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Sensitivity Analyses on the Earth Surface Potential of Multiple Rods under Lightning Strokes in Ionized, Dispersive and Two-Layer Earths

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Abstract — In this paper, touch and step voltages of different arrangements of rods including single rod, double, triple and quadruple rods under lightning strikes are evaluated. In all analyses, an efficient modeling approach called improved multi-conductor transmission line is adopted. Also, two typical lightning currents namely first and subsequent stroke currents are injected to the mentioned grounding rods. This paper consists of two parts.

In the first part, the touch and step voltages of different arrangements of rods considering dispersion and ionization of lossy earth are computed. Firstly, sensitivity analyses show that from single rod to quadruple rods, both voltages are reduced especially in poorly resistive earths. The relative reduction percentage is on average 81.5% and 69.5% respectively for the touch and step voltages. In addition, when the arm length is increased, considerable reduction of both voltages especially in poorly resistive earths under subsequent stroke current is achieved so that the length variation from 10m to 40m, leads to the reduction of 85% and 91% respectively for the touch and step voltages on average. Depending on the earth resistivity and lightning waveform, after a starting length this reduction is slightly continued.

In the second part, the above analyses are extended to two-layer earth and sensitivity analysis versus the resistivity of the two layers is carried out. The simulation results show that both voltages have middle values between the counterparts in single-layer earths.

Index Terms — Ionization, dispersion, multiple rods, lightning stroke, touch and step voltage.

I. INTRODUCTION

Multiple vertical rods as a typical grounding system are used in distribution, and telecommunication systems and installed in earths of low resistivity [1, 2]. Fig. 1 shows different arrangements of vertical rods including single rod, double, triple, and quadruple rods. Proper design of such devices is strictly dependent upon the complex nature of earth such as ionization nonlinear phenomenon [3], frequency dependence of electrical parameters (dispersion) [4] and so on.

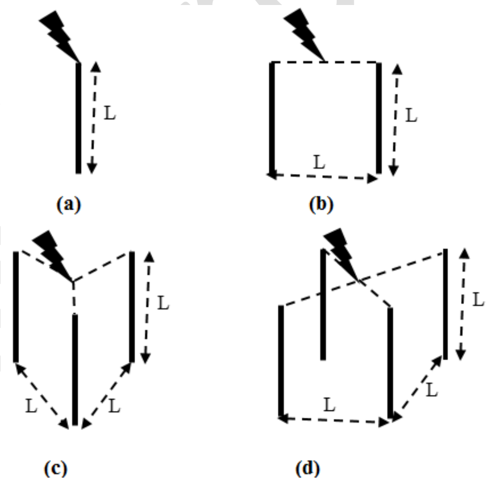

Fig. 1. Different arrangements of vertical rods, (a): single rod, (b): double rods, (c): triple rods, and (d): quadruple rods.

More recently a number of researches is focused on the effects of dispersion and ionization, separately and simultaneously to evaluate the lightning performance of grounding systems [5-9], transmission lines [10], absorbed energy of arresters [11], and effective length /area of grounding systems [12-14].

More recently, grounding potential rise (GPR) of multiple vertical rods buried in ionized, dispersive and two-layer earth has been investigated [15], but the earth surface potential (ESP) was not addressed. The only research on the ESP is found in [16-20] focusing on the only-ionized [16, 17], non-ionized [18-20] and only-dispersive [21] earths. This parameter is importance since two well-known quantities namely touch and step voltages are extracted from it. The touch voltage is defined voltage difference between the lightning current injection point and the arbitrary point separated 1 m, while the step voltage is the voltage difference between two feet of personnel separated 1 m on the earth surface. The two quantities are significant in terms of the safety of human beings and electrical and electronic equipment [22]. Fig. 2 shows definition of these two quantities schematically.

The above-mentioned notes motivate the authors to investigate ESPs of such grounding systems comprehensively. To this aim, the improved multi-conductor transmission line model (IMTL) [23] is first adopted. This approach was proposed by Guardado et al [24]. Since it is inherently in frequency domain, it can be then easily used to incorporate frequency dependence of the earth (dispersive soil) [21]. This was shown via comparison with experiment and full-wave methods [22]. It was finally improved to include ionization and dispersion simultaneously [23]. After then, its application was investigated for grounding potential rise (GPR) [23] effective length of counterpoise wire [25] and earth surface potential (ESP) of single rod [26].

