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Dynamic Harmonic Analysis of Long Term Over Voltages Based on Time Varying Fourier Series in Extended Harmonic Domain

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ABSTRACT

Harmonics have become an important issue in modern power systems. The widespread penetration of non-linear loads to emerging power systems has turned power quality analysis into an important operation issue under both steady state and transient conditions. This paper employs an Extended Harmonic Domain (EHD) based framework for dynamic analysis of long term analysis over voltages during the transients caused by inrush currents while large power factor capacitors are located at transformer secondary side. In such cases, a combination of capacitor and inductive system impedance may lead to parallel resonance circuits of high impedance. As a significance of the developed method, it is fully frequency domain dependent solution technique which uses time dependent Fourier series, orthogonal bases and matrix operators as addressed in EHD. The proposed method has been successfully tested on several networks and the obtained results are compared to those of a time-domain software, followed by discussion on results.

KEYWORDS

Capacitor bank, Extended harmonic domain, Inrush current, Transformer, Windowed fast Fourier transform.

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1. Introduction

Harmonic analysis and power quality assessment have become important issues in modern power systems [1]. Transformers are one of the most important components in power systems [2]. Saturation characteristic affects the magnetization current in both steady state and transient. The steady-state magnetization current is about 1 to 2% of the rated current but it may reach 10 to 20 times of the transformers' rated value when the transformer is energized [3]. Note that since the core is designed to operate close to the knee point, small over-excitation will cause a rise in harmonic generation. Moreover, inrush current may cause long-term over-voltages in the system when large power factor capacitors are located at the secondary side. In this case a combination of capacitor and inductive system impedance may lead to a parallel resonance circuit of high impedance [4].

Harmonic analysis in the time-domain, requires extra tools like Windowed Fast Fourier Transform (WFFT), which allows the calculation of the harmonic content by sliding a FFT window [5]. Regardless, numerical errors such as Gibbs oscillation and the picket-fence effect are considerable for fast transients [6].

Very great approach for steady-state harmonic analysis is the harmonic domain (HD), which models the coupling of harmonics in the nonlinear systems very accurately [7]. This approach has been successfully applied to power electronic systems and flexible AC transmission system (FACTS) devices [8-18]. It should be noted that an alternative hybrid time-frequency domain method in order to compute the steady-state response of an electrical system is presented in [19].

The HD has been further extended to incorporate the dynamic analysis of harmonics during transient states. The method is called Extended Harmonic Domain (EHD) or Dynamic Harmonic Domain (DHD). As shown in [20], the EHD is a powerful method that contributes to the accurate assessment of power quality. This approach provides the calculation of harmonic content step-by-step. One of the main salient features of this method is its straightforward initialization in comparison with the time domain. Combining this method and companion circuit modeling leads to a powerful analytical technique called dynamic companion circuit modeling [13]. The DHD has been used in order to exact harmonic analysis of FACTS devices, synchronous machines and transmission lines [13], [20-25].

In this paper, the EHD method is used for the dynamic harmonic analysis of long term over voltages during the transient period. Comparing EHD and WFFT results, shows that WFFT may lead to significant error during the transient state. Time domain results of EHD are compared with those are obtained through EMTP software.

2. Extended Harmonic Domain

The main idea behind EHD is that function $x(t)$ can be approximated by time dependent Fourier series as shown in Eq. (1):

$$x(t) = \sum_{n=-\infty}^{\infty} X_n(t)\, e^{j\, n\, \omega\, t} \tag{1}$$

The complex Fourier coefficients $X_n(t)$ are a function of time given by:

$$X_n(t) = \frac{1}{T_0}\int_t^{t+T_0} x(\tau)\, e^{-j\, n\, \omega_0\, \tau}\, d\tau \tag{2}$$

where $\omega_0 = \frac{2\,\pi}{T_0}$, $\tau = [t, t+T_0]$ and T_0 is time period. By considering only first h harmonics, Eq. (1) can be rewritten in matrix form as follows:

$$x(t) = \boldsymbol{G}^T(t)\, \boldsymbol{X}(t) \tag{3}$$

where

$$\boldsymbol{G}(\tau) = \begin{bmatrix} e^{-j\,h\,\omega_0\,t} \\ \vdots \\ e^{-j\,\omega_0\,t} \\ 1 \\ e^{j\,\omega_0\,t} \\ \vdots \\ e^{j\,h\,\omega_0\,t} \end{bmatrix}, \qquad \boldsymbol{X}(t) = \begin{bmatrix} X_{-h}(t) \\ \vdots \\ X_{-1}(t) \\ X_0(t) \\ X_1(t) \\ \vdots \\ X_{-h}(t) \end{bmatrix} \tag{4}$$

The derivative of $x(t)$ with respect to t is given by Eq. (5).

$$\dot{x}(t) = \dot{\boldsymbol{G}}^{\boldsymbol{T}}(t)\boldsymbol{X}(t) + \boldsymbol{G}^T(t)\, \dot{\boldsymbol{X}}(t) \tag{5}$$

