

Performance Analysis of Decomposition of Time Reversal Imaging with Signal Averaging and Autoencoder Denoising

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Abstract:

This study investigates two denoising strategies for improving decomposition of time reversal microwave imaging under severe noise: signal averaging and a denoising autoencoder. The imaging process is based on singular value decomposition of the multistatic data matrix, where dominant singular values represent the signal subspace associated with physical scatterers. At low signal-to-noise ratios, the signal and noise subspaces merge, which can mask the dominant modes and degrade target detection and image reconstruction. To evaluate the denoising performance, complex additive white Gaussian noise is introduced into simulated multistatic data over a two-dimensional imaging domain containing point-like scatterers. Signal averaging suppresses uncorrelated noise through repeated measurements, whereas the denoising autoencoder learns a mapping from noisy data to the corresponding noise-free representation. Their effects are assessed through singular-value separation and reconstructed decomposition of time reversal images over challenging noise conditions. The results show that both methods improve the recoverability of the dominant signal modes, but the denoising autoencoder provides stronger subspace separation when the noise level is severe. In particular, at a signal-to-noise ratio of minus five decibels, the autoencoder preserves the dominant singular values and enables reliable image reconstruction with substantially fewer repeated observations than signal averaging. These findings indicate that learned denoising can provide an efficient alternative to repeated-measurement averaging for robust time-reversal microwave imaging in low-signal-to-noise environments.

Keywords: Decomposition of time reversal, signal averaging, denoising autoencoder, multistatic data matrix, microwave imaging

**Artificial intelligence declaration: This study uses a denoising autoencoder as the artificial intelligence method for multistatic-data denoising; the method and its literature sources are described in Section III.*

1. Introduction

Time-Reversal (TR) microwave imaging is a valuable method used in non-destructive testing, aerial imaging, geophysical imaging, and especially in biomedical tomography applications [1, 2, 3]. TR employs Singular Value Decomposition (SVD) on the Multistatic Data Matrix (MDM), which plays a crucial role in image formation during the reconstruction process. The TR method incorporates two selective focusing sub-methods: signal subspace and noise subspace techniques [4, 5]. These sub-methods involve the Decomposition of Time Reversal Operator (DORT) using dominant singular values in the signal subspace and Time Reversal Multistatic classification (TR-MUSIC) based on singular values in the noise subspace [6].

The theoretical aspects of signal and noise subspace techniques are detailed in [7]. In the TR sub-methods image formation process, various destructive effects such as clutter, side lobes, and noise were studied [4]. Additive White Gaussian Noise (AWGN) in the image formation process for both forward and inverse problems is caused by the merging of signal and noise subspaces [1]. Depending on the SNR levels, target detection and focusing scenarios can fail [8]. We are studying the DORT method, which operates on the signal subspace under challenging SNR levels where the signal and noise subspaces merge. When the dominant singular values in the signal subspace are not identified after applying SVD, the detection scenario fails. In addition, we aim to find an efficient and significant denoising approach that can be applied to MDM in the frequency domain to separate signal and noise subspaces under challenging SNRs.

Recently, researchers have been extensively studying the use of Deep Neural Networks (DNNs) in solving both forward and inverse electromagnetic problems, as well as in characterization tasks [9, 11, 12, 13]. DNNs offer many benefits, especially in improving computational efficiency and reducing noise in images and data [14, 10]. Several types of DNNs, such as Generative Adversarial Networks (GANs), Convolutional Neural Networks (CNNs), and Denoising Autoencoders (DAs), are commonly used for tasks like denoising images and data [15, 16, 17, 18]. A popular dataset for testing DNN models is the MNIST library, which contains images of hand-written digits from zero to nine [19].

Total Variation Denoising (TVD) and Non-Local Means (NLM) are other common effective techniques for reducing noise in complex signals [20, 21]. TVD preserves important features like edges by minimizing the total variation of the signal, making it suitable for signals with distinct transitions. Conversely, NLM denoises by averaging similar patches within the signal, regardless of their spatial location, and is especially effective for signals with spatially or temporally varying noise. Both methods are valuable for denoising complex matrices across various SNR levels.

We adopted a Denoising Autoencoder (DA) architecture to denoise the Multistatic Data Matrix (MDM), particularly under conditions where additive white Gaussian noise (AWGN) is synthetically added in complex form to emulate realistic electromagnetic imaging noise. Our prior study [22] demonstrated that a DA model can effectively disentangle the signal and noise subspaces in such conditions, significantly improving the DORT method's imaging performance. That work showed that even at low SNR levels (e.g., -5 dB), the DA architecture preserves the dominant singular values associated with physical targets, outperforming classical approaches in terms of detection accuracy and subspace separability.

Another practical denoising application is the Signal Averaging Technique (SAT), which is detailed in the KEYSIGHT application note [23], many advantages and challenges of this method is highlighted in [23, 24]. This method is applied to a wide range of fields, including speech recognition and microwave device measurement [25]. SAT uses N measurements to remove noise from the signal in both the time and frequency domains [24]. Nowadays, this method considered a classical denoising method.

The TR imaging method relies on the decomposition of the MDM into orthogonal signal and noise subspaces using singular value decomposition (SVD). In low-SNR scenarios, these subspaces may merge, severely degrading the performance of the DORT method by masking the dominant singular vectors associated with true scatterers. To address this, our recent work [22] proposed the use of a DA, demonstrating that it effectively restores the orthogonality of signal and noise subspaces and preserves the dominant singular modes for robust image formation.

In the current study, we extend this concept by conducting a detailed performance analysis comparing the DA model against the classical Signal Averaging Technique (SAT). Both methods were applied to a synthetic 2D noisy environment involving two point-like scatterers swept across a grid with multiple SNR levels. Our results confirm that the DA model significantly outperforms SAT in separating signal and noise subspaces, especially under challenging conditions such as -5 dB SNR, where SAT requires extensive repetitions to reach comparable accuracy. This superior performance positions the DA as a more efficient and scalable denoising method for real-time DORT-based imaging.

This paper is organized as follows. In Section II, the computational TR imaging method, DORT sub-method, and theoretical aspects of the creation of MDM is presented. In Section III, the denoising method, including the DA model and denoising SAT considering the AWGN effect on image formation, is shown. Additionally, the application of the denoising method on MDM under challenging SNRs is represented. In Section IV, the imaging setup in a two-dimensional grid, considering two-point-like scatterers in image formation, is described. In Section V, results and discussion regarding the AWGN effect on MDM and the study on denoising performance, while considering the advantages of the denoising methods, are presented. Finally, in Section VI, we conclude our work.