In this paper to consider the rise time and magnitude of the lightning current, two typical lightning current waveforms namely first and subsequent stroke currents are then injected to the mentioned grounding systems.

The simulation results show that in dispersive and ionized earths when the number of arms is increased, both voltages are decreased especially in poorly resistive earth, so that from single rod to quadruple rods, the relative reduction percentage is on average 81.5% and 69.5% respectively for the touch and step voltages. In addition, increasing the arm length from 10m to 40m, decreases the touch and step voltages respectively 85% and 91% on average.

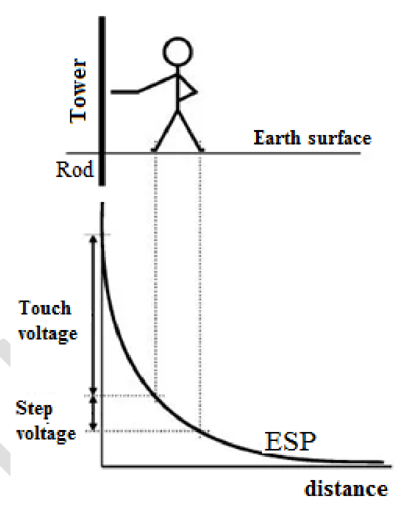

Fig. 2. Definition of the touch and step voltages [22].

To extend the above analyses, the effects of ionization and dispersion in two-layer earths are finally investigated. The simulation results show that in such earths, both voltages are placed between the counterparts in single-layer earths. This fact is practically of importance especially when the first layer is less resistive than the second layer.

This paper is organized as follows. Section II gives modeling principles of analysis. In section III, computation of the earth surface potential (ESP) and accordingly touch and step voltages of different arrangements of rods considering ionization and dispersion of the earth are carried out. Section IV is focused on extracting formulae of the touch and step voltages and their application in two-layer earths. Finally, conclusions are given in section V.

II. MODELING PRINCIPLES

As mentioned in the previous section, all analyses in this paper are based on the IMTL and the lightning current, the dispersion and ionization of the earth are imported in to it somehow [23]. Here, two typical waveforms for the lightning current namely the first and subsequent return strokes, which are characterized by Heidler's functions [18] are used. They are expressed in (1) and (2). All parameters in (1) are listed in Table 1. Note that the sum of two Heidler's functions is used for the subsequent stroke current. Fig. 3 shows the wave shapes related to the first and subsequent return stroke currents which are used in this study.

$$i(t) = (I_0/\eta)\exp(-t/\tau_2)(t/\tau_1)^n / \left[1+(t/\tau_1)^n\right] \tag{1}$$

where

$$\eta = \exp\left[-(\tau_1/\tau_2)(n\tau_2/\tau_1)^{1/n}\right] \tag{2}$$

Table 1. Parameters of lightning currents adopted from [27].

Current	I_0(kA)	n	τ_1(μs)	τ_2(μs)
First stroke	28	2	1.8	95
Subsequent stroke	10.7	2	0.25	2.5
	6.5	2	2	230

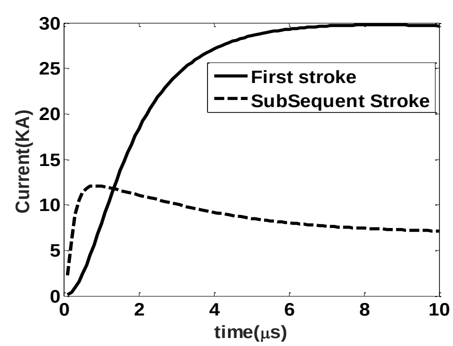

Fig. 3. First and subsequent stroke currents in this study.