$\dot{\boldsymbol{G}}^{\boldsymbol{T}}(t)$ can be shown as $\boldsymbol{G}^T(\tau)\, \boldsymbol{D}(j\,h\,\omega_0)$, where $\boldsymbol{D}(j\,h\,\omega_0)$ is given by:

$$\boldsymbol{D}(j\,h\,\omega_0) = j\omega_0 \begin{bmatrix} -h & & & & & & \\ & \ddots & & & & & \\ & & -1 & & & & \\ & & & 0 & & & \\ & & & & 1 & & \\ & & & & & \ddots & \\ & & & & & & h \end{bmatrix} \tag{6}$$

Another important equation which needs to be transferred to EHD is product of two function. In general, these functions could be time varying based on the system charactristics. The product of two functions is given by (7).

$$a(t)\, x(t) = \boldsymbol{G}^T(t)\, [\boldsymbol{A}]\, \boldsymbol{X}(\tau) \tag{7}$$

In Eq. (7), $[\boldsymbol{A}]$ has Toeplitz structure and it is formed based on harmonic content of $a(t)$ as below.

$$[\boldsymbol{A}] = \begin{bmatrix} A_0 & A_{-1} & \cdots & A_{-h} & & & \\ A_1 & \ddots & \ddots & & \ddots & & \\ \vdots & \ddots & A_0 & A_{-1} & & \ddots & \\ A_h & & A_1 & A_0 & A_{-1} & & A_{-h} \\ & \ddots & & A_1 & A_0 & \ddots & \vdots \\ & & \ddots & & \ddots & \ddots & A_{-1} \\ & & & A_h & \cdots & A_1 & A_0 \end{bmatrix} \tag{8}$$

2.1. State Space Equation

Consider the following state space equation.

$$\begin{aligned} \dot{x}(t) &= a(t)x(t) + b(t)u(t) \\ y(t) &= c(t)x(t) + e(t)u(t) \end{aligned} \tag{9}$$

In Eq. (9), $a(t)$, $b(t)$, $c(t)$ and $e(t)$ have a period of T_0. Using Equations (3)-(7) and dropping $\boldsymbol{G^T(t)}$ yields Eq. (10).

$$\begin{aligned} \dot{\boldsymbol{X}}(t) &= \{\boldsymbol{A} - \boldsymbol{D}(j\, h\, \omega_0)\}\, \boldsymbol{X}(t) + \boldsymbol{B}\, \boldsymbol{U} \\ \boldsymbol{Y}(t) &= \boldsymbol{C}\, \boldsymbol{X}(t) + \boldsymbol{E}\, \boldsymbol{U} \end{aligned} \tag{10}$$

These two equations are the main equations in EHD methodology and their solution provide a complete information about dynamics of each harmonic.
Eq. (10) is the transformation of (9) into the EHD, where the state variable in (9) is $x(t)$ and in (10) are the harmonics of $x(t)$. By comparing (9) and (10) one can observe that EHD transforms linear time periodic (LTP) system to linear time invariant (LTI) system. A particular case of (10) is the steady state which is reduced to a set of algebraic equations.

Solving Eq. (10) needs a numerical integration method. In this paper, ode45 solver in Matlab® with time step of 10^{-5}s is used.

2.2. Steady State Response

Steady state response can be easily reached by EHD method. In the steady state condition derivative of the state variables in Eq. (10) is equal to zero which means $\boldsymbol{0} = (\boldsymbol{A} - \boldsymbol{D})\boldsymbol{X} + \boldsymbol{BU}$. This equation can be rewritten as follows:

$$\begin{aligned} \boldsymbol{X} &= \{\boldsymbol{D}(j\, h\, \omega_0) - \boldsymbol{A}\}^{-1}\, \boldsymbol{B}\, \boldsymbol{U} \\ \boldsymbol{Y} &= \boldsymbol{C}\, \boldsymbol{X} + \boldsymbol{D}\, \boldsymbol{U} \end{aligned} \tag{11}$$

Eq. (11) shows one of the main salient features of EHD method against time domain simulations. In the steady state all the above matrices are constant and therefore the solution for state variables is straightforward. However, in the presence of non-linear components, an iterative process is required. It should be noted that, Eq. (11) can be used as initial condition for Eq. (10) for dynamic analysis purposes.

3. Transformer Model

There are different models for transformers which are selected based on their application and conditions of the study. For harmonic and transient studies, modeling the magnetizing branch is essential. Fig. 1 depicts a single-phase transformer equivalent circuit for harmonic analysis in which all the secondary parameters are transferred to the primary. The model includes a parallel branch that represents the losses and magnetization in the core. The magnetization branch can be represented by piecewise linear approximation or polynomial approximation [1].