2. Computational Time Reversal Imaging

In the computational TR imaging method, the TR operator T is self-adjoint and plays crucial role in both the computation process and image formation. The TR operator plays a key role in the back-propagation process [5], as illustrated in

$$T_{N_\alpha \times N_\alpha}(\omega) = K(\omega)K(\omega)^\dagger \leftrightarrow T_{N_\alpha \times N_\alpha}(t) = K(t) * K(-t). \quad (1)$$

In Eq. (1), K represents the MDM, with its theoretical aspects detailed in [6, 4]. $\dagger$ denotes the transpose conjugate operator. In this context, we address the inverse scattering potential problem, where $t_k(\omega)$ represents a set of input signals in the frequency domain (ω) for k transceivers. The measured response of the transmitted signal $U^{in}(\mathbf{r})$, is denoted as $U^{(s)}(\mathbf{r})$ that it is expressed as

$$U^{(in)}(\mathbf{r}) = \sum_{k=1}^{N_\alpha} t_k(\omega) G_0(\mathbf{r}, \boldsymbol{\alpha}_k), \quad (2)$$

$$U^{(s)}(\mathbf{r}) = \sum_{m=1}^M \mathcal{V}_m G(\mathbf{r}, \mathbf{X}_m) U^{(in)}(\mathbf{X}_m). \quad (3)$$

$\mathcal{V}_m$ denotes the amplitude of the m -th point-like scatterer within the imaging medium, and $U^s(\mathbf{r})$ represents the linearized form of the Lippmann-Schwinger integral equation, as detailed in [6]. The term $r_j(\omega)$ specifies the MDM in the context of the inverse scattering potential. N_α indicates the number of transceivers at position α , as illustrated in Fig. 4. The indices i and j represent the transceiver numbers distributed around the imaging domain. In the computational TR algorithm, $r_j(\omega)$ is included Green's function response within the imaging domain. Consequently, the MDM utilized in the TR imaging back-propagation process is derived from the decomposition of $r_j(\omega)$, as described in

$$r_j(\omega) = \sum_{k=1}^{N_\alpha} \left\{ \sum_{m=1}^M \mathcal{V}_m G(\boldsymbol{\alpha}_j, \mathbf{X}_m) G_0(\mathbf{X}_m, \boldsymbol{\alpha}_k) \right\} t_k(\omega). \quad (4)$$

In addition, according to TR theory, if the background scattering potential V_0 is known, it can be computed as background Green's function $G_0(\mathbf{r}, \mathbf{r}')$ [4]. $G(\mathbf{r}, \mathbf{r}')$ represents the Green's function for the

existing point-like scatterer, and $\mathbf{X}_m$ denotes the location of the m -th point-like scatterer. In this problem, $G(\mathbf{r}, \mathbf{r}')$ represents the known composite Green's function, which, for simplicity, can be replaced by the 2D free-space Green's function. The scattered wave field is calculated from the linear form of the Lippmann-Schwinger scattering equation for discrete scatterer systems [6]. In the following steps of TR imaging, SVD is applied to K . The pseudo spectrum will be calculated in signal subspace [1, 6]. Thus, MDM is specified from as Eq. (4) and is shown in

$$K_{j,k}(\omega) = \sum_{m=1}^M \mathcal{V}_m G(\boldsymbol{\alpha}_j, \mathbf{X}_m) G_0(\mathbf{X}_m, \boldsymbol{\alpha}_k). \quad (5)$$

Computational TR imaging is used for target detection. This detection is based on selective focusing using SVD on the MDM for determining singular value and eigenvector of MDM [4]. This method is operated on the signal and noise subspaces mathematical technique [7]. The SVD process is repeated until Eq. (6) is converged [8]. In Eq. (6), σ denotes the dominant singular value, and $t^*(\omega)$ represents the time-reversal input signal in the frequency domain. The iterative process ensures that the dominant singular value and it is associated singular vector are accurately determined.

$$Kt(\omega) = \sigma t^*(\omega) \quad (6)$$

2.1. Signal and Noise Subspace Technique

The TR imaging method is based on the distinction between the signal and noise subspaces, as outlined in the signal subspace and noise subspace techniques [1, 3, 26]. The theoretical aspect for differentiating these subspaces is discussed in detail in [7]. In this study, the signal and noise subspaces are determined by applying SVD to the MDM. The threshold ϵ in Eq. (7), derived from the SVD of the MDM, serves to separate the signal and noise subspaces [3, 6, 4]. We assume that the MDM under ideal conditions is noise-free. Furthermore, after applying SVD, the signal and noise according to the number of targets in a 2D imaging domain are separated as follows: $C^N = \mathcal{S} \oplus \mathcal{N}$, where C represents the vector space of MDM and $\oplus$ signifies the direct sum of signal and noise subspaces [4]. Additionally, the signal subspace ($\mathcal{S}$) is orthogonal to the noise subspace ($\mathcal{N}$) within the vector space, denoted as $\mathcal{S} \perp \mathcal{N}$ [6].

$$\begin{aligned} \mathcal{S} &= \text{span}\{u_j, \sigma_j > \epsilon\} \\ &\approx \text{span}\{g(\mathbf{x}_m), m = 1, 2, \dots, M_0\} \\ \mathcal{N} &= \text{span}\{u_j, \sigma_j \leq \epsilon\} \\ &\approx \text{span}\{g(\mathbf{x}_m), m = M_0 + 1, M_0 + 2, \dots, N_\alpha\} \end{aligned} \quad (7)$$

It is important to note that ideal condition is $M_0 \leq \text{Rank}(K)$. On the other hand, M_0 is the number of dominant singular values in signal subspace.

2.2. Decomposition of Time Reversal Operator

The DORT method is one of the selective focusing methods that is operated in signal subspace [27]. The MDM is an invertible matrix and defines the TR operator as self-adjoint, it is expressed in Eq. (1) [6, 1, 2]. After applying SVD on the MDM and substituting decomposed singular vectors into the TR operator, we achieved: $T(\omega) = U(\omega)\Lambda(\omega)V^\dagger(\omega)V(\omega)\Lambda(\omega)U^\dagger(\omega)$, where $V^\dagger(\omega)V(\omega) = 1$. Consequently, we simplified the expression to result in $T(\omega) = U(\omega)\lambda(\omega)U^\dagger(\omega)$, with $\lambda(\omega) =$

$[\sigma_1^2(\omega), \sigma_2^2(\omega), \dots, \sigma_{N_\alpha}^2(\omega)]$, representing a real diagonal matrix [6]. The dimension of MDM is equal to the number of transceivers (N_α) that are used for imaging co-located around the 2D imaging domain. We assumed M_0 target exists in 2D imaging domain. Thus, we used SVD to the MDM, next signal subspace is separated from noise subspace, and the vector space has the same form. As K is a reciprocal matrix in complex form for this reason after the $\dagger$ operator does not change the main vector spaces (C^N). In Eq. (8), it is shown that the transpose conjugate does not change vector space of the MDM.

$$\begin{aligned} K: C^N &\rightarrow C^N & K v_j &= \sigma_j u_j \\ K^\dagger: C^N &\rightarrow C^N & K^* u_j &= \sigma_j v_j \end{aligned} \quad (8)$$