In the case of dispersion, among different models [28] with the aim of better prediction [29, 30], the causal model proposed by Visacro et al [31] is used , that is

$$\rho(f) = \rho_0\left(1+h(\rho_0)\times\left(f/1\text{MHz}\right)^{\gamma}\right)^{-1} \tag{3}$$

$$\varepsilon_r(f) = \varepsilon'_{r\infty} + \frac{\tan(\pi\gamma/2)\times 10^{-3}\times h(\rho_0)\times(f/1\text{MHz})^{\gamma}}{(2\pi\varepsilon_0\rho_0)} \tag{4}$$

Where ρ_0 is the low-frequency resistivity of dispersive earth, and f is the frequency in Hz. Also, $\gamma = 0.54$,

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$\varepsilon'_{r\infty} = 12$ and $h(\rho_0) = 1.26 \times \rho_0^{0.73}$ where ρ_0 is the low-frequency resistivity of dispersive earth, and f is the frequency in Hz. Also, $\gamma = 0.54$, and $\varepsilon'_{r\infty} = 12$.

The ionization as a nonlinear phenomenon in the lossy earth is taken place when the induced electric field of earth exceeds from its critical value (E_C). It is expressed as below [32]

$$E_c = 241\rho^{0.215} \quad (5)$$

In (5), E_c, and ρ are respectively critical electric field in kV/m, and resistivity of the earth in Ωm.

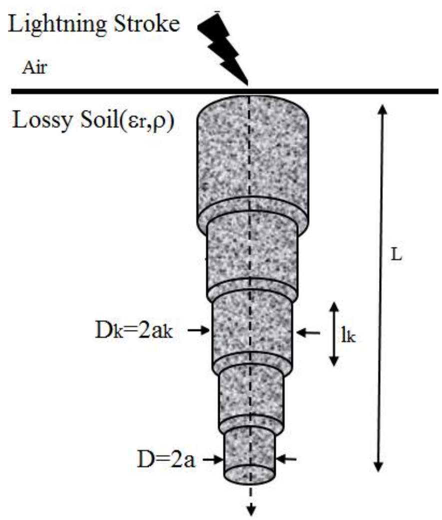


Fig. 4. Schematic of ionization phenomenon as gradually increased radius in single rod.

When the ionization phenomenon is occurred, it is here modelled as gradually increased radius. This fact is shown in Fig. 4. for instance, the rod of length L is divided into N segments of length l_k=L/N, k=1, 2, ...,N. The segment length L_k should be satisfied in the relation $l_k<\lambda/10$, where λ is wavelength in free space. The very short segment improves the accuracy, but the run time increases. Here, l_k=0.5m results in satisfied results.

Prior to analysis using IMTL, each segment of single rod is represented as transmission line model [23].

$$\frac{d}{dx}V_{MTL,k} + Z[\rho(f), \varepsilon_r(f), a_k]I_{MTL,k} = 0 \quad (6)$$

$$\frac{d}{dx}I_{MTL,k} + Y[\rho(f), \varepsilon_r(f), a_k]V_{MTL,k} = 0 \quad (7)$$

Where $\rho(f), \varepsilon_r(f)$ are parameters of dispersive soil and a_k is the kth segment radius. Also, Z and Y are respectively series impedance and parallel admittance of each segments and nonlinearly dependent on the soil parameters, i.e., ρ, ε_r and the radius of each segment (a_k). The above corresponding equations of different segments are connected to each other via Kirchhoff's voltage low (KVL) and Kirchhoff's current low (KCL). At first, the radius of kth segment is assumed to be the original radius of the rod and the MTL equations are solved so that the voltage and current at each frequency inside the spectral content [20]. Then the leakage current is computed as current difference of the kth and (k+1) th segments, that is

$$I_{Lk} = I_k - I_{k-1} \quad (8)$$

The electric field on the kth segment is computed as

$$E_k = \frac{\rho(f)I_{Lk}}{2\pi a L_k} \quad (9)$$

If E_k is greater than E_c, the radius of each segment is increased as

$$a_k = E_k / E_c \quad (10)$$

With the new radius, the new voltage and current at each segment is computed up to $E_k < E_c$. After the iteration is finished, the ESP is computed as follows

$$ESP(x, y, z) = \sum_{k=1}^{N} R_m(x_1, y_1, z_1)_i \times I_{Lk} \quad (11)$$

Where N is the total number of segments, and R_m is the transfer resistance between the kth segment and the point (x_1, y_1, z_1) at each point $(x, y, 0)$ on the earth surface ($z = 0$) as below

$$R_m = \frac{\rho(f)}{8L\pi}[F_1(x^+, Ax^-) - F_1(x^-, Ax^-) + F_1(x^+, Ax^-) - F_1(x^+, Ax^+)] \quad (12)$$

