

Ergodic Capacity Approximations in Distributed Reconfigurable Intelligent Surface Channels

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Abstract:

This paper analyzes the Ergodic Capacity (EC) as a key performance metric for reconfigurable intelligent surface (RIS)-aided communication channels. We classify these channels into Centralized RIS (C-RIS) and Distributed RIS (D-RIS) configurations. A primary challenge in this area is deriving tractable closed-form EC expressions, especially for D-RIS, as existing methods often yield complex results that are impractical for system design. To address this gap, we propose three novel approximations. First, we derive a precise closed-form EC expression for D-RIS systems using Meijer-G functions, which also serves as a general framework that includes C-RIS as a special case. Second, we introduce two efficient numerical approximations based on the Gauss-Laguerre method, providing an excellent balance between high accuracy and low computational complexity. All proposed methods are rigorously validated against Monte Carlo simulations. Furthermore, using these approximations, we provide new insights into how system parameters, such as the number of RIS elements and transmit power, directly impact the achievable EC.

Keywords:

Distributed RIS, Ergodic capacity, Laguerre approximation, Relay channel, RIS.

1. Introduction

Reliable, high-data-rate connectivity is a fundamental requirement for 5G and 6G networks. This exponentially growing demand is particularly difficult to meet in environments with blockages and severe fading. These challenges have motivated the search for new, intelligent solutions that can overcome the limitations of wireless channels particularly with slow fading and poor scattering conditions, where multiplexing and diversity gains are difficult to achieve. In this context, Reconfigurable Intelligent Surfaces (RIS) have emerged as a paradigm shift. This technology is a scalable structure, as the designer can arbitrarily increase the number of its reflecting elements, consisting of a network of reflectors that dynamically manipulate electromagnetic waves to create a smart and programmable radio environment and to provide more favorable propagation characteristics [1].

While previous solutions, such as complex multi-input multi-output (MIMO) systems, improve reliability and data rate, they often come with high cost and power consumption. A RIS, however, offers a more efficient alternative. Its passive elements do not require dedicated power sources. By intelligently controlling the phase of reflected signals using channel state information, a RIS strengthens the received signal power at the intended destination [2]. This process enhances overall system reliability and data rate in a cost-effective and energy-efficient manner [3]. The performance of these RIS-aided systems is often evaluated using fundamental communication metrics, with channel capacity being a key performance criterion [1]-[5]. In wireless environments, the channel power gain is subject to fast fluctuations due to fading. This makes the Ergodic Capacity (EC) as the main performance metric over these varying conditions [4]. The EC is defined as the average of the instantaneous channel capacity across all possible channel realizations [4], [5]. This metric is especially important for RIS-aided channels where the instantaneous CSI is not known at transmitters, providing a key measure for the long-term expected performance of the system.

RIS-aided channels can be classified into two main categories: Centralized RIS (C-RIS) and Distributed RIS (D-RIS). In a C-RIS configuration, all the phase-shifting elements are located on a single surface, and this is the standard setup referred to as "RIS" in most literature. In a D-RIS setup, also known as a multi-RIS system, the elements are distributed across multiple, potentially distant surfaces [5], [6]. D-RIS offers greater flexibility in coverage enhancement and improved spatial diversity compared to C-RIS, making it more suitable for practical deployment scenarios such as urban and indoor environments.

Deriving a tractable, closed-form expression for the EC in such systems is a significant challenge. The primary difficulty stems from the complex statistical distribution of the composite source-RIS-destination (S-RIS-D) channel gain. This challenge is more severe in D-RIS systems. Unlike in C-RIS, where channels are often assumed to be statistically identical, the individual S-RIS-D channels in a D-RIS setup can have different statistical properties, making a unified analysis intractable.

Due to its simpler implementation and earlier introduction, research has primarily focused on C-RIS rather than D-RIS. The majority of studies for both configurations have concentrated on optimizing beamforming schemes to maximize the received signal-to-interference-plus-noise ratio (SINR) [7], [8], with other common metrics being sum rate [9], [10] and energy efficiency [3]. Studies specifically addressing the EC in RIS-aided systems are limited; examples of related work include [4], [7], [8], [11-16]. As a significant work, [8] proposed a Gamma distribution to approximate the composite S-RIS-D channel gain distribution. This approximation was shown to match Monte Carlo simulations closely for metrics like EC, outage probability, and outage capacity. More recent studies are as follows. Statistically optimal beamforming in a single-user scenario is conducted in [12], and extending this work to a multi-user scenario is achieved in [13, 14]. Optimal EC in a RIS-assisted system with an unmanned aerial vehicle (UAV) is studied in [7]. Generalization of channel models to the Nakagami-m fading model is also of interest and has been studied in [15, 16].

Research on EC for D-RIS systems is even scarcer. Related works include [6, 10]. In [10], the sum rate in a multi-RIS system is optimized. Meanwhile, [6] considers a multi-RIS mmWave system and exploits statistical channel state information (CSI) for beamforming.

The limited research on EC for D-RIS systems, despite their importance for future wireless networks, provides a primary motivation for this work. Furthermore, while prior analyses for RIS-aided systems have derived EC expressions, these often rely on complex special functions or integral forms that are computationally intensive and offer limited analytical insight for system design. Approximations are essential as they provide tractable forms that enable rapid performance evaluation and design insights without relying on complex numerical integration. Therefore, accurate and mathematically tractable EC approximations are needed to provide deeper insights into system parameters.

This paper proposes three novel approximations for the EC in both C-RIS and D-RIS aided systems. Several recent works have analyzed the ergodic capacity of RIS-assisted systems using Meijer-G functions [17], [18], [19]. However, those work are limited to a single RIS, i.e., C-RIS. In particular, [18] employs a Gaussian approximation based on the central limit theorem (CLT). In contrast, our work addresses the general D-RIS architecture and provides a unified framework that includes C-RIS as a special case. Moreover, we adopt a Gamma approximation, which offers superior accuracy for cascaded RIS channels, our Meijer-G-based expression is exact, and we further propose two simple Gauss-Laguerre quadrature-based approximations for reduced computational complexity. First, we extend the approach from [8] to D-RIS systems, deriving a closed-form EC expression based on Meijer-G functions. It is emphasized that our analysis is general, and the C-RIS configuration is treated as a particular instance of the derived D-RIS expressions. Subsequently, we propose two simpler numerical approximations. Both are based on the Gauss-Laguerre method for computing integrals. By sacrificing the negligible performance gain of Approximation 1, Approximation 2 achieves significantly higher computational efficiency, as its Laguerre roots can be easily obtained from a standard lookup table for rapid evaluation.

The accuracy of all proposed approximations is validated against Monte-Carlo simulations. Furthermore, the analysis provides valuable insights into the impact of key system parameters, such as the number of RIS elements, fading severity, and transmit power, on the achievable EC. The main contributions of this paper are summarized as follows:

- We derive a tractable closed-form EC expression for D-RIS systems using Meijer-G functions, with C-RIS emerging as a special case.
- We develop two Gauss-Laguerre quadrature-based approximations that circumvent the computational burden of Meijer-G functions while maintaining high accuracy.
- We characterize the sensitivity of the EC to variations in system parameters, including the number of RIS elements, fading severity, and transmit power.