Now we are using free-space 2D Green's function (steering vector) $G_0(\mathbf{r}, \mathbf{r}', \omega) = \frac{j}{4} H_0(K|\mathbf{r} - \mathbf{r}'|)$ and then in vector form of Green's response for the 2D imaging domain is expressed in

$$\begin{aligned} g(\bar{X}_m, \omega) &= \\ [G(\bar{X}_m, \bar{R}_1, \omega), G(\bar{X}_m, \bar{R}_2, \omega), \dots, G(\bar{X}_m, \bar{R}_{N_\alpha}, \omega)]^T. \end{aligned} \quad (9)$$

In Eq. (9), $\bar{X}_m$ is the target location and $\bar{R}_i$ is the position vector of i -th transceivers that are located around a 2D homogeneous imaging domain that includes point-like scatterer. The MDM can be written for α_j and β_k positions of transceivers that are co-located around a 2D imaging domain.

$$K_{j,k} = \sum_{m=1}^M \tau_m G(\bar{X}_m, \alpha_j) G(\bar{X}_m, \beta_k) \quad (10)$$

τ_m is the scattering strength of the m 'th existing target, and j, k are elements of the MDM. The noise-free version of the MDM after decomposition can be expressed as

$$K = \overbrace{\sum_{m=1}^{M_0} \kappa_m g(\mathbf{x}_m) g^T(\mathbf{x}_m)}^{K_0} + \overbrace{\sum_{m=M_0+1}^M \kappa_m g(\mathbf{x}_m) g^T(\mathbf{x}_m)}^{\Delta K}. \quad (11)$$

The DORT method is synthetically used for back-propagating incident wave field and for image formation through calculating pseudo spectrum using Eq. (12) as described in [6].

$$D(\mathbf{r}, \omega) = \sum_{j=1}^{N_\alpha} u_p^*(k) G_0(\mathbf{r}, \alpha_j) = g_{0\alpha}^T(\mathbf{r}) u_p^* \quad (12)$$

2.3. Composite DORT Method

As the column vectors of these singular values are independent, eigenvectors from transceivers at α and β positions are combined in Eq. (13) to plot the results of the DORT imaging functional. Eq. (13) is called the composite DORT method in sub-selective focusing imaging [6].

$$D_c(\mathbf{r}, \omega) = g_{0\beta}^T(\mathbf{r}) u_m^* + v_m^T g_{0\alpha}(\mathbf{r}), \quad m = 1, 2, \dots, N_{\sigma>0} \quad (13)$$

The number of nonzero singular values that are used for image reconstruction is given by $N_{\sigma>0} \leq \min(M_0, N_\alpha, N_\beta)[4, 1, 6]$.

2.4. Signal-to-Noise Ratios

The SNR in dB for the noisy version of the MDM is defined as

$$20\log_{10} \frac{\|K\|_F}{\|k\|_F}. \quad (14)$$

where $\|\cdot\|_F$ denotes the Frobenius norm of a matrix [1]. We considered $K_{noisy} = K + k$, where k is the complex AWGN added to the noise-free version of K . After adding AWGN to the MDM and considering the noise effects, the noisy version of MDM is written as

$$K_{noisy} = \overbrace{\sum_{m=1}^{M_0} \kappa_m g(\mathbf{x}_m) g^T(\mathbf{x}_m)}^{K_0} + \overbrace{\sum_{m=M_0+1}^M \kappa_m g(\mathbf{x}_m) g^T(\mathbf{x}_m)}^{\Delta K}. \quad (15)$$

In Eq. (15), the separated subspaces cannot be explicitly expressed because the signal and noise subspaces are merged, depending on the SNR level. We used the free-space Green's function $G_0(\bar{X}_m, R)$ instead $G(\bar{X}_m, R)$ [6, 29, 2]. In this study, we used Eq. (12) and Eq. (13) in the image formation process. SNR is the quantitative noise-control parameter, while DORT reconstruction quality is the final imaging-performance indicator, the iterative denoising is stopped when the reconstructed image reaches a clear response.

3. Denoising Methods

3.1. Denoising Autoencoder

The DA model is a well-known DNN used for denoising images and data [28, 10]. The DA is used for data reduction, denoising, and GAN network application [15]. The DA is one of the unsupervised deep neural networks that is used for denoising because it has a special layer as shown in Fig. 1. We are using a special DA model in [19] and changed the specification of this model so that it is applied to our dataset. The encoder section of this sequential deep neural network encompasses an input layer, Batch Normalization, and LeakyReLU. Output data from this section is directed to the middle-hidden layer, positioned centrally, to facilitate subsequent decoding. The decoder layer deciphers data from the middle-hidden layer, employing Batch Normalization and LeakyReLU. This iterative process continues until the loss function achieves its minimum[16].

The dataset was generated using MATLAB simulations for a wide range of SNR levels, from negative to high values. The dataset was generated using MATLAB simulations for a wide range of SNR levels, from negative to high values. Specifically, a single ideal MDM corresponding to one fixed scatterer location was considered as the reference, and 15 different SNR levels were generated by independently adding complex AWGN to this MDM. The resulting noisy MDMs were concatenated to form the dataset used for training the DA. Since the original MDM is an 11×11 complex-valued matrix, its real and imaginary components were separated and rearranged to construct an 11×12 real-valued input representation. In the present implementation, no independent training/testing split was performed; therefore, the dataset was constructed for the purpose of evaluating the denoising capability of the DA across the considered SNR range. The dataset is creating by mounted point-like scatterer with step size $\frac{\lambda}{5}$ at 2D grid with dimension $30\lambda \times 30\lambda$.