Where

$$F_1(t, u) = \ln(t + \sqrt{t+u}) \quad (13)$$

$$x^+ = x - x_1 + L; \qquad x^- = x - x_1 - L \quad (14)$$

$$Ax^+ = \sqrt{(y-y_1)^2 + (z+z_1)^2}; \quad (15)$$

$$Ax^- = \sqrt{(y-y_1)^2 + (z-z_1)^2} \quad (16)$$

Fig. 6, show the iteration process of the improved MTL. Finally, via inverse fast Fourier transform in (11), earth surface potential in time domain is computed.

In dealing with multiple rods, the analysis is the same as single vertical rod except that the series inductance L_k for computing Z in (6) is changed as below

$$L_k = \frac{\mu_0 l_k}{2\pi}\left(\ln\frac{2l_k}{a} - 1\right) + \sum_{m=1, k\neq m}^{N} L_{km} \quad (17)$$

Where N is the total number of parallel segments, and L_{km} is mutual inductance between kth and mth segments of length l_k as shown in Fig. ٦ and computed as (18) [13]. All parameters in (18) are defined in Fig. 5.

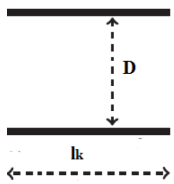


Fig. 5. Schematic of two parallel segments of length l_k.

$$L_{km} = \frac{\mu_0}{2\pi}\left\{ l_k \ln \frac{l_k + \sqrt{l_k^{\,2} + D^2}}{D} - \sqrt{l_k^{\,2} + D^2} + D \right\} \quad (18)$$

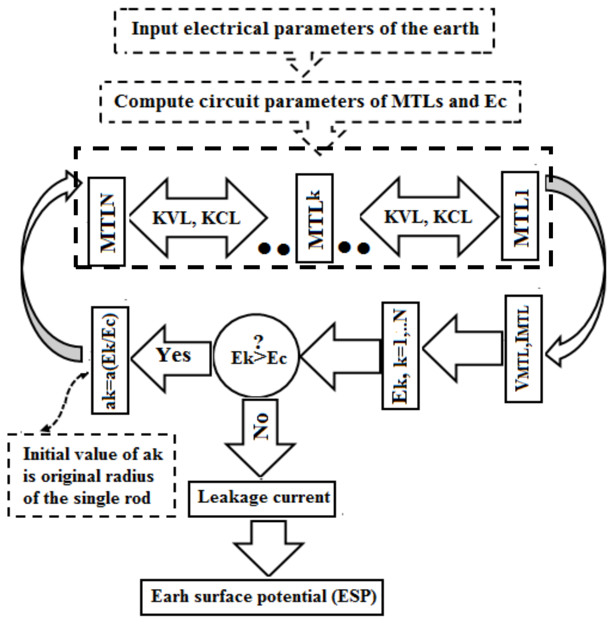


Fig. 6. Iteration process in the improved MTL algorithm.

III. Sensitivity Analyses

In this section, sensitivity analyses on the earth surface potential are carried out versus the number and length of rods and their effects on the touch and step voltages are accordingly investigated.

1. Number of Arms

To know how the number of rods affects the touch and step voltages, ESPs for single rod, double, triple, and quadruple rods having length L=3m and spacing 3m are computed and shown in Figs. 7 and 8 respectively for the first and subsequent stroke currents. As qualitatively seen from these figures, when the number of rods is increased, the slope of ESPs is decreased. Therefore, it is expected that the touch and step voltages to be decreased. Touch and step voltages of different arrangements of rods are numerically computed and tabulated in Tables 2 and 3 respectively for the touch and step voltages. From these tables, the touch and step voltages under subsequent stroke current is less than those of the first stroke current. Also, when the number of rods is increased, both voltages are considerably reduced on average up to 81.5% and 69.5% respectively for the touch and step voltages.