To the best of our knowledge, this is the first work to provide tractable closed-form EC approximations for D-RIS systems with a unified framework covering both C-RIS and D-RIS configurations.

The remainder of this paper is organized as follows. Section II details the system and signal model for both C-RIS and D-RIS configurations. Section III introduces the EC and the equivalent Gamma channel model. The three proposed EC approximations are derived and presented in Section IV. Section V validates the accuracy of these approximations against Monte Carlo simulations through numerical results. Finally, the paper is concluded in Section VI.

The following notation is used in this manuscript: lowercase and uppercase boldface letters denote vectors and matrices, respectively, and $E(\cdot)$, ^{T} , and $|\cdot|$ denote mathematical expectation, transpose, and absolute value or determinant operators. A Rayleigh random variable (RV) X with the scale parameter σ is denoted as [20]

$$X \sim \text{Rayleigh}(x: \sigma^2) = \frac{x^2}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}, \quad x \geq 0 \quad (1)$$

A Gamma RV X with the shape-rate parameterization α and λ is denoted as [20]

$$X \sim \text{Gamma}(x: \alpha, \lambda) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}, x \geq 0 \quad (2)$$

where $\Gamma(\alpha)$ is the Gamma function.

2. System Model

As shown in Fig. 1, the D-RIS wireless network consists of the source (S), the destination (D) and the group of RIS nodes. Nodes S, and D use a single antenna but each node of RIS may include multi reconfigurable reflectors. Total number of RIS reflectors is L . Each packet of the signal is transmitted by Node S, reflected by RIS nodes and received by Node D.

The channel gain between Node S and l th node of RIS is denoted by $\tilde{h}_{Sl}$, and the channel gain between the l th node of RIS and Node D is denoted by $\tilde{h}_{lD}$. The following channel gain vectors

$$\tilde{\mathbf{h}}_S = [h_{S1}e^{-j\phi_{S1}}, \dots, h_{SL}e^{-j\phi_{SL}}]^T \quad (3)$$

$$\tilde{\mathbf{h}}_D = [h_{1D}e^{-j\phi_{1D}}, \dots, h_{LD}e^{-j\phi_{LD}}]^T \quad (4)$$

where $h_{Sl} = |\tilde{h}_{Sl}|$, $h_{lD} = |\tilde{h}_{lD}|$, are complex random vectors with independent zero-mean complex Gaussian distributed components [8], meaning the channel gain random variables (RVs) h_{Sl} or h_{lD} , $l = 1, \dots, L$, are Rayleigh distributed RVs [20], i.e.,

$$h_{Sl} \sim \text{Rayleigh}\left(x: \frac{\sigma_{Sl}^2}{2}\right) \quad (5)$$

$$h_{lD} \sim \text{Rayleigh}\left(x: \frac{\sigma_{lD}^2}{2}\right) \quad (6)$$

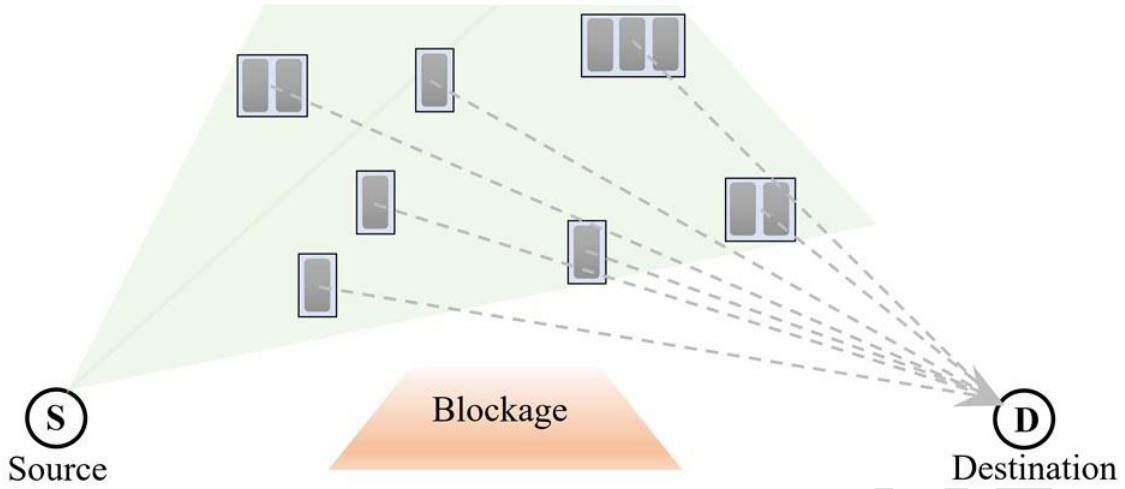


Fig. 1. D-RIS system including several panels in different geographical places with total L RIS elements.

where $\sigma_{S_l}^2 = E(h_{S_l}^2)$, $\sigma_{D_l}^2 = E(h_{D_l}^2)$. Also, ϕ_{S_l} or ϕ_{D_l} , $l = 1, \dots, L$, are distributed according to a uniform pdf within interval $[0, 2\pi]$. The elements of vectors $\mathbf{h}_S = [h_{S1}, \dots, h_{SL}]^T$ or $\mathbf{h}_D = [h_{1D}, \dots, h_{LD}]^T$ are not necessarily i.i.d but independent and their pdf parameters are determined mainly based on the distances between nodes and the path-loss exponent. Direct link between nodes S and D is not considered in the modeling due to severe path-loss and/or shadowing [11]. A block fading channel model is considered where the system channels are constant in time duration, named block, but independently varies block to block [21]. The received SNR in Node D is given as

$$\gamma_D = \rho_S \left| \sum_{l=1}^L h_{S_l} h_{lD} e^{-j(\phi_{S_l} + \phi_{lD} - \theta_l)} \right|^2 \quad (7)$$

where $\rho_S = P_S/N_0$, is the transmit power by Node S over the noise power in Node D, and θ_l is the phase designed by the RIS node at the l th reflector. Since the phase information ϕ_{S_l} and ϕ_{lD} are available in the corresponding RIS (see e.g., [3]), the optimum phase designed by RIS is obtained as $\theta_l = \phi_{S_l} + \phi_{lD}$, $l = 1, \dots, L$, yielding the received SNR in Node D as

$$\gamma_D = \rho_S \left(\sum_{l=1}^L h_{S_l} h_{lD} \right)^2 = \rho_S (\mathbf{h}_S^T \mathbf{h}_D)^2 \quad (8)$$

The global CSI available to adjust ideal phase compensation, as also considered in the seminal work [3], can be acquired through training-based channel estimation, where pilot signals are transmitted and the cascaded channel is estimated at the receiver and fed back to the RIS controller. The associated overhead scales with the number of RIS elements and the channel coherence time. The assumed continuous phase compensation in this work, as considered in many studies, e.g., [3], provides an upper bound on the actual achievable EC. Practical RIS implementations typically employ discrete phase shifts; however, as the number of quantization levels increases, the discrete phase shifts converge to continuous phase shifts, and the achievable EC approaches this bound. In practice, at least 3-bit quantization is sufficient for the performance to become indistinguishable from the continuous case [22].