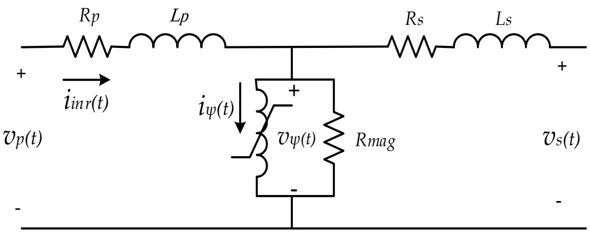

Fig. 1. Transformer equivalent circuit for harmonic studies

In this paper the following equation is used to describe the magnetization branch.

$$i_\psi(t) = a\psi(t) + b\psi(t)^n \tag{12}$$

It is worth noting that $i_\psi(t)$ should be calculated at each time step in the form of harmonic component where $\psi(t)$ and $\psi(t)^n$ become harmonic vectors $\boldsymbol{\Psi}$ and $\boldsymbol{\Psi}^n$, respectively. The term $\boldsymbol{\Psi}^n$ is calculated by harmonic convolution (denoted here with symbol ⊗) [1]. It can be easily shown that:

$$\boldsymbol{\Psi}^n = \boldsymbol{\Psi} \otimes \ldots \otimes \boldsymbol{\Psi} = \mathrm{T}^{n-1}\boldsymbol{\Psi} \tag{13}$$

where T is formed by harmonic content of ψ as shown in Eq. (8).
Eq. (14) describes an unloaded single phase transformer in time domain.

$$\begin{bmatrix} \dfrac{di_{inr}(t)}{dt} \\ \dfrac{d\psi(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{R_p + R_{mag}}{L_p} & 0 \\ R_{mag} & 0 \end{bmatrix} \begin{bmatrix} i_{inr}(t) \\ \psi(t) \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L_p} & \dfrac{R_{mag}}{L_p} \\ 0 & -R_{mag} \end{bmatrix} \begin{bmatrix} v_p(t) \\ i_\psi(t) \end{bmatrix} \tag{14}$$

Eq. (14) has a representation in EHD as follows:

$$\dot{\boldsymbol{X}}(t) = ([\boldsymbol{A}] - [\boldsymbol{D_d}])\boldsymbol{X}(t) + \boldsymbol{BU}(t) \tag{15}$$

where

$$\boldsymbol{X}(t) = \begin{bmatrix} \boldsymbol{I_{inr}}(t) \\ \boldsymbol{\Psi}(t) \end{bmatrix}; [\boldsymbol{A}] = \begin{bmatrix} -\frac{R_p + R_{mag}}{L_p}\boldsymbol{U_I} & \boldsymbol{0} \\ R_{mag}\boldsymbol{U_I} & \boldsymbol{0} \end{bmatrix}$$

$$[\boldsymbol{D_d}] = \begin{bmatrix} \boldsymbol{D} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{D} \end{bmatrix}; \ \boldsymbol{B} = \begin{bmatrix} \frac{1}{L_p}\boldsymbol{U_I} & \frac{R_{mag}}{L_p}\boldsymbol{U_I} \\ \boldsymbol{0} & -R_{mag}\boldsymbol{U_I} \end{bmatrix} \quad (16)$$

$$\boldsymbol{U}(t) = \begin{bmatrix} \boldsymbol{V_p}(t) \\ \boldsymbol{I_\Psi}(t) \end{bmatrix}$$

where $\boldsymbol{U_I}$ is identity matrix and $\boldsymbol{0}$ is matrix of zeroes. It should be noted that both $\boldsymbol{U_I}$ and $\boldsymbol{0}$ have dimension of $(2h+1) \times (2h+1)$. These equations are the basics for modeling three phase transformers.

3.1. Long Term over Voltages

The application of power factor correction capacitors in industrial systems introduces a capacitive element and this will always result in a harmonic resonance. The most common situation is a parallel resonance where the shunt connected capacitor bank appears to be in parallel with the system impedance with respect to the harmonic source. Long term over voltages in capacitor banks due to transformer energizing is a common problem in low voltage distribution systems. This phenomenon shortens capacitor banks useful life. A typical configuration where long term over voltages may occur is shown in Fig. 2.

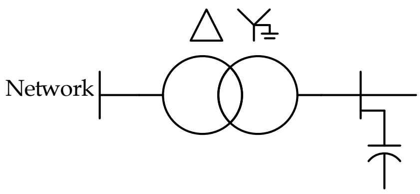

Fig. 2. Three-phase infeed transformer energization, with capacitor bank

In this case, equivalent circuit will be as shown in Fig. 3 in which magnetization branch acts as a harmonic source. This harmonic source is time varying and depends on source voltage and transformer magnetization characteristic. For the case of simplicity resistance which models the core loss is not shown in Fig .3.