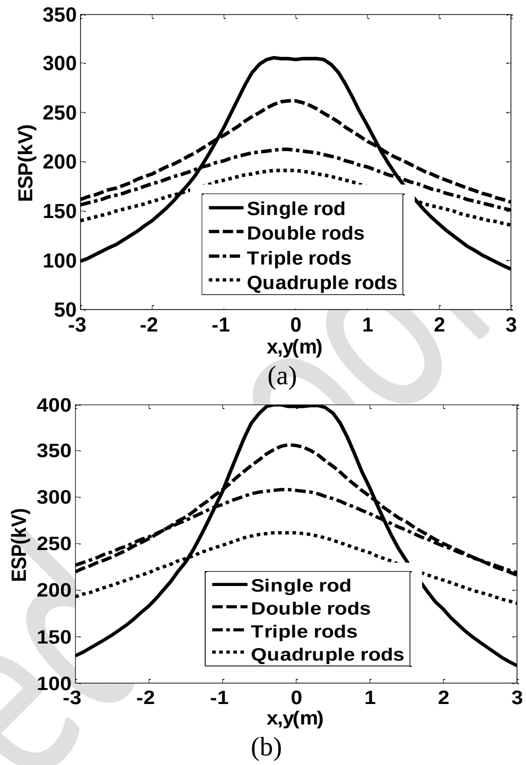
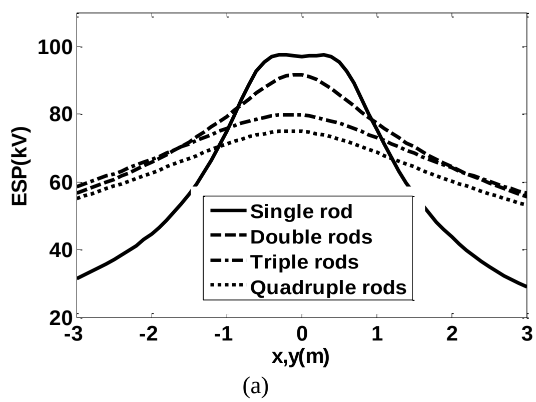


Fig. 7. ESP of multiple rods under first stroke current in both-affected earth for (a): $\rho_0 = 100\Omega m$ and (b): $\rho_0 = 1000\Omega m$.



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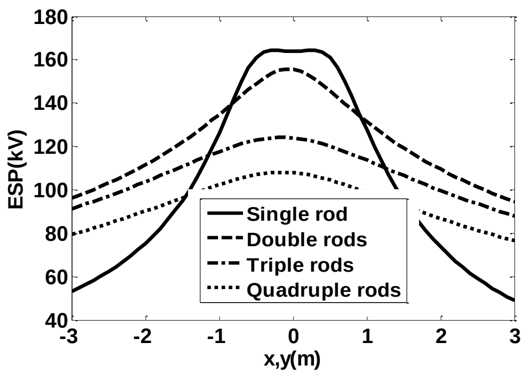


(b)

Fig. 8. ESP of multiple rods under subsequent current in both-affected earth for (a): $\rho_0 = 100\Omega m$ and (b): $\rho_0 = 1000\Omega m$.

Table 2. Comparing the touch voltages (kV) of different vertical rods with L=3m.

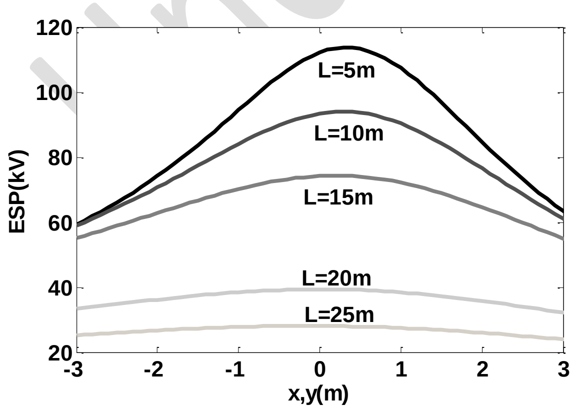
Lightning Current		**First Stroke**		**Subsequent Stroke**	
$\rho_0 (\Omega m)$		100	1000	100	1000
Rod	Single	100	110	30	40
	Double	40	50	10	20
	Triple	20	25	5	10
	Quadruple	18	23	4.5	8
Reduction (%)		82%	79%	85%	80%

Table 3. Comparing the step voltages (kV) of different vertical rods with L=3m.