3- EC and RIS Equivalent Channel Model

Considering a fast block fading channel the ergodic capacity is defined as follows [5]

$$C_{SLD} = E(\log(1 + \gamma_D)) \quad (9)$$

where γ_D is defined in (8). By defining RV X_l as

$$X_l = h_{Sl}h_{lD} \quad (10)$$

Eq. (9) is written as

$$C_{SLD} = E\left(\log\left(1 + \rho_S(\sum_{l=1}^L X_l)^2\right)\right) \quad (11)$$

To calculate (11), firstly, pdf of each term in the summation is required. In [8] a Gamma pdf is proposed to approximate distribution of X_l , i.e., $X_l \sim \text{Gamma}(x: \alpha_l, \lambda_l)$, and it is shown the approximation is very tight with the order of $1E-7$. To obtain shape-rate parameters, i.e., α_l, λ_l , the first and the second moments are matched to those of the Gamma distribution;

$$E(X_l) = E(h_{Sl})E(h_{lD}) = \frac{\pi}{4} \sigma_{Sl} \sigma_{lD} = \frac{\alpha_l}{\lambda_l} \quad (12)$$

$$E(X_l^2) = E(h_{sl}^2)E(h_{ld}^2) = \sigma_{sl}^2 \sigma_{ld}^2 = \frac{\alpha_l + \alpha_l^2}{\lambda_l^2} \quad (13)$$

yielding

$$\alpha_l = \frac{\pi^2}{16 - \pi^2} \triangleq \alpha_0 \quad (14)$$

and

$$\lambda_l = \frac{4\alpha_0}{\pi} \frac{1}{\sigma_{sl}\sigma_{ld}}. \quad (15)$$

Using the pdf of the summation over X_l s, i.e.,

$$X = \mathbf{h}_S^T \mathbf{h}_D = \sum_{l=1}^L X_l \quad (16)$$

is obtained by L times convolution over pdf of X_l which is cumbersome, instead, we can approximate pdf of X by Gamma distribution. This idea is already proposed in [8] for i.i.d RVs with Gamma distribution. Gaussian distribution is another option to approximate pdf of X . However, since X is always positive and L is not necessarily high, the central limit theorem is not hold, then, Gamma approximation is preferred, i.e.,

$$X \sim \text{Gamma}(x: \alpha, \lambda) \quad (17)$$

Using the same approach to obtain (14) and (15) accompanied with the fact of independency of X_l with respect to (wrt) each other, α and λ are obtained as follows

$$\alpha = \alpha_0 \frac{(\sum_{l=1}^L \sigma_{sl}\sigma_{ld})^2}{\sum_{l=1}^L \sigma_{sl}^2 \sigma_{ld}^2} \quad (18)$$

$$\lambda = \frac{4\alpha_0}{\pi} \frac{\sum_{l=1}^L \sigma_{sl}\sigma_{ld}}{\sum_{l=1}^L \sigma_{sl}^2 \sigma_{ld}^2} \quad (19)$$

where α_0 is the constant defined in (14). Note that the analysis made in this paper is general and covers all systems ranging from L distantly separated single-element RIS nodes to a single RIS with L elements, as well as intermediate configurations. For instance, for a system with two RIS panels each with $L_i, i =$

1,2 elements, $L_1 + L_2 = L$, we have $\sigma_{Sl} = \sigma_{S(i)}, \sigma_{lD} = \sigma_{(i)D}, i = 1, l \leq L_1$ and $i = 2, L_1 < l \leq L_2$.

Therefore α and λ in (18) and (19) are respectively simplified to

$$\alpha = \alpha_0 \frac{(\sum_{i=1}^2 L_i \sigma_{S(i)} \sigma_{(i)D})^2}{\sum_{i=1}^2 L_i \sigma_{S(i)}^2 \sigma_{(i)D}^2}$$

$$\lambda = \frac{4\alpha_0}{\pi} \frac{\sum_{i=1}^2 L_i \sigma_{S(i)} \sigma_{(i)D}}{\sum_{i=1}^2 L_i \sigma_{S(i)}^2 \sigma_{(i)D}^2}$$

4- EC Approximation

Using (11), (16) and (17), we have

$$C_{SLD} = \int_0^\infty \log(1 + \rho_S x^2) \text{Gamma}(x; \alpha, \lambda) dx \quad (20)$$

where α and λ are given in (18) and (19), respectively.

4-1- Closed form approximation

Applying (2) in (20) yields

$$C_{SLD} = \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^\infty \log(1 + \rho_S x^2) x^{\alpha-1} e^{-\lambda x} dx \quad (21)$$

Integral (21) can be solved using Meijer-G functions. We have the following proposition.

Proposition 1: a closed form approximation for the EC of a D-RIS with L reflectors is given as follows

$$C_{SLD} = \frac{2^{\alpha-1}}{\Gamma(\alpha)\sqrt{\pi}} G_{4,2}^{1,4} \left(\frac{4\rho}{\lambda^2} \left| \frac{1}{2} - \frac{\alpha}{2}, 1 - \frac{\alpha}{2}, 1, 1 \right. \right)_{1,0} \quad (22)$$

where $G(\cdot)$ is Meijer-G function and α and λ are given in (18) and (19), respectively.

Proof: in order to prove (22) we need the following two properties of Meijer-G functions [23]

$$x^\epsilon G_{p,q}^{m,n} \left(x \left| \begin{matrix} a_1, \dots, a_p \\ b_1, \dots, b_q \end{matrix} \right. \right) = G_{p,q}^{m,n} \left(x \left| \begin{matrix} a_1 + \epsilon, \dots, a_p + \epsilon \\ b_1 + \epsilon, \dots, b_q + \epsilon \end{matrix} \right. \right), \quad (23)$$

$$\int_0^\infty e^{-\lambda x} G_{p,q}^{m,n} \left(\rho x^2 \left| \begin{matrix} a_1, \dots, a_p \\ b_1, \dots, b_q \end{matrix} \right. \right) dx = \frac{1}{\lambda \sqrt{\pi}} G_{p+2,q}^{m,n+2} \left(\frac{4\rho}{\lambda^2} \left| \begin{matrix} 0, \frac{1}{2}, a_1, \dots, a_p \\ b_1, \dots, b_q \end{matrix} \right. \right). \quad (24)$$

Then, we need to write the log-function as a Meijer-G function as follows

$$\begin{aligned} x^{\alpha-1} \log(1 + \rho_S x^2) &= x^{\alpha-1} G_{2,2}^{1,2} \left(\rho_S x^2 \left| \begin{matrix} 1, 1 \\ 1, 0 \end{matrix} \right. \right) = \rho_S^{-\frac{\alpha-1}{2}} (\rho_S x^2)^{\frac{\alpha-1}{2}} G_{2,2}^{1,2} \left(\rho_S x^2 \left| \begin{matrix} 1, 1 \\ 1, 0 \end{matrix} \right. \right) = \\ &\rho_S^{-\frac{\alpha-1}{2}} G_{2,2}^{1,2} \left(\rho_S x^2 \left| \begin{matrix} \frac{\alpha+1}{2}, \frac{\alpha+1}{2} \\ \frac{\alpha+1}{2}, \frac{\alpha-1}{2} \end{matrix} \right. \right) \end{aligned} \quad (25)$$

where the last equality used the property of (23). Using (25) in (22) we get

$$C_{\text{SLD}} = \frac{\lambda^\alpha \rho_S^{-\frac{\alpha-1}{2}}}{\Gamma(\alpha)} \int_0^\infty e^{-\lambda x} G_{2,2}^{1,2} \left(\rho_S x^2 \left| \begin{matrix} \frac{\alpha+1}{2}, \frac{\alpha+1}{2} \\ \frac{\alpha+1}{2}, \frac{\alpha-1}{2} \end{matrix} \right. \right) dx \quad (26)$$