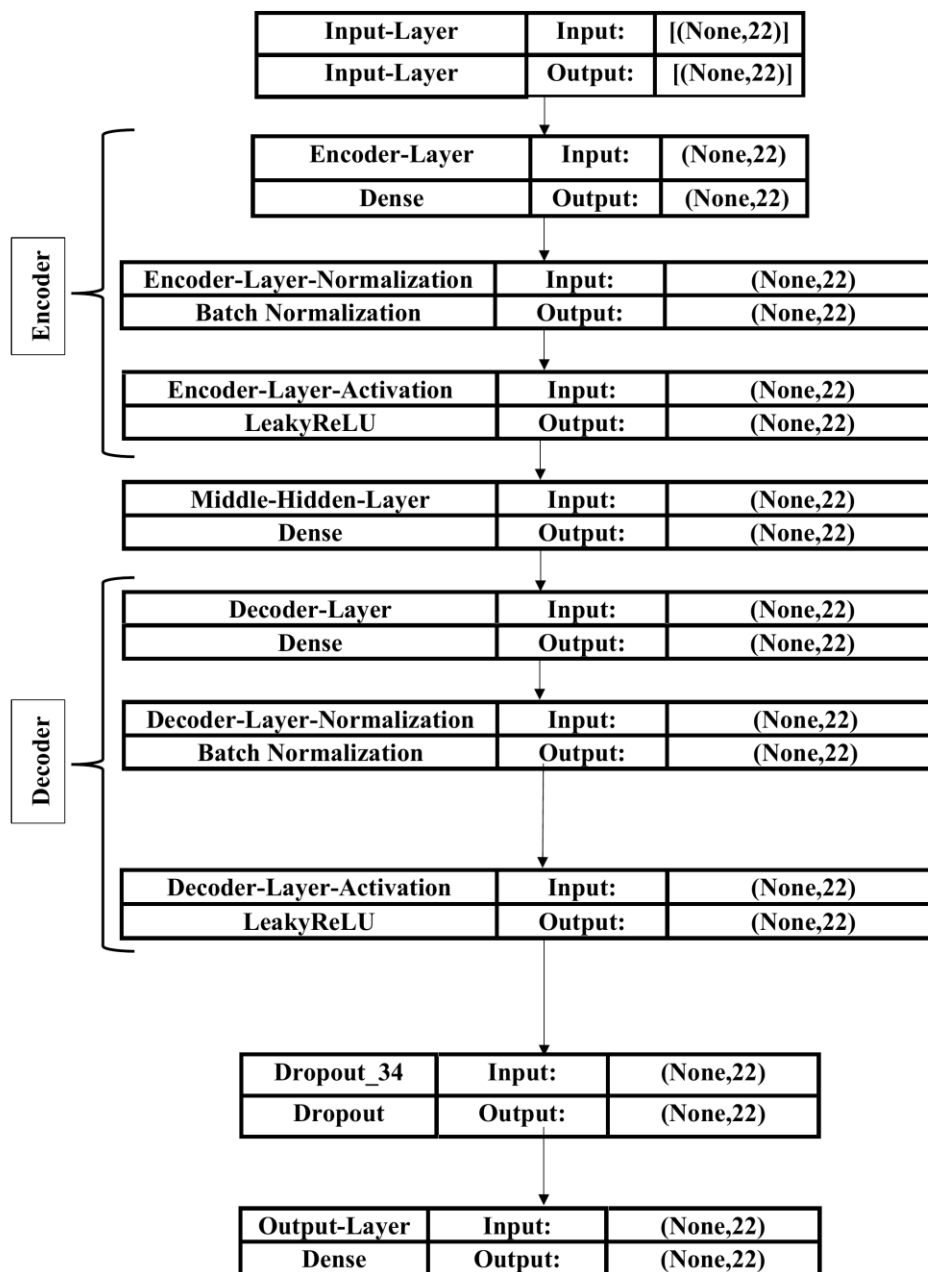


Fig. 1. Sequential architecture of DA model.

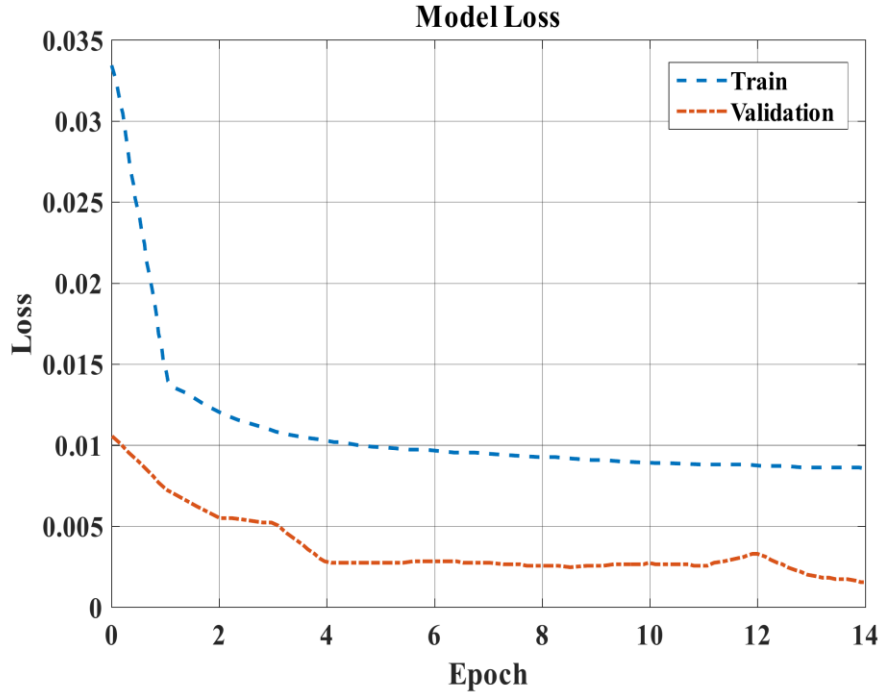


Fig. 2. Model loss of DA.

This dataset for every new matrix is

$$K_{new} = [\Re(K), \Im(K)]_{11 \times 22} \quad (16)$$

K_{new} is the MDM that real and imaginary parts of an element are separated and concatenated. At each location of target multiple SNRs measured, so for every location we have multiple MDM for converging to realistic conditions. The number of data used for biasing the DA model is

$$n = 11 \times 22 \times \frac{30\lambda \times 30\lambda}{\frac{\lambda}{5}} \times N. \quad (17)$$

Therefore, we implement a deep neural network in Fig. 1. This deep neural network is applied to our created dataset. In Eq. (17), N is the number of SNR levels measured at each location of a point-like scatterer in a 2D grid.

Thus, X_{train} is the noisy dataset from N levels of SNR, where 15 SNR levels were measured, and Y_{train} is the dataset of the noise-free signals. The optimizer used for this deep neural network optimization is Adam [16]. The DA was implemented for 15 iterations, and the model loss was plotted in Fig. 2. The number of data points used for X_{train} is $3,853,396 \times 22$, and the same holds for the noise-free data used for Y_{train} . Additionally, for X_{test} and Y_{test} , each set used $680,064 \times 22$ data.

3.2. Signal Averaging Technique

The Signal Averaging Technique (SAT) is a classical denoising method widely used in real measurement systems such as Vector Network Analyzers (VNA) [23]. In our method, multiple simulations are

simulated at a single SNR level for each location of a point-like scatterer. This method is a similar approach used in the DA model. The SAT formula is applied in the frequency domain at random locations within a 2D domain and is expressed as

$$K_{denoised} = \frac{\sum_{i=1}^N K_{noisy(x,y)}(m,l)}{N}. \quad (18)$$

Here, i represents the number of repeated simulated MDM instances at each point for a specific SNR level. Also m and l are shown as an index of MDM elements. The parameter N signifies the number of simulations performed for one SNR level across a 2D grid at each location. The choice of N for random locations at a given SNR level provides a freedom degree in the simulation process.

This iterative process consists of multiple AWGN while maintaining a constant signal Frobenius norm, as detailed in Eq. (14). Consequently, following the TR algorithm, the MDM is specified. Subsequently, the SAT method is applied to the noisy MDM for computing $K_{denoised}$ as shown in Eq. (18). The SNR level is calculated using the formula in Eq. (14).

In our study, the MDM for a specific SNR level is generated and repeated a specified number of times, as illustrated in Fig. 3. It is noteworthy that the repetition count N is determined through MATLAB simulations, ensuring the target is reliably detected.