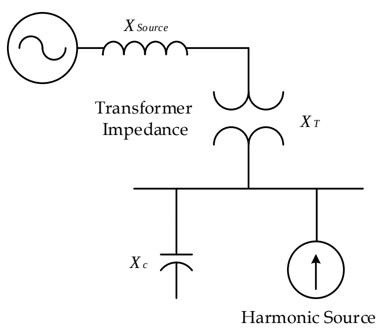

Fig. 3. Equivalent circuit of Fig. 2

If the ensuing parallel circuit is tuned to a harmonic frequency component of the inrush current then a voltage magnification will take place. In the long range, if over-voltages occur frequently then the life expectancy of the capacitors will be very much reduced. The resonant frequency of this parallel capacitive and inductive combination system can be calculated using any one of the following formulas [19]:

$$f = \sqrt{X_C / X_L} \times f_0 \quad (17)$$

$$f = \sqrt{MVA_{SC} / MVA_{CAP}} \times f_0 \quad (18)$$

$$f = 1/2\pi\sqrt{LC} \quad (19)$$

$$f = \sqrt{(kV_{LL})^2 / (MVA_{CAP} \times X_{SC})} \times f_0 \quad (20)$$

Where

$$X_C = (kV_{LL})^2 / MVA_{CAP} \text{ in ohms} \quad (21)$$

$$X_L = (kV_{LL})^2 / MVA_{SC} \text{ in ohms} \quad (22)$$

MVA_{SC}	Three phase short circuit MVA at capacitor bank
MVA_{CAP}	Three phase Mvar of the capacitor
L	Circuit inductance in henries
C	Circuit capacitance in farads
kV_{LL}	Line to line voltage in kV
X_{SC}	Short circuit reactance at capacitor bank in ohms

4. Simulation Results

In order to access the effectiveness and precision of EHD approach in analyzing of long term over voltages, three single phase transformer as depicted in Fig. 1 are connected as a transformer bank which is shown in Fig. 2. Also a capacitor bank is connected at the secondary side in order to provide power factor correction. Eq. (12) is used to describe magnetization branch of each single phase transformer in which a, b and n are 0.7576, 1.03×10^7 and 19, respectively.
The other transformer parameters are $r_1 = 0.096\Omega$, $l_1 = 0.9mH$ and $r_{mag} = 612.86\Omega$.

In this case, consider a three phase transformer connected in delta-wye and fed with a balanced voltage with magnitude of 100V and 60Hz. Also assume a star connected capacitor bank at secondary side which has a value of 886.6μF per phase. This value leads to third harmonic resonant (180Hz) according to Eq. (19)..