Applying property of (24) to (26) yields

$$C_{\text{SLD}} = \frac{\lambda^{\alpha-1} \rho_S^{-\frac{\alpha-1}{2}}}{\Gamma(\alpha) \sqrt{\pi}} G_{4,2}^{1,4} \left(\frac{4\rho_S}{\lambda^2} \left| \begin{matrix} 0, \frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha+1}{2} \\ \frac{\alpha+1}{2}, \frac{\alpha-1}{2} \end{matrix} \right. \right) \quad (27)$$

Finally, applying the following equality into (27) and using property (23) completes the proof.

$$\lambda^{\alpha-1} \rho_S^{-\frac{\alpha-1}{2}} = 2^{\alpha-1} \left(\frac{4\rho_S}{\lambda^2} \right)^{-\frac{\alpha-1}{2}} \quad (28)$$

Meijer-G functions can be evaluated for any given parameters using recursive properties and their relation with generalized hyper geometric functions. Nevertheless, the EC can be further approximated using numerical approaches.

4-2- Gauss-Laguerre approximation

Gauss-Laguerre approximation of the EC, proposed in this paper, is based on generalized Gauss-Laguerre quadrature integration of (21). Firstly, we need the following theorem.

Theorem 1, (generalized Gauss-Laguerre integration) [23]: for a given analytic function $f(x)$ and $\alpha > -1$ we have

$$\int_0^\infty x^\alpha e^{-x} f(x) dx \approx \sum_{i=1}^n w_i f(x_i) \quad (29)$$

where x_i is the i th root of Laguerre polynomial $L_n(x)$ and the weight w_i is given by

$$w_i = \frac{\Gamma(n+\alpha+1)x_i}{n!(n+1)^2 \left(L_{n+1}^{(\alpha)}(x_i)\right)^2} \quad (30)$$

There is an explicit form for the Laguerre polynomial $L_n^{(\alpha)}(x)$ as follows

$$L_n^{(\alpha)}(x) = \sum_{k=0}^n \binom{n+\alpha}{n-k} \frac{(-1)^k}{k!} x^k \quad (31)$$

where

$$\binom{n+\alpha}{n-k} \triangleq \frac{\Gamma(n+\alpha+1)}{(n-k)! \Gamma(k+\alpha+1)} \quad (32)$$

Using Theorem 1, (21) can be numerically approximated which by some algebraic manipulations we get the following proposition

Proposition 2, (numerical approximation 1): the EC of a D-RIS with L reflectors is obtained using a generalized Gauss-Laguerre based approximation as follows

$$C_{\text{SLD}} \approx \frac{\binom{n+\alpha-1}{n}}{(n+1)^2} \sum_{i=1}^n \frac{x_i}{\left(L_{n+1}^{(\alpha-1)}(x_i)\right)^2} \ln \left(1 + \frac{\rho_s}{\lambda^2} x_i^2\right), \quad (33)$$

where x_i is the i th root of $L_n^{(\alpha-1)}(x)$, and

$$\binom{n+\alpha-1}{n} = \frac{\Gamma(n+\alpha)}{n! \Gamma(\alpha)}. \quad (34)$$

Proof: using Theorem 1, the proof is straightforward, then, omitted for the sake of brevity.

The EC approximation in Proposition 2 is appropriate; however, finding roots of $L_n^{(\alpha-1)}(x)$, i.e., $x_i, i = 1, \dots, n$ depends on value α which is cumbersome; because α , as given in (18), is a function of average channel gains of the network. If α be an integer, roots of $L_n^{(\alpha-1)}(x)$ can be given in a lookup table which is much straightforward; because the generalized Laguerre polynomials for integer orders are well-tabulated as classical orthogonal polynomials. Their roots are fixed, real, distinct, and can be pre-computed offline for standard values of n and integer $\alpha - 1$. This allows for instantaneous root acquisition in practice, eliminating the need for iterative numerical root-finding algorithms during system execution. Thus, in the following proposition, we present a slightly different version of the EC approximation presented in (33).

Proposition 3, (numerical approximation 2): the EC of a D-RIS with L reflectors is obtained using a generalized Gauss-Laguerre based approximation as follows

$$C_{\text{SLD}} \approx \frac{(n+m)!}{n!(n+1)^2\Gamma(\alpha)} \sum_{i=1}^n \frac{x_i^{\alpha-m}}{\left(L_{n+1}^{(m)}(x_i)\right)^2} \ln\left(1 + \frac{\rho_S}{\lambda^2} x_i^2\right), \quad (35)$$

where $m = [\alpha]$, i.e., the integer part of α , and x_i is i th root of $L_n^{(m)}(x)$.

Proof: rewriting (21) as

$$C_{\text{SLD}} = \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^\infty \log(1 + \rho_S x^2) x^{\alpha-m-1} x^m e^{-\lambda x} dx \quad (36)$$

since $\alpha - m - 1 > -1$, Theorem 1 can be applied in (36) to obtain

$$C_{\text{SLD}} \approx \frac{1}{\Gamma(\alpha)} \sum_{i=1}^n w_i x_i^{\alpha-m-1} \ln\left(1 + \frac{\rho_S}{\lambda^2} x_i^2\right), \quad (37)$$

where x_i is i th root of $L_n^{(m)}(x)$, i.e., $L_n^{(m)}(x_i) = 0$ and

$$w_i = \frac{(n+m)!x_i}{n!(n+1)^2 \left(L_{n+1}^{(m)}(x_i) \right)^2} \quad (38)$$

Replacing (38) in (37) completes the proof.

As mentioned above, roots of $L_n^{(m)}(x)$ can be obtained offline and saved in a table. Also, using (31), we have

$$L_n^{(m)}(x) = \sum_{k=0}^n \binom{m+n}{m+k} \frac{(-1)^k}{k!} x^k. \quad (39)$$

4-3- Centralized RIS

As mentioned earlier, by the C-RIS, we mean the classical RIS where all RIS reflectors are located in a single surface. In this case, since $\sigma_{sl} = \sigma_S$ and $\sigma_{ld} = \sigma_D$ parameters α and λ are directly obtained using (18) and (19), respectively, as follows

$$\alpha = L\alpha_0 \quad (40)$$

$$\lambda = \frac{4\alpha_0}{\pi} \frac{1}{\sigma_S \sigma_D} \quad (41)$$

The EC in the C-RIS is obtained using Propositions 1, 2 and 3, driven so far with the parameters α and λ given in (40) and (41).