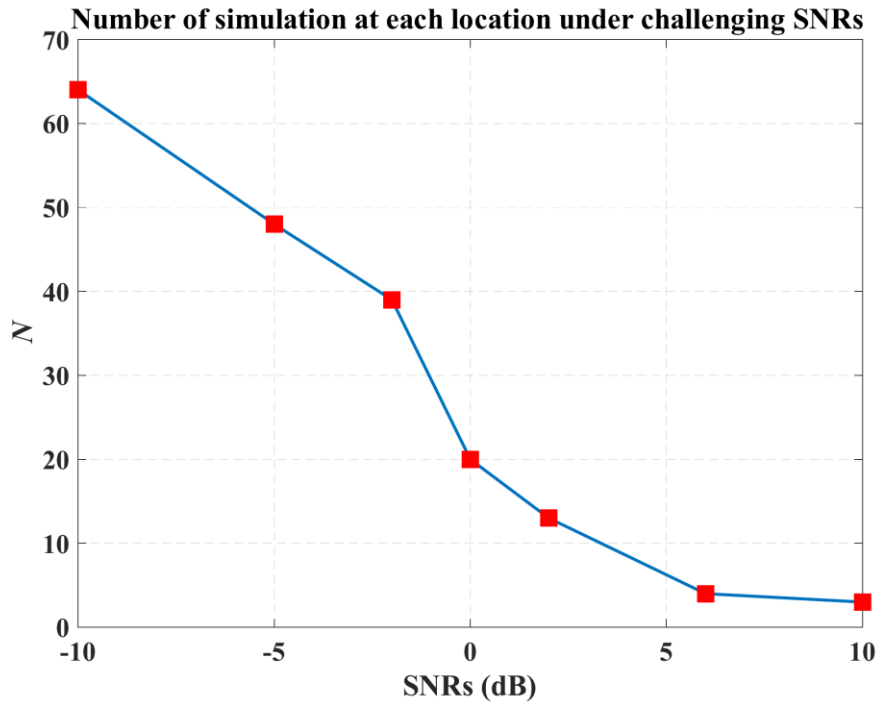


Fig. 3. The number of simulations for the proposed MDM using SAT for denoising under challenging SNRs for target detection is performed at random locations on the 2D grid.

In 3 the minimum SNR threshold of SNR in the horizontal axis is -10dB. In simulation using the SAT method, it is necessary to perform 63 repetitions for averaging. It is clear that increasing SNRs while

decreasing the number of measurements (N) poses a trade-off between measurement efficiency and reconstruction quality.

4. Imaging Setup

This section outlines the setup of the imaging domain. We assumed a 2D grid with 11 transceivers located uniformly on the x -axis with a separation distance of 2λ as shown in Fig. 4. Consequently, the MDM is created with dimensions of 11×11 . Initially, we considered two point-like scatterers within this grid [6]. Within this 2D domain of interest, we defined a grid with dimensions $x = 30\lambda$ and $y = 30\lambda$.

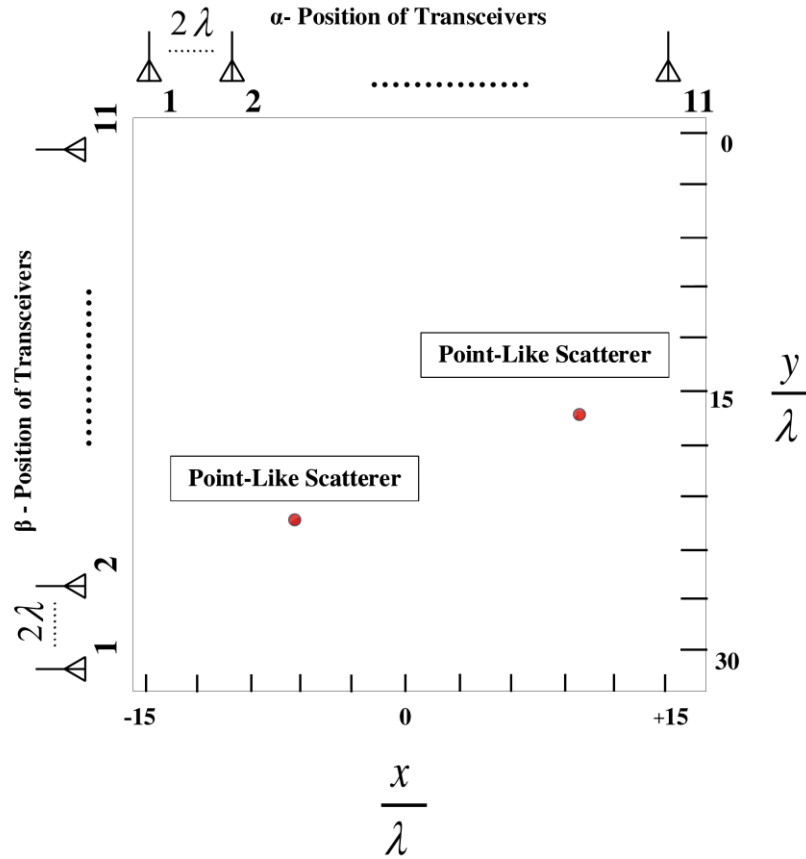


Fig. 4. Time-reversal imaging setup for the distribution of array in α and β positions.

Two point-like scatterers imply that the scatterer's dimensions are significantly smaller than the wavelength (λ). Each transceiver is assumed to be a point source capable of both emitting the incident wave and measuring the scattered wave within the domain. According to the TR algorithm, the MDM matrix K is produced, which is symmetric in the frequency domain because of the reciprocity theory [5, 8, 2]. We considered the 2D free space Green's function for reducing complexity in the image reconstruction process.

After the MDM is formed for the 11 transceivers located on the x -axis (α positions), we proceed to place the transceivers on the y -axis (β positions). The TR-DORT back-propagate the measured scattered wave fields based on the DORT method. The DORT-Imaging function Eq. (13) is employed to combine images

derived from independent dominant singular values of the α and β positions. This imaging system is illustrated in Fig. 4.

4.1. Composite DORT Imaging Method in α and β Positions $\alpha\beta$

In this section, we considered the α and β transceiver positions in the two states of implementing the DORT imaging method in the signal sub-space. Initially, the imaging algorithm is implemented for the α position of the array. After reconstructing the image using the dominant singular value in Eq. (12), the same process is repeated for the β position, and the result is substituted into Eq. (13).

4.2. Flowchart of Denoising Approach Applied to the DORT Method

The flowchart of this study is presented in Fig. 5. The Input SNR level and Output SNR level from the denoising method shown in Fig. 5. Those SNR levels are calculated using Eq. (14) after selecting a random location on the 2D grid illustrated in Fig. 4. Therefore, the terms 'Input SNR' and 'Output SNR' mentioned in the captions of the resulting figures refer to the process described in this flowchart.

