
<td>Calculation of Switching</td>
</tr>
<tr>
<td>Function Shift</td>
</tr>
<tr>
<td>Is Switching Function Shifted?</td>
</tr>
<tr>
<td>No</td>
</tr>
<tr>
<td>Calculation of Switching</td>
</tr>
<tr>
<td>Function in DHD</td>
</tr>
</tbody>
</table>

**Fig. 3.** Harmonic content of switching function using harmonic elimination technique

**Fig. 4.** Proposed procedure to calculate switching function

**Fig. 5.** Back to Back VSC-HVDC connected to two separate AC systems

**Fig. 6.** Harmonic magnitude of receiving end currents

**Fig. 7.** Time domain response of receiving end currents

**Fig. 8.** Harmonics and Time domain response of sending end currents
Fig. 9. Harmonic content of sending end current

Fig. 10. Time domain response of sending end currents

Fig. 11. Harmonic content of receiving end current

Fig. 12. Time domain response of receiving end currents

Fig. 13. Time domain response of sending end currents

Fig. 14. Sending end active powers

Fig. 15. Receiving end active powers

Fig. 16. Sending end reactive powers
Since the system is completely balanced, Total Harmonic Distortion (THD) of receiving end currents are the same and all equal to 36%. THD for sending end currents is equal to 33.36%. It should be noted that according to Eq. (10) and keeping in mind that DC side voltage can be represented by Fourier series as Eq. (1) and since switching functions and DC side voltage contain only odd and even harmonics, respectively; the fifth harmonic is not completely eliminated at AC sides.

6- Conclusion
This paper described the dynamic harmonic analysis of VSC-HVDC systems by employing the Dynamic Harmonic Domain method and solving state space model. In the extended VSC-HVDC model, AC and DC sides are connected through switching function. Moreover, a general procedure in order to calculate the switching function by using time domain and FFT is proposed. The proposed solution approach for analyzing VSC-HVDC systems is fully frequency dependent, which provides a step-by-step procedure for following the harmonics evolution with respect to the time. It should be mentioned that according to the proposed representation for VSC-HVDC systems, an equivalent impedance is obtained which depends on switching procedure and electrical parameters and it could be used for resonance analysis. The proposed solution and extended equations were applied to a test system followed by the discussion on results.
References


Please cite this article using:

DOI: 10.22060/eej.2016.816