5- Numerical Results

In the considered D-RIS system, Nodes S and D are located at coordinates (-1,0) and (1,0), respectively, and L RISs are distributed uniformly in the region $x \in (-0.5, 0.5), y \in (0.5, 1)$. The path-loss exponent between any pair of nodes is 3. The average channel power gain between two nodes A and B is modeled as $\sigma_{AB}^2 = d_{AB}^{-3}$, where d_{AB} is the normalized distance by a reference distance [24]. The Gauss-Laguerre approximation order is considered as $n = 2$. All results are averaged over 1E8 Monte Carlo trials

implemented in Python. Python was used for its open-source scientific libraries and efficient special function handling.

Fig. 2 shows EC vs. SNR(dB) from simulations, Meijer-G approximation and two Gauss-Laguerre-based approximations for $L \in \{2, 8\}$. The trend of EC improvement wrt SNR is evident. The three approximations presented in this paper behave very closely to the simulation results. Fig. 3 presents the EC versus SNR for different pathloss exponents $\nu \in \{2, 3, 4\}$, where higher EC corresponds to lower ν , and the close behavior of the three approximations with respect to simulations is also evident.

To evaluate the accuracy of the proposed approximations against the simulation results, the relative error (RE) is defined as follows:

$$RE \triangleq \frac{|EC_{\text{simulation}} - EC_{\text{approximation}}|}{EC_{\text{simulation}}} \quad (42)$$

Here, $EC_{\text{simulation}}$ denotes the ergodic capacity obtained from Monte Carlo simulations, where the expectation in (11) is computed by averaging over a large number of channel realizations, and $EC_{\text{approximation}}$ denotes the approximated ergodic capacity obtained from the proposed analytical expressions.

In Fig. 4 RE vs. SNR for Meijer-G and two Gauss-Laguerre-based approximations is shown for $L = 2$. It is seen that the Meijer-G approximation is very tight with an RE on the order of $O(10^{-4})$ and the RE for other two approximations is less than 2%. Similar results are shown in Fig. 5 for $L = 8$, where the tightness is significantly improved for all approximations.

Fig. 6 shows the EC vs. number of RISs, i.e., L , for simulations, the Meijer-G approximation and two Gauss-Laguerre-based approximations for $SNR \in \{5, 15\}$. The trend of EC improvement wrt L and the close agreement of the approximations with the simulation result are evident. In Fig. 7, the RE versus L is shown for the Meijer-G and two Gauss-Laguerre-based approximations at $SNR = 5$. It is seen that the Meijer-G approximation is very tight with an RE on the order of $O(10^{-4})$ and the RE for the other two approximations is on the order of $\sim O(10^{-3})$ which is acceptable for EC approximation. Similar results

are shown in Fig. 8 for $\text{SNR} = 15$ where the approximations maintain a similar order of tightness as observed in Fig. 7.

The aforementioned results were for $n = 2$, i.e., the worst case. Fig. 9 shows the RE versus n for the two Gauss-Laguerre-based approximations at $\text{SNR} = 5$ dB and $L = 2$. It is seen that for $n > 5$, these approximations perform as well as the Meijer-G approximation. Similar results are shown for $\text{SNR} = 15$ dB in Fig. 10, where all performances converge for $n > 7$. This demonstrates that with just a few terms, i.e., a small n , the two Gauss-Laguerre-based approximations perform as well as the Meijer-G approximation. This is particularly promising for Approximation 2, as its implementation is much simpler than that of Approximation 1 and the Meijer-G-based one.

Regarding the complexity of the three approaches to compute EC, the Meijer-G-based expression has high computational cost. In our Python-based implementation, it is approximately 10 times slower than Approximation 1. Approximation 1 requires $O(n)$ operations. Approximation 2 uses pre-computed roots from a lookup table, making it slightly faster for $n < 10$ and approximately twice as fast for $10 < n < 20$.

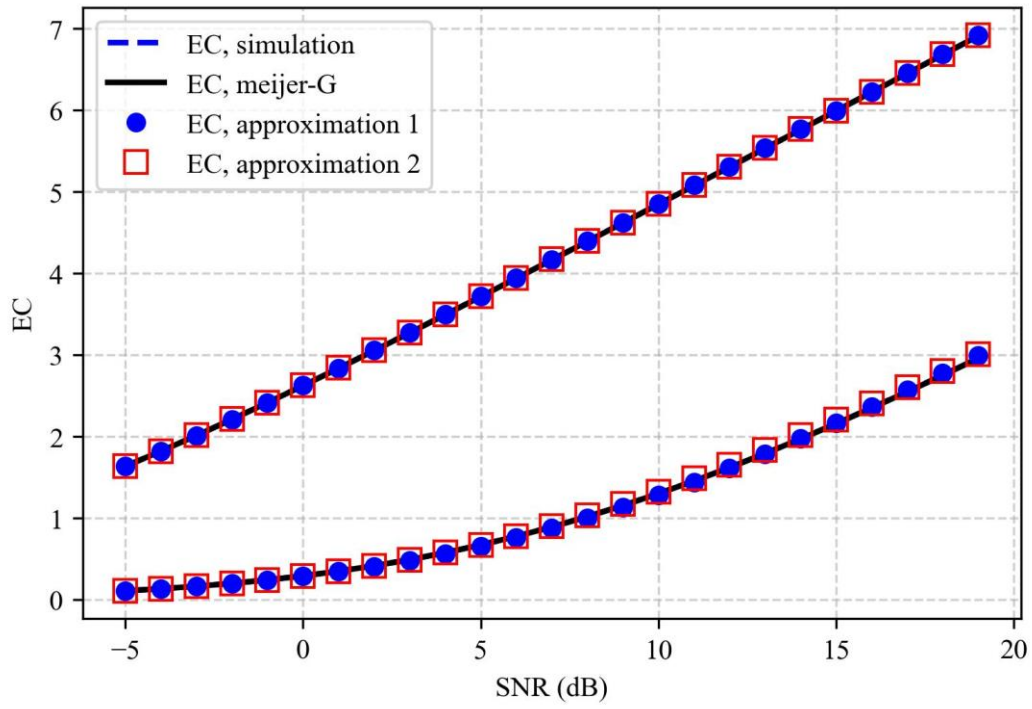


Fig. 2. EC vs. SNR for $L \in \{2, 8\}$.