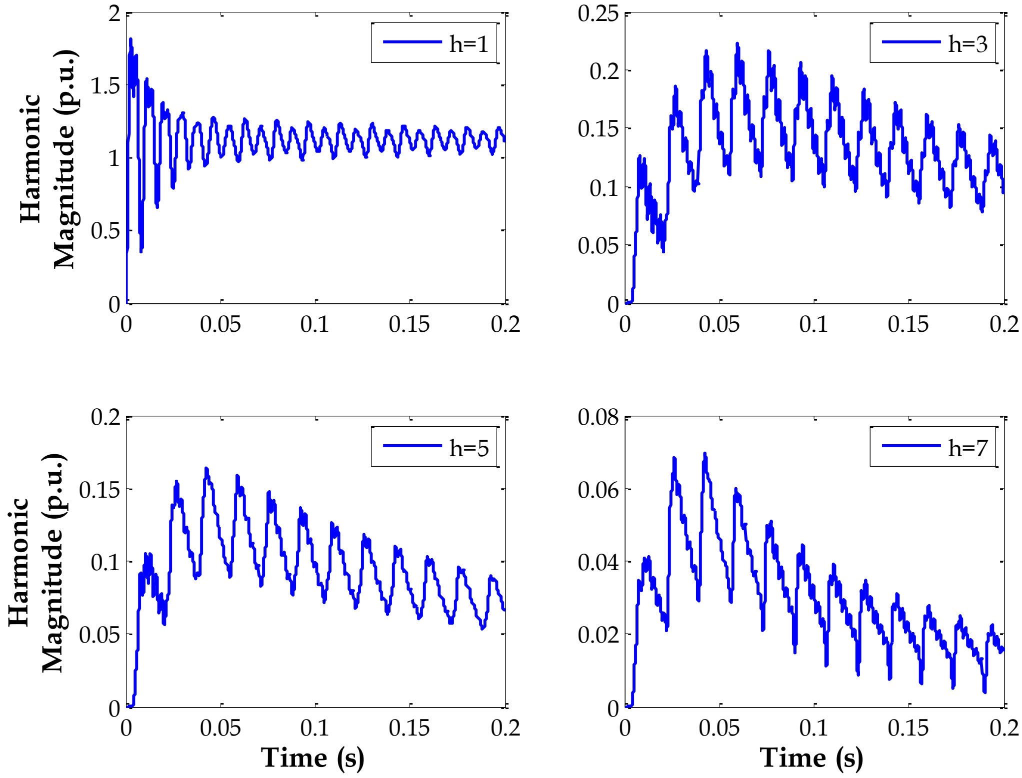

Fig. 4. Harmonic content of phase *a* capacitor voltage

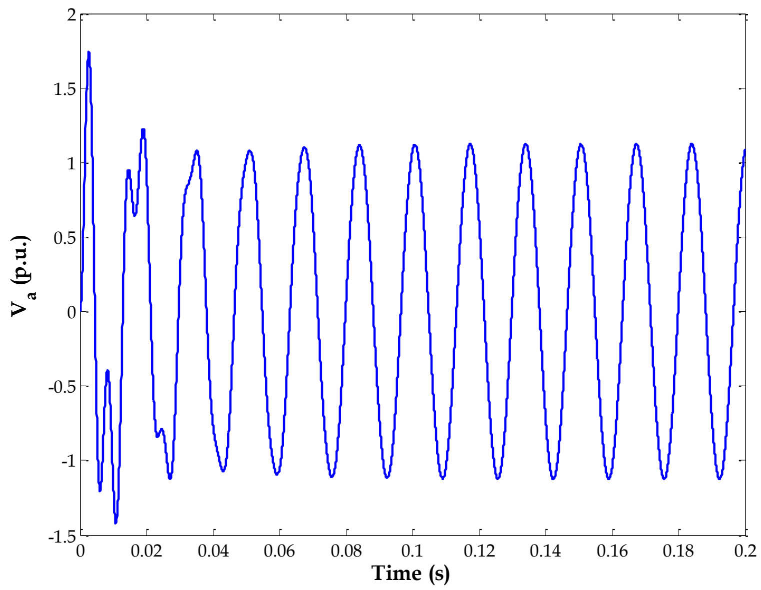

Fig. 5. Time domain response of phase *a* capacitor voltage

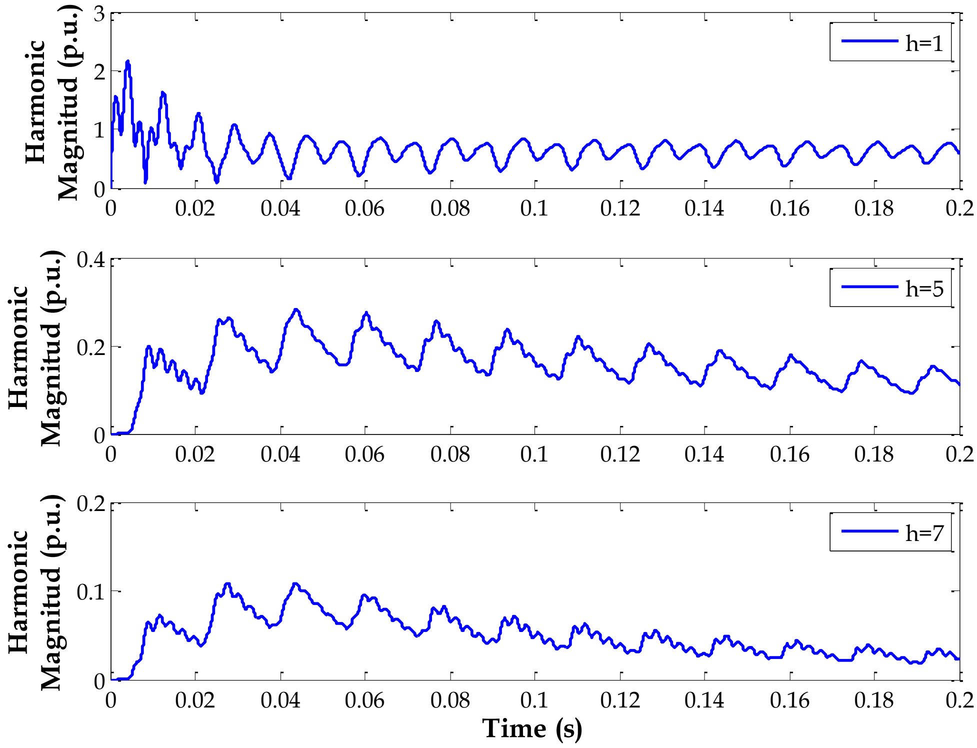

Fig. 6. Harmonic content of phase *a* current

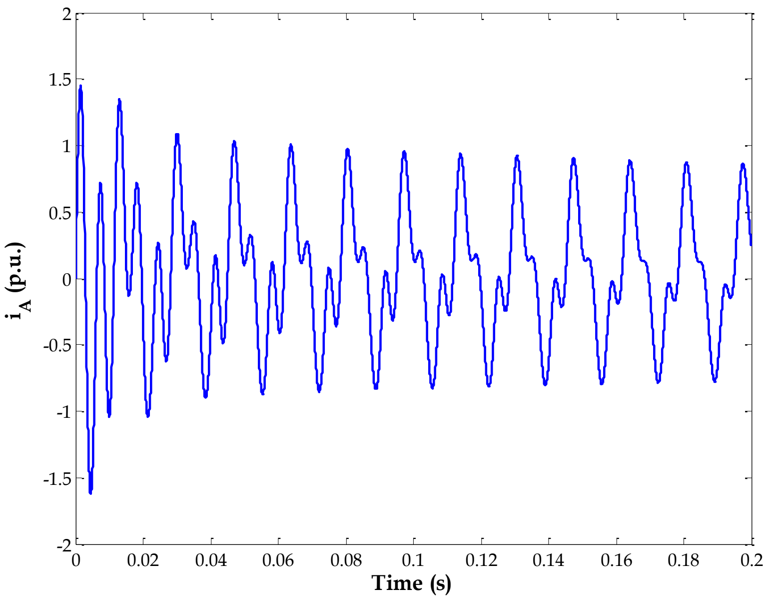

Fig. 7. Time domain response of phase *a* inrush current

It should be noted that all initial conditions are set to zero and base values for voltage and apparent power are 100V and 10KVA, respectively.

4.1. Harmonic content and Time Response

Harmonic content of phase *a* voltage of the secondary side capacitor is depicted in Fig. 4 for nine selected harmonics. Also secondary voltage time domain response is depicted in Fig. 5

Time domain representation in Fig. 5 shows large over voltage on the secondary side, resulting from the interaction between inrush currents with significant harmonic pollution and the parallel resonant circuit formed by the capacitor bank and transformer. It should be noted that these voltages have values above 50% of their nominal value during the first peak of the transient in all three phases. Their rate of attenuation is also quite slow, particularly in phase *C*, where after the 15th cycle the voltage is still 15% above the rated value. Table I lists the peak values of three phase capacitor bank voltages.

This table emphasizes on the significant over voltages during the first cycle.

Table I. Peak value of three phase secondary voltages (p.u.)

phase	Peak value (p.u.)
a	1.743
b	1.514
c	1.680

Harmonic content of phase *a* current and time domain response of three phase primary inrush currents are depicted in Figs 6 and 7, respectively. As Figs. 5 and 7 imply three phase waveforms are significantly distorted.

4.2. Comparison of Harmonics with WFFT and EHD

In some power system analysis, one must require to compute the harmonic content of a signal during a disturbance. One of the most used techniques is WFT also known as short-time Fourier transform (STFT), sliding Fourier transform or Windowed Fast Fourier Transform (WFFT). WFFT is a tool which uses FFT by a sliding window in a period as shown in Fig. 8.