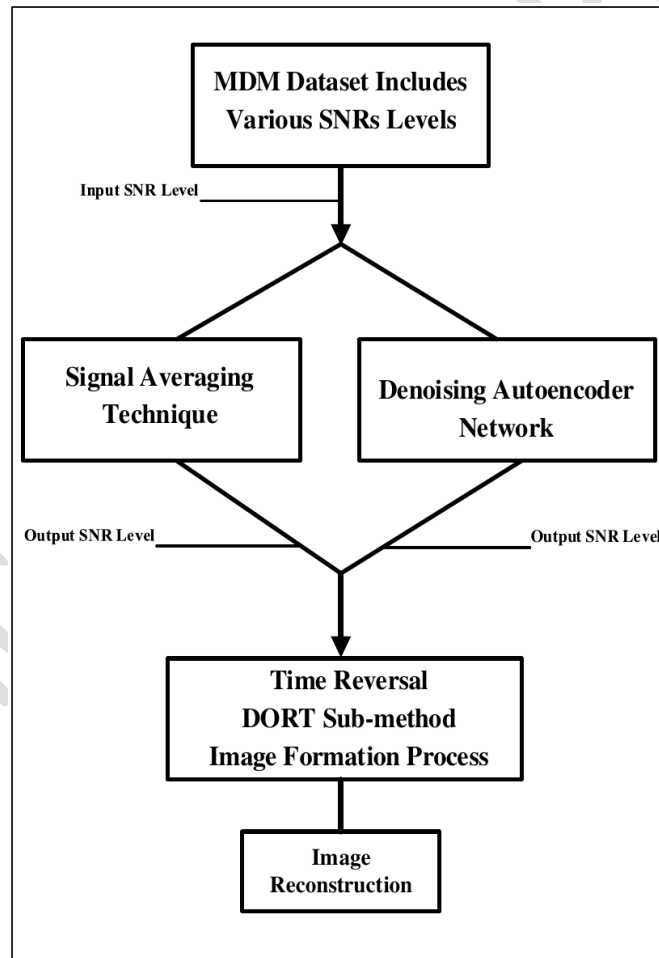


Fig. 5. Time-reversal image formation denoising process flowchart.

5. Results and Discussion

We considered three states of the MDM: noise-free, noisy, and under challenging SNR levels, where signal and noise subspaces are merged, evaluating the effectiveness of the two denoising methods described previously. The DORT imaging method relies on dominant singular values within the signal subspace. Initially, the number of singular values discussed theoretically in Section II, Part B, is examined across these three states.

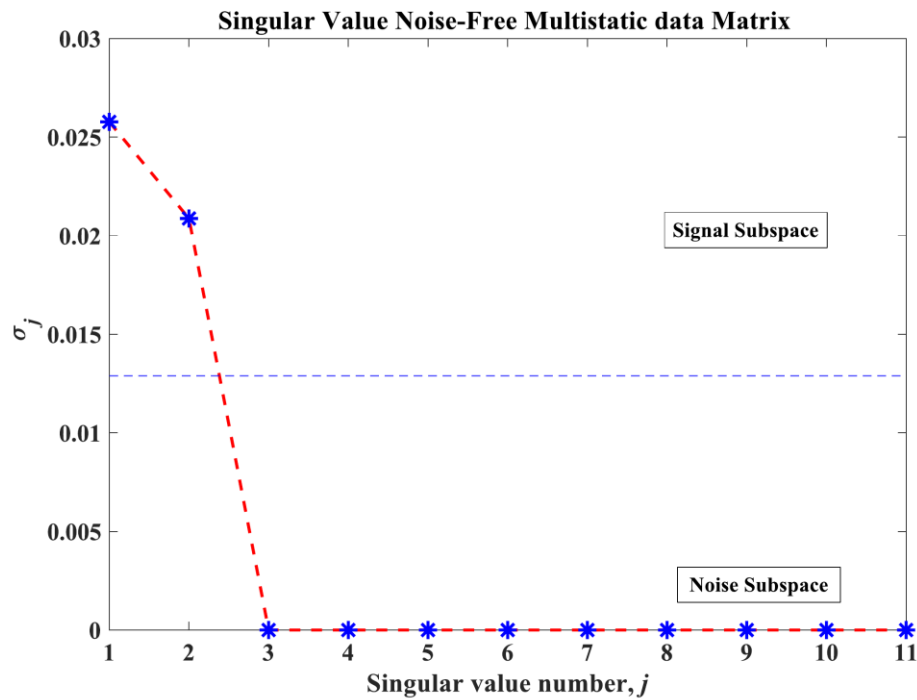


Fig. 6. Singular value number of noise-free version of MDM.