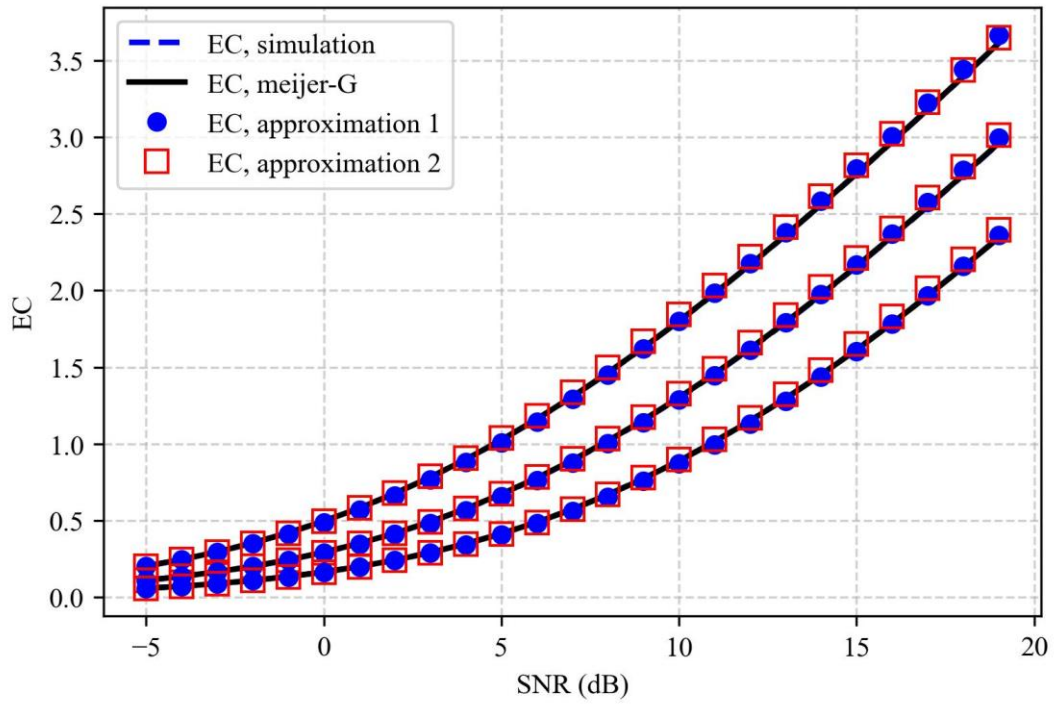


Fig. 3. EC vs. SNR for $L = 2$ and different pathloss exponent: $\nu \in \{2, 3, 4\}$ where higher EC corresponds to lower ν .

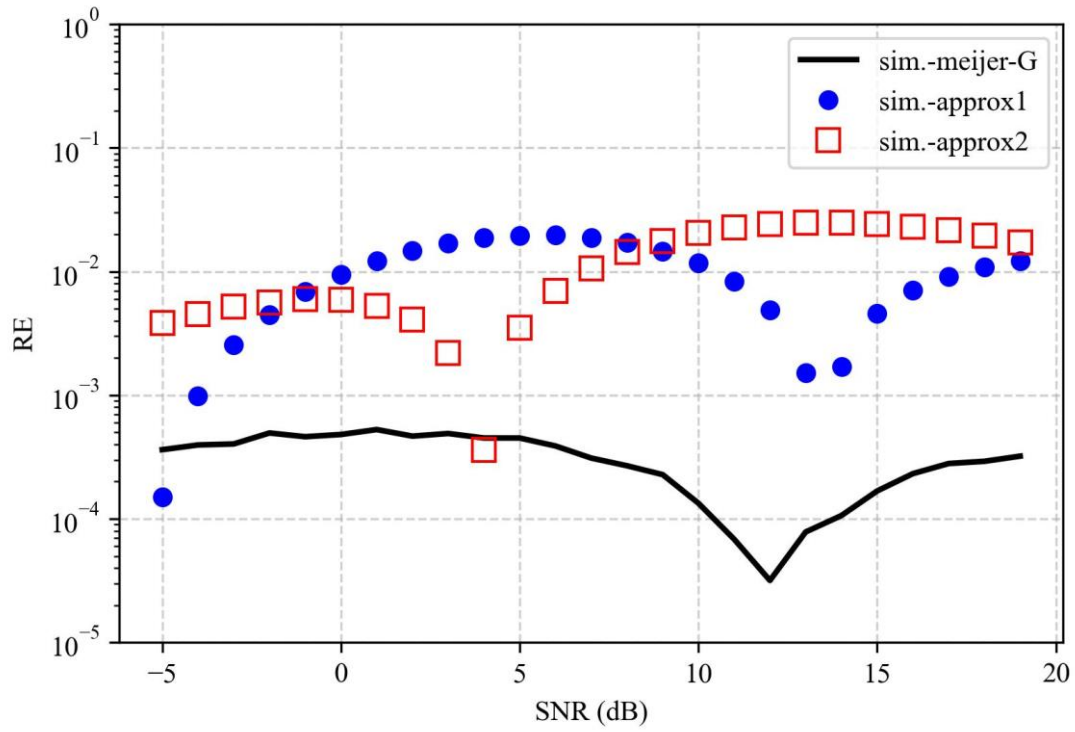


Fig. 4. RE wrt simulation results vs. SNR for Meijer-G and two Gauss-Laguerre based approximations in $L = 2$.

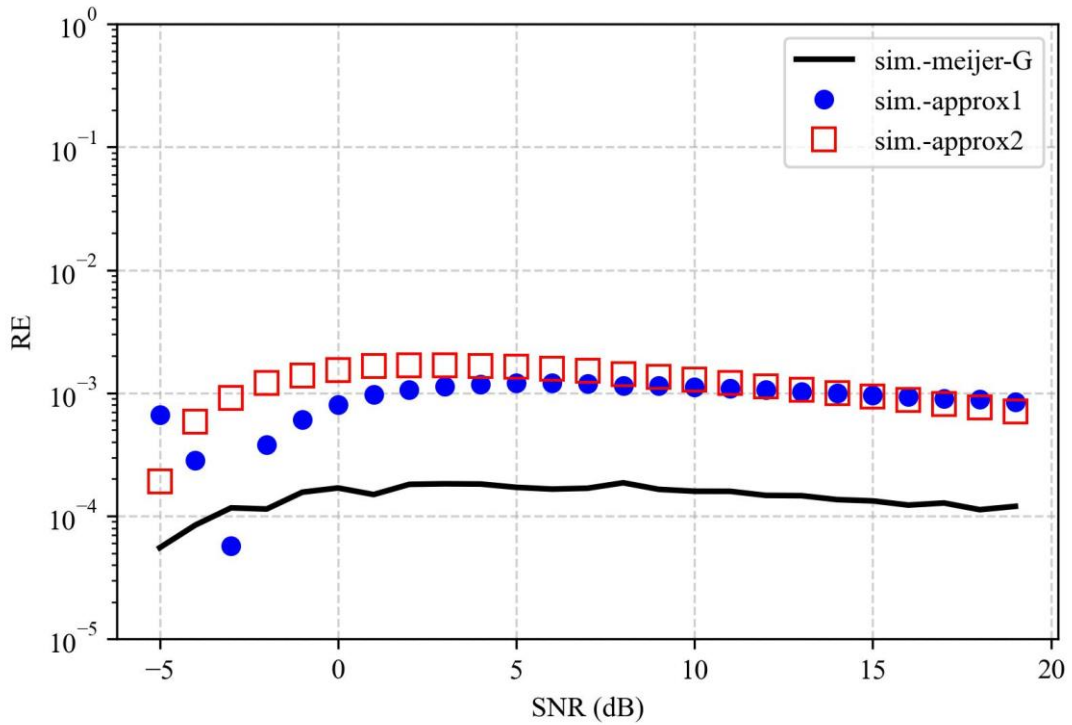


Fig. 5. RE wrt simulation results vs. SNR for Meijer-G and two Gauss-Laguerre based approximations in $L = 8$.