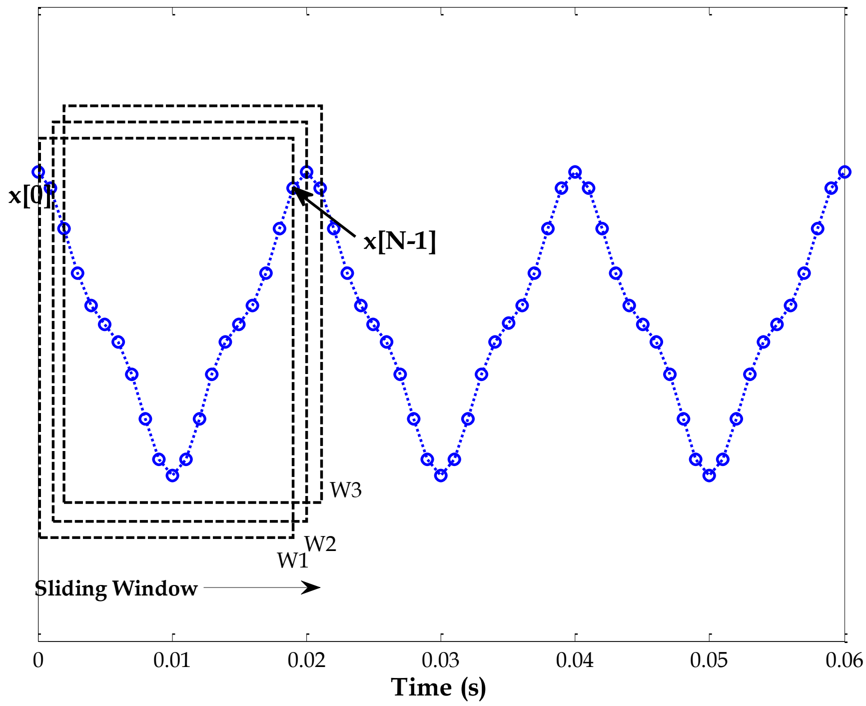

Fig. 8. Sliding window over a discrete signal

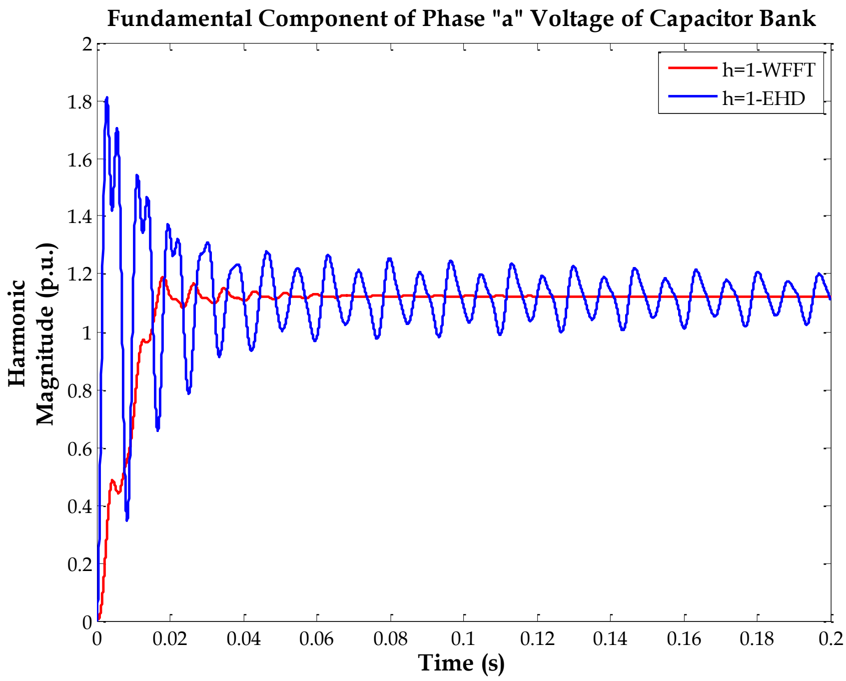

Fig. 9 .Comparison of WFFT and EHD

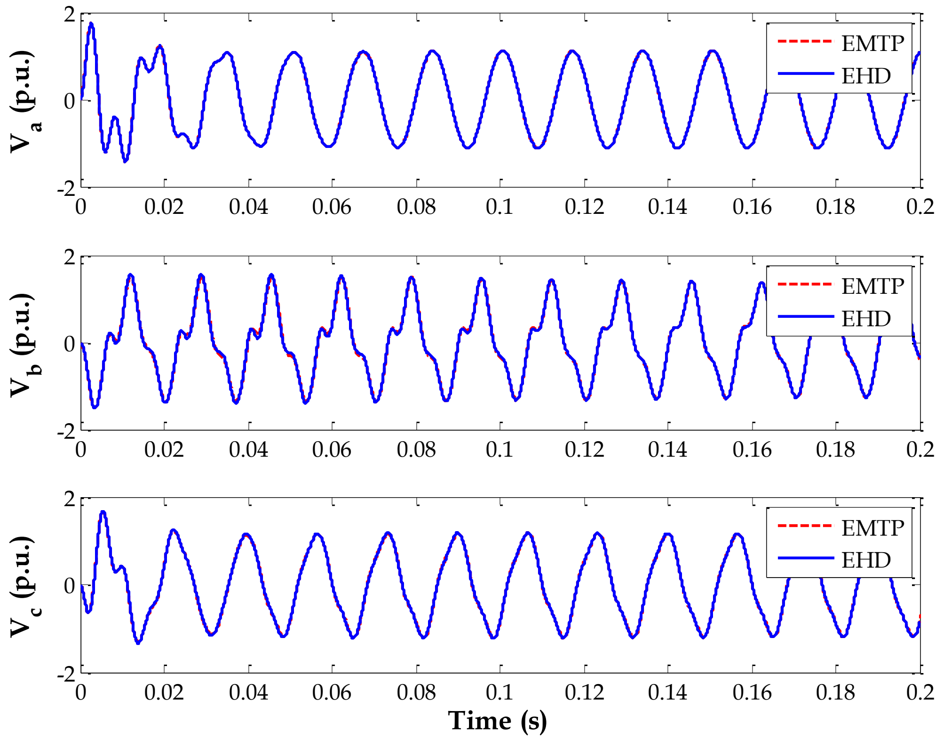

Fig. 10. Secondary side voltages

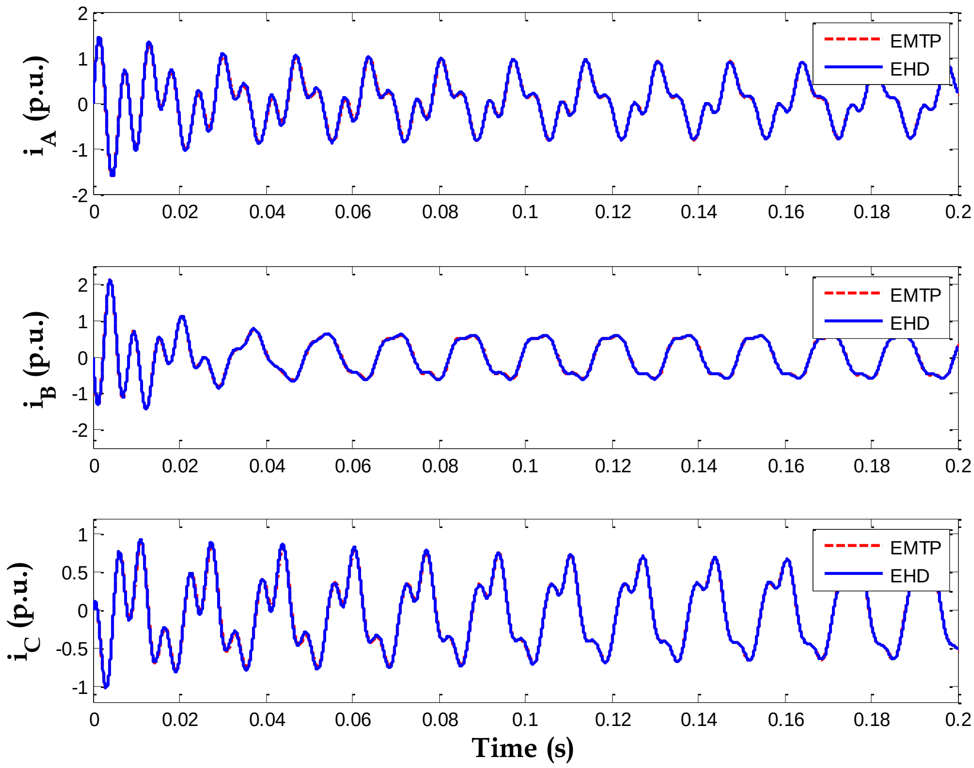

Fig. 11. Three phase inrush currents

Fig. 9 provides a comparison between WFFT and EHD for calculating magnitude variation of fundamental component of phase *a* voltage of the capacitor bank. As this figure implies WFFT shows that steady state response is achieved after 0.073s, but EHD indicates that voltage remains in transient period since the harmonic magnitude is not constant which can be clearly seen in Figs. 10 and 11. However, results are exactly same when steady state response is achieved.

4.3. EMTP Results

In order to validate EHD time domain results (calculated by Eq. (3)), Electro Magnetic Transient Program (EMTP) is used. Time domain response of three phase secondary side voltages and three phase primary currents are depicted in Figs. (10) and (11), respectively. It should be noted that 11 harmonics are used in EHD. As these figures imply both results match very well. Table II shows the effects of this parameter on the required time for a 0.2 sec simulation.