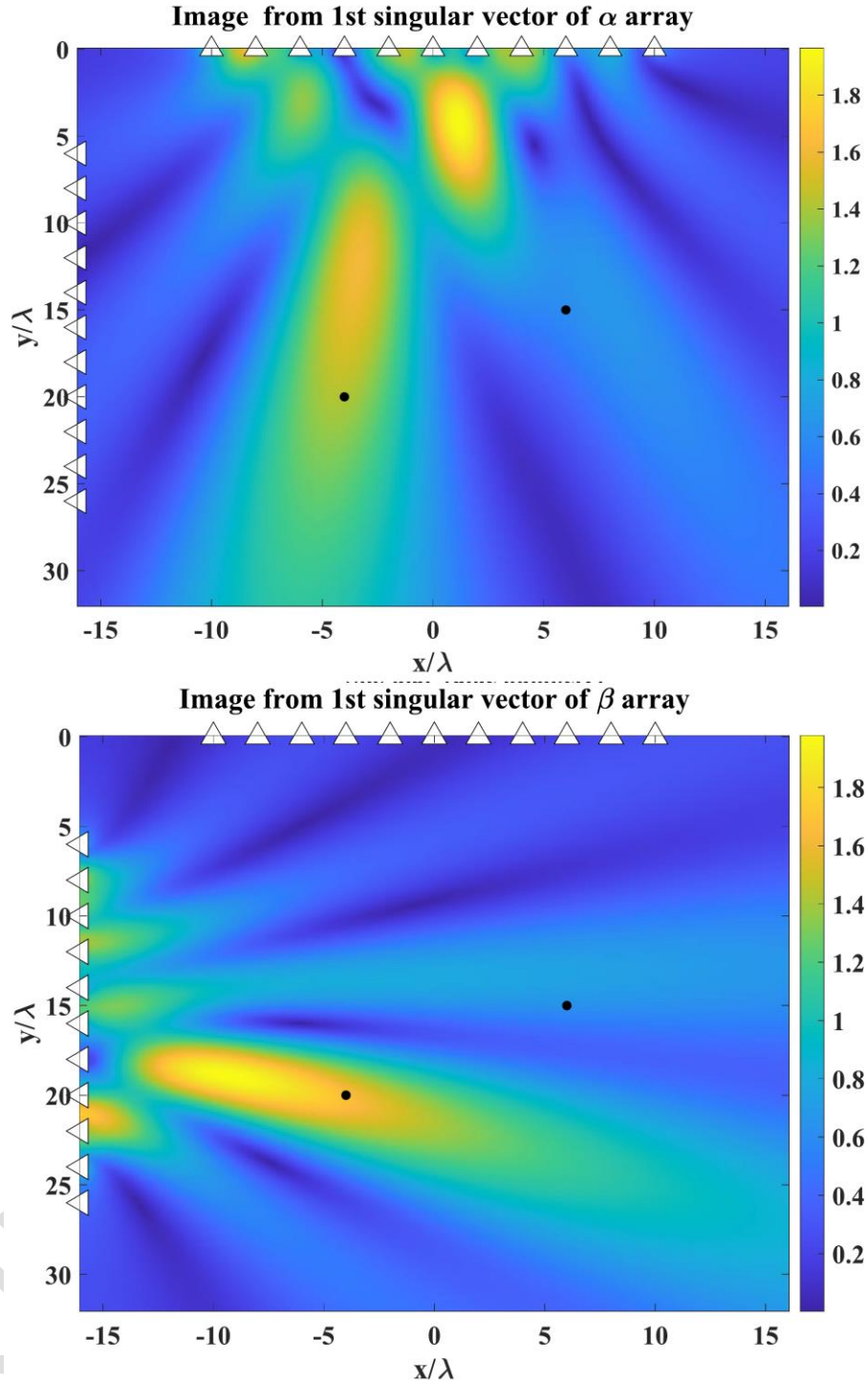


Fig. 7. Utilizing the DORT method using first and second dominant singular values for image reconstruction from noise-free MDM in the signal sub-space for the targets at location $(-4, 20)$ and $(6, 15)$.

In the noise-free version of the MDM containing two point-like scatterers in a 2D imaging domain, we anticipate that applying SVD will reveal two dominant singular values in the signal subspace. Fig. 6 illustrates the singular values obtained after simulating and using SVD to the noise-free version of MDM.

In addition, the signal subspace has an ideal distance from the noise subspace in Fig. 6. Therefore, our expectation for an ideal noise-free MDM was valid. Using DORT image function Eq. (12) for image formation in α position is shown Fig. 7. This sequence of simulations is repeated for β position without changing the target location, and the results from the DORT method are depicted in Fig. 8.

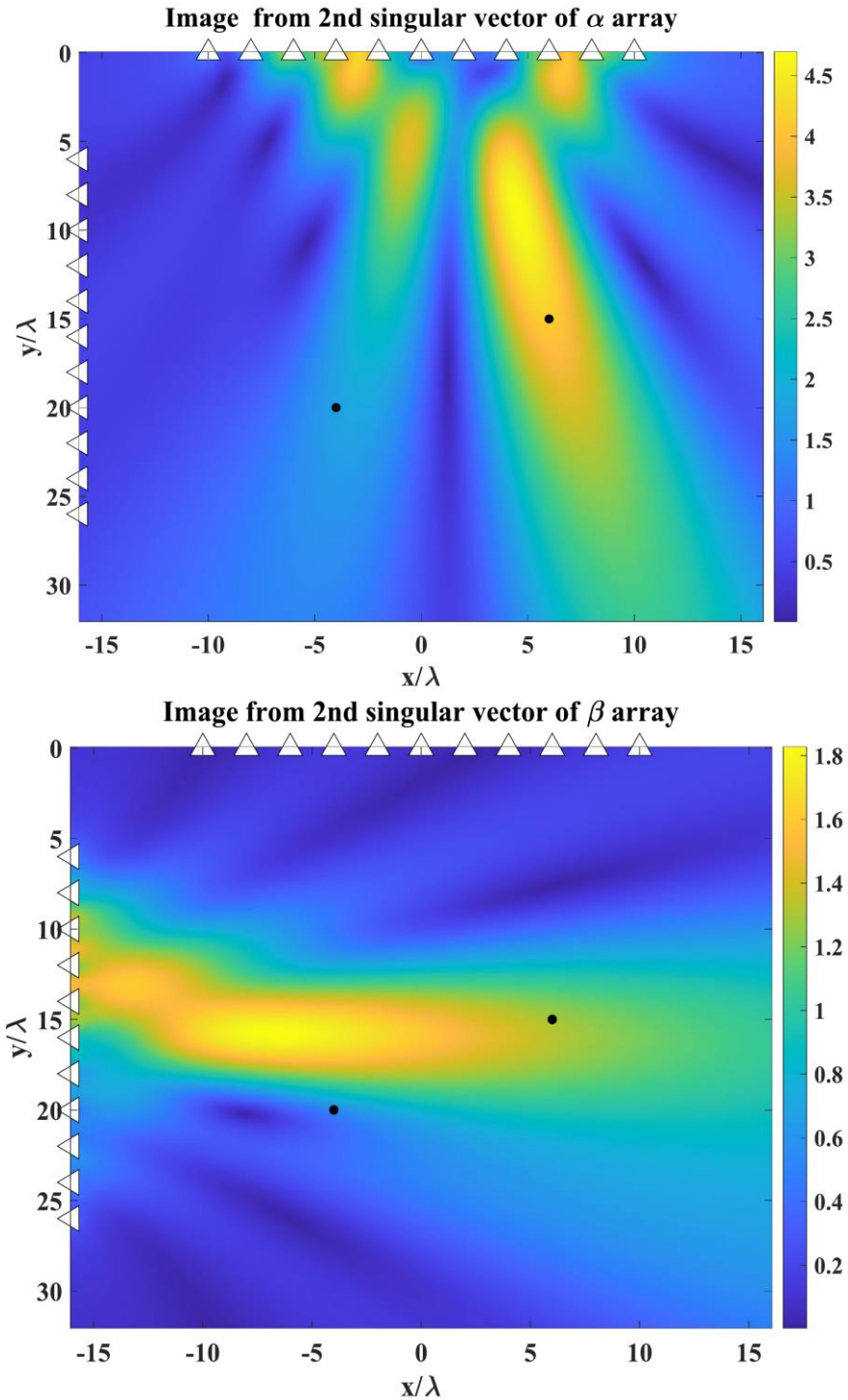


Fig. 8. Utilizing the DORT method using first and second dominant singular values for image reconstruction from noise-free MDM in the signal sub-space for the targets at location $(-4,20)$ and $(6,15)$.