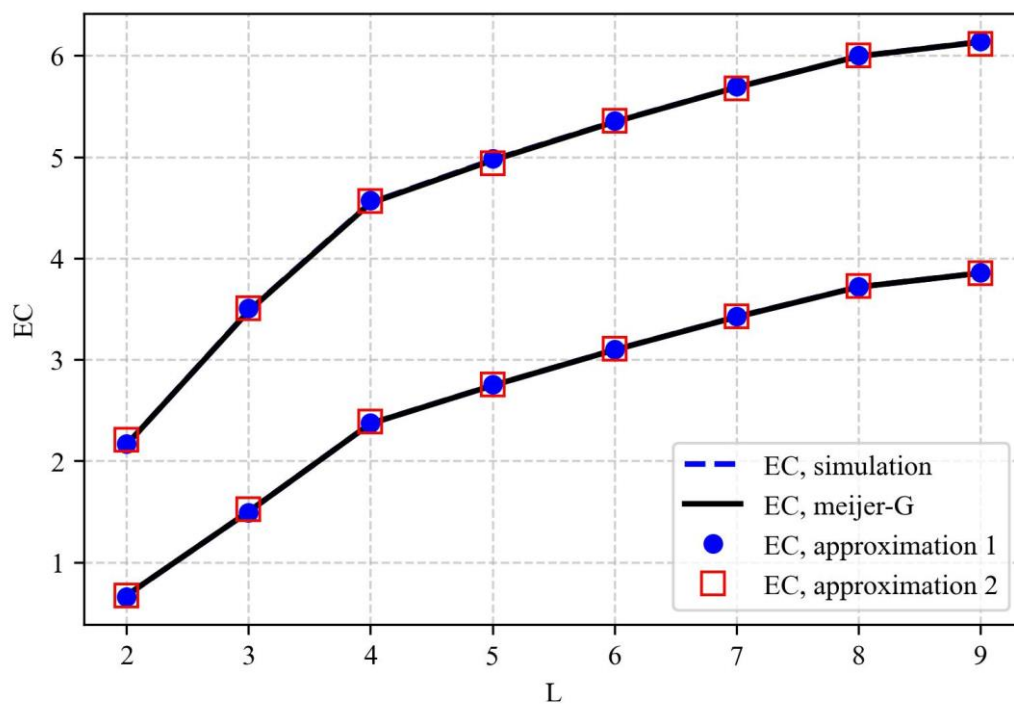


Fig. 6. EC vs. L for $\text{SNR} \in \{5, 15\}$.

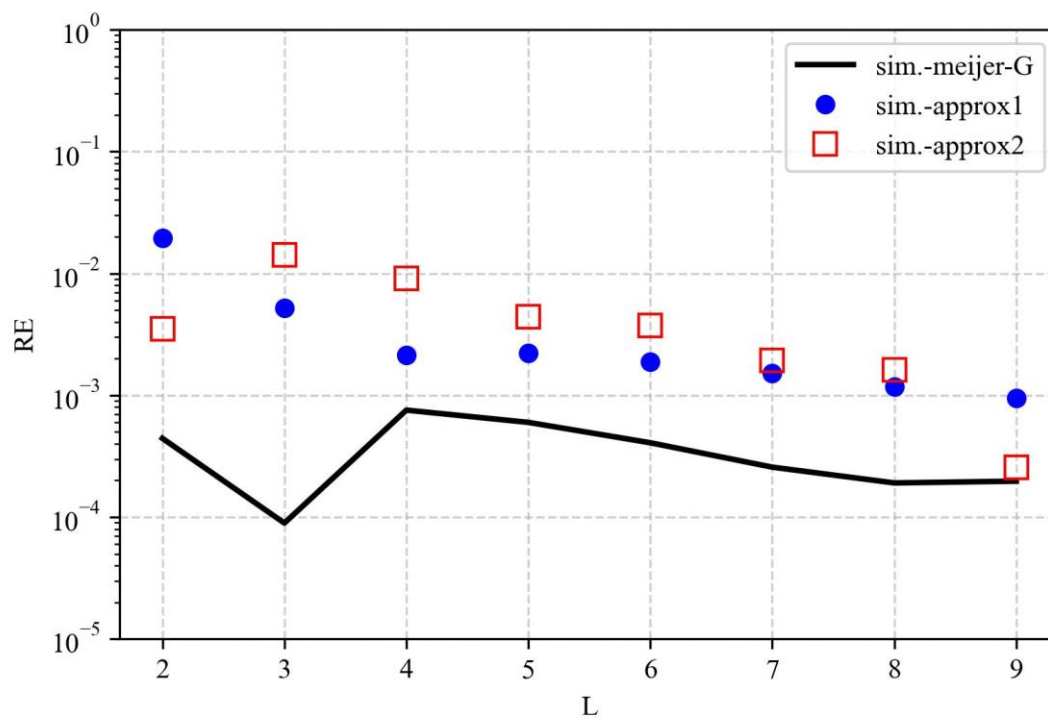


Fig. 7. RE wrt simulation results vs. L for Meijer-G and two Gauss-Laguerre based approximations in $\text{SNR} = 5$.

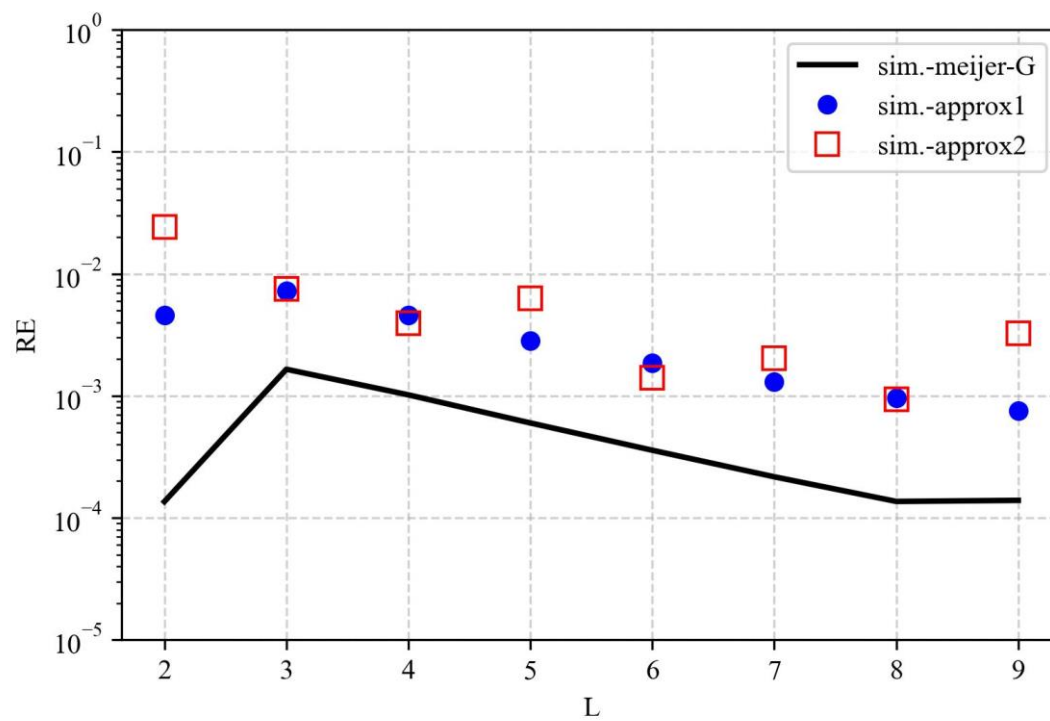


Fig. 8. RE wrt simulation results vs. L for Meijer-G and two Gauss-Laguerre based approximations in $\text{SNR} = 15$.