Table II. Effects of number of harmonics on required simulation run time

Number of Harmonics	Required Time for 0.2s Simulation
3	2.467
7	6.269
9	9.841
11	14.394

As this table shows higher values for number of selected harmonics greatly affects the required simulation run time. Fig. 12 depicts the phase *a* inrush current waveform for different numbers of selected harmonics.

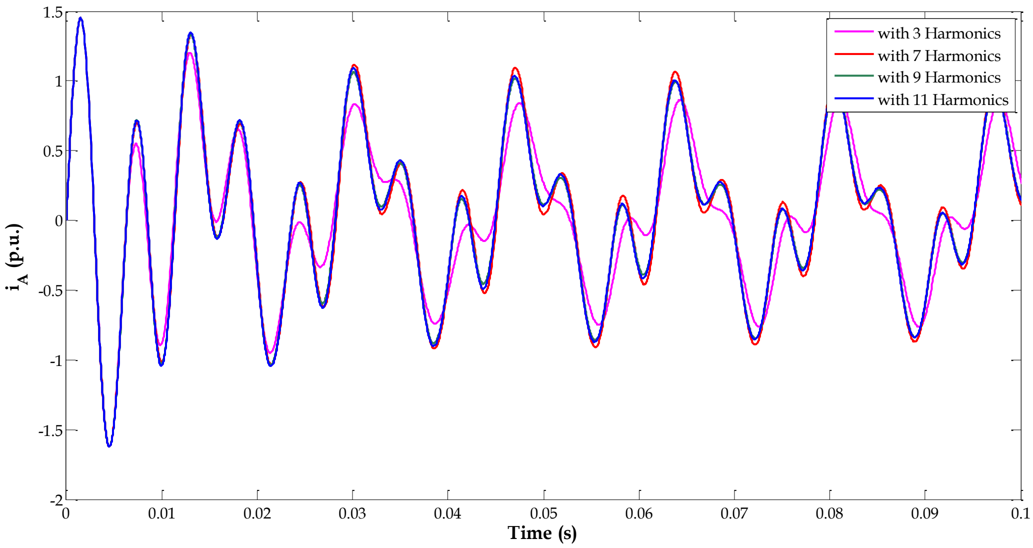

Fig. 12. Effects of the number of selected harmonics on the inrush current waveform

According to results of Fig. 12 and Table II, a compromise between accuracy and required simulation time is essential.

5. Conclusion

This paper described the dynamic harmonic analysis of long term over voltages by using the Extended Harmonic Domain method. The proposed solution is fully frequency dependent, which provides step-by-step procedure for following the harmonics evolution with respect to time. The proposed solution were applied to a test system and results were compared with those are obtained through time domain software. It was shown that if transformer and capacitor bank are energized simultaneously, significant over voltages with high harmonic distortion may take place which can reduce useful life of transformer and capacitor bank.

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