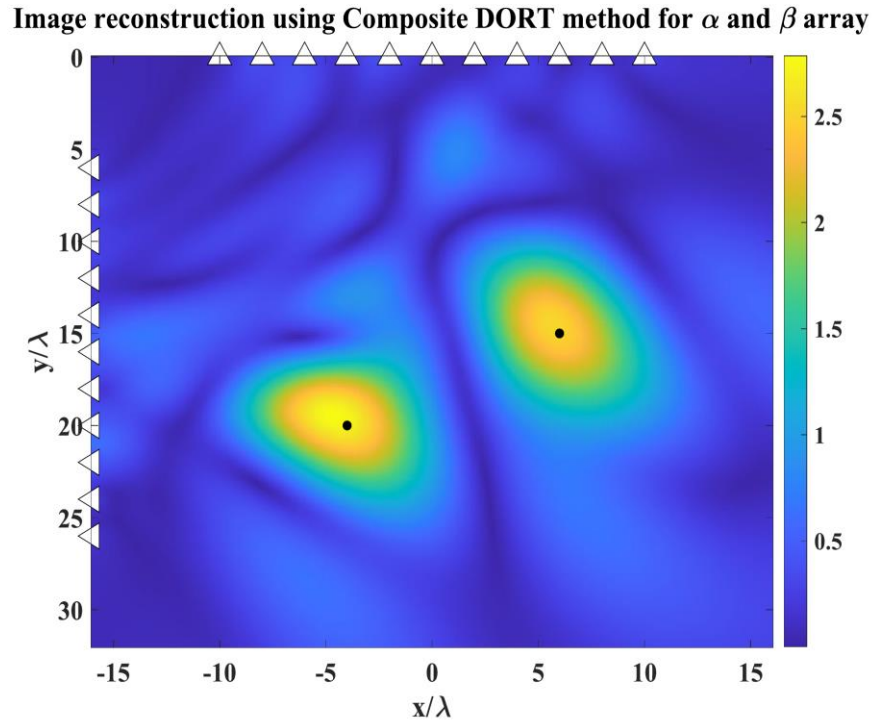


Fig. 9. Utilizing the composite DORT method using first and second dominant singular values for image reconstruction from noise-free MDM in the signal sub-space for the targets at location $(-4,20)$ and $(6,15)$.

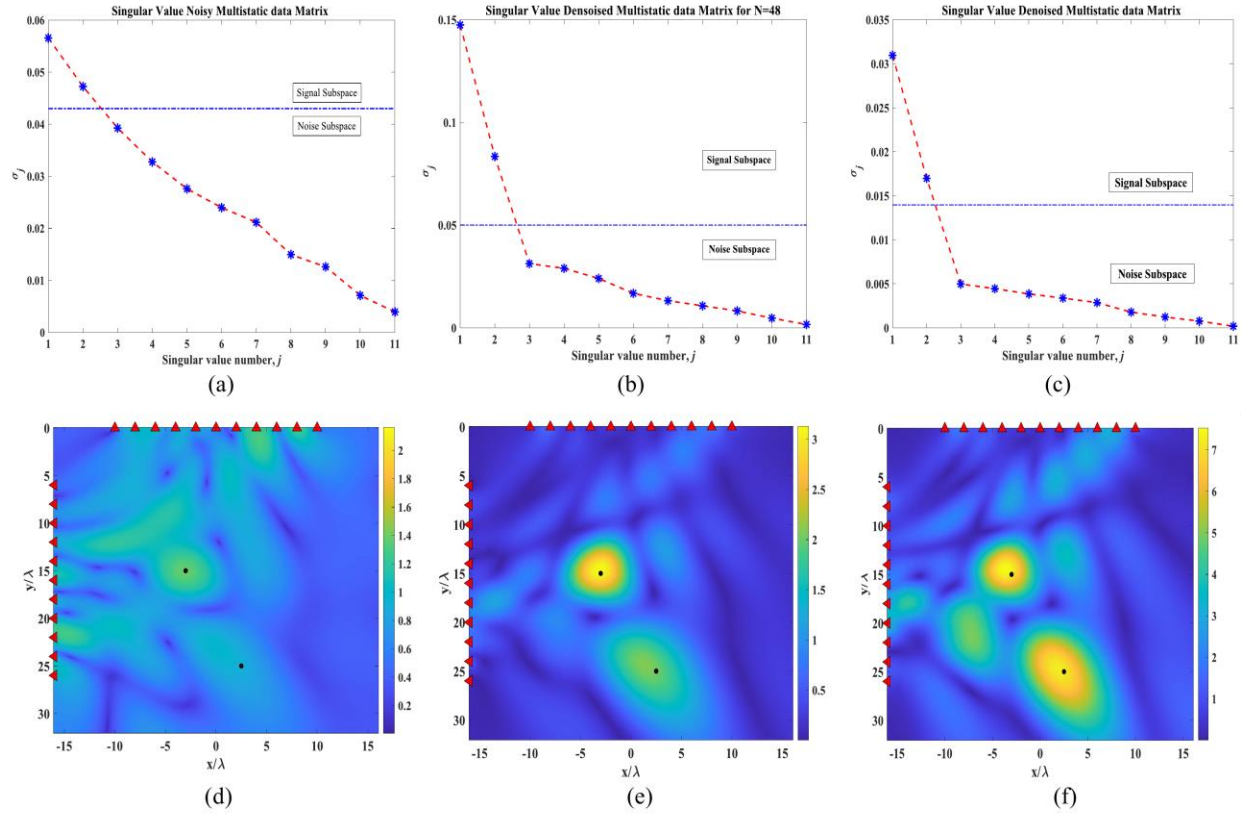


Fig. 10. Singular-value spectra and reconstructed images for the noisy and denoised MDM: (a) noisy MDM, (b) signal-averaging denoised MDM, (c) denoising-autoencoder denoised MDM, and the corresponding reconstructed images (d)-(f).

The composite DORT method combines the reconstructions obtained from the independent dominant singular values of the α - and β -array configurations. As shown in Fig. 9, the two point-like targets are clearly detected for the noise-free MDM. For the noisy MDM, AWGN causes the signal and noise subspaces to merge, which degrades the reconstruction at challenging SNR levels. Figure 10(a) shows the singular-value spectrum of the noisy MDM, whereas Figs. 10(b) and 10(c) show the corresponding spectra after signal averaging and denoising-autoencoder processing, respectively. The reconstructed images obtained from these three cases are shown in Figs. 10(d)-10(f). After denoising, the separation between the dominant signal singular values and the noise subspace is improved, and the point-like targets are recovered more clearly. The DORT imaging functional in Eq. (12) is used for the individual-array reconstructions, while the composite formulation in Eq. (13) combines the α - and β -array contributions for the final image reconstruction.

6. Conclusion

In summary, we presented approaches for multi-target detection using the DORT method under challenging SNR conditions. We proposed two approaches that apply our denoising purpose for separating signal subspace from noise subspace. Firstly, we applied SAT, a classical denoising method for SNR levels equal to -5dB. This method needed 48-the measurements for noise reduction and separate signal and noise sub-spaces from each other under this challenging SNR level.

It is pointed out in the simulation results that the SNR threshold for target detection is -10dB. The DA model is not effective for denoising at SNR levels lower than -10dB.

The comparison between SAT and DA is not entirely balanced, as SAT requires repeated measurements, while DA benefits from extensive pre-training. However, final results show DA outperforms classical SAT. Sequential measurements are inefficient, requiring 48×11 measurements for SNR -5dB. In contrast, our pre-trained DA model needed only one simulated data point to separate signal and noise subspaces, verifying its impressive advantage over SAT as a classical denoising method.

In addition, the final detection results derived from the MDM output of the DA, utilizing the composite DORT method for the TR image formation process, demonstrate superior performance compared to MDM derived from SAT, particularly when applied to this qualitative method. The proposed method can be considered for experimental validation in future studies.

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