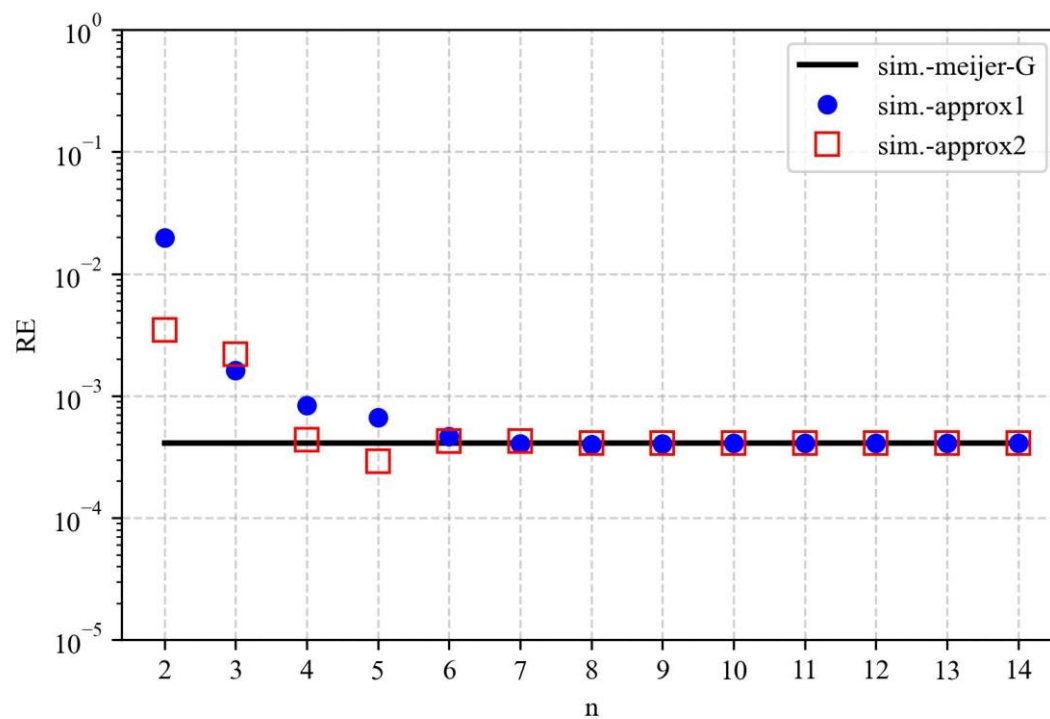


Fig. 9. RE wrt simulation results vs. order of approximation (n) for Meijer-G and two Gauss-Laguerre based approximations in $\text{SNR} = 5, L = 2$.

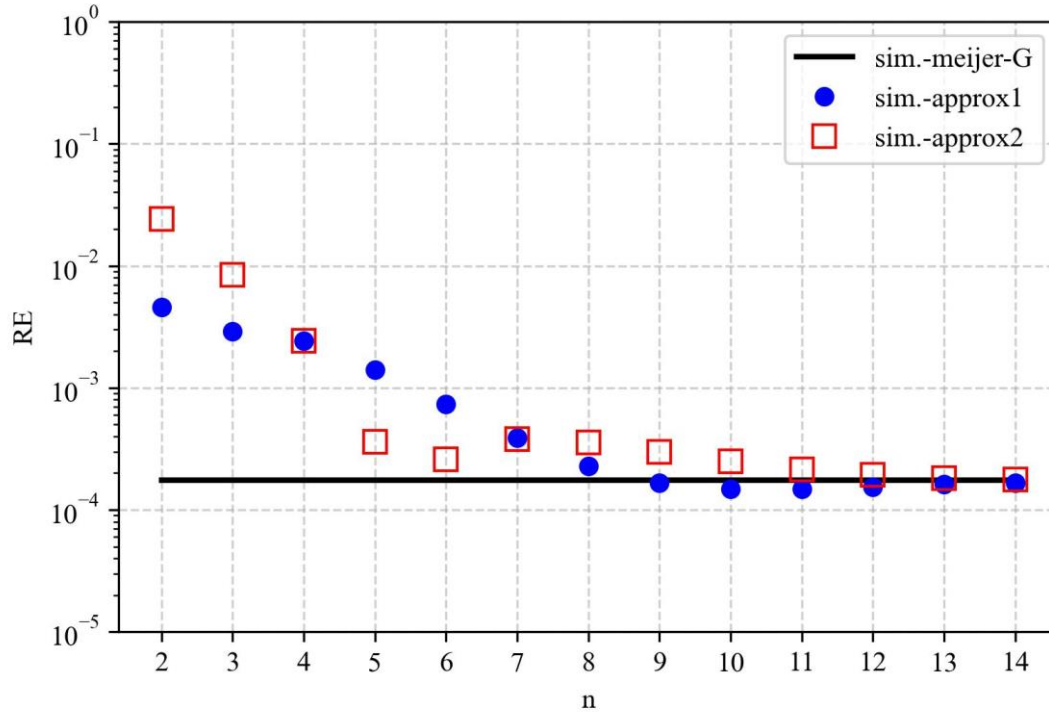


Fig. 10. Relative error wrt simulation results vs. order of approximation (n) for Meijer-G and two Gauss-Laguerre based approximations in $\text{SNR} = 15, L = 2$.

5- Conclusion

This paper has addressed the analytical challenge of evaluating the EC in both C-RIS and D-RIS systems by proposing three novel, tractable approximations. We derived a precise closed-form EC expression for D-RIS using Meijer-G functions, which also models C-RIS as a special case, and introduced two low-complexity numerical approximations based on Gauss-Laguerre quadrature. Extensive Monte Carlo simulations validated all methods, showing that the Meijer-G approximation achieves exceptional accuracy (relative error, $\text{RE} \sim O(10^{-4})$), while the numerical approximations provide reliable performance ($\text{RE} < 2\%$) with much simpler implementations. In particular, the numerical methods match the Meijer-G accuracy at a low approximation order ($n > 5$), confirming their efficiency especially for the simpler-to-implement Approximation 2. In summary, this work provides a unified analytical framework for EC

evaluation in C-RIS and D-RIS systems. The derived approximations enable rapid performance analysis and offer valuable design insights, facilitating the efficient optimization of future RIS-aided wireless networks. From a design perspective, the proposed framework enables rapid and accurate performance evaluation of RIS-assisted systems, facilitating link budget analysis and system design without relying on time-consuming simulations. The derived approximations also provide valuable insights into the impact of key system parameters, such as the number of RIS elements and transmit power, on the achievable EC. Practical impairments such as imperfect CSI, hardware non-idealities, phase quantization errors, correlated fading can be considered as extensions of this work. In particular, extending the work to the Rician fading model is interesting and practical due to the line-of-sight path, which is probable in realistic scenarios. The derived expressions provide a useful upper bound for the achievable EC under such practical conditions, and extending the proposed framework to incorporate these impairments is a promising direction for future research. Investigating analytical upper or lower bounds for the EC also remains an open topic for further investigation.

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