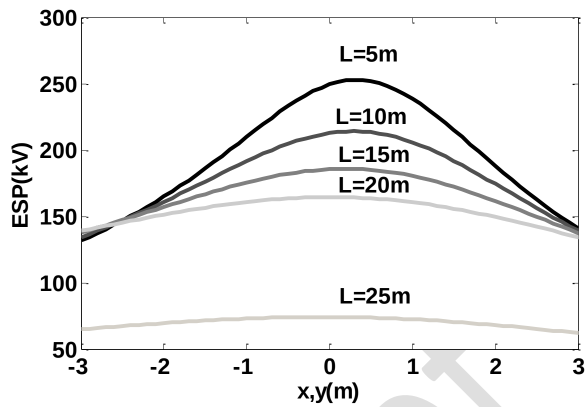
Lightning Current		**First Stroke**		**Subsequent Stroke**	
$\rho_0 (\Omega m)$		100	1000	100	1000
Rod	Single	50	100	33	45
	Double	35	60	15	30
	Triple	20	35	10	20
	Quadruple	15	34	9.5	13
Reduction (%)		70%	66%	71%	71%

2. *Length of Arms*

In the previous section, it is assumed that the length of the arm is constant. Now, the effect of length on the touch and step voltages is investigated.

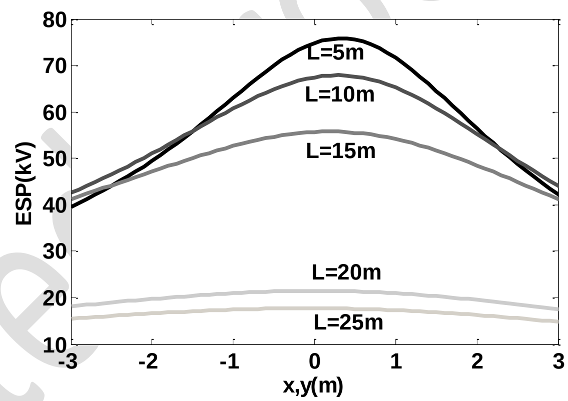


(a)



(b)

Fig. 9. ESP of quadruple rods under first stroke current with variable length, (a): $\rho_0 = 100\Omega m$ and (b): $\rho_0 = 1000\Omega m$.



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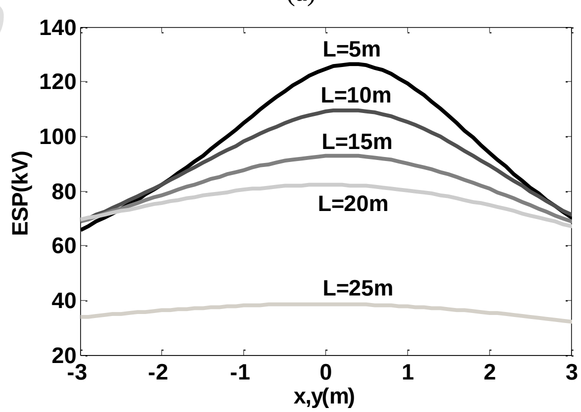

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Fig. 10. ESP of quadruple rods under subsequent current with variable length, (a): $\rho_0 = 100\Omega m$ and (b): $\rho_0 = 1000\Omega m$.

Figs 9 and 10 show the variations of ESP of the quadruple rods for different values of the rod length respectively for the first and subsequent stroke currents. As can be qualitatively observed from these figures, when the rod length is increased, slope of ESPs is considerably decreased which evidently leads to reduction of the touch and step voltages. Quantitative results for both voltages versus different values of the

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rod length are tabulated in Tables 4 and 5 respectively for the touch and step voltages.

Table ٤. Touch voltage (kV) of quadruple rods lossy earth.

Lightning Current		**First Stroke**		**Subsequent Stroke**	
$\rho_0 (\Omega m)$		100	1000	100	1000
L(m)	10	3	7	2	4
	15	2	6	1.53	2
	20	1	4	0.48	1.98
	٢٥	0.78	2	0.4	0.89
	٣٠	0.62	1.44	0.3	0.75
	٣٥	0.53	1.23	0.24	0.65
	٤٠	0.5	1.17	0.22	0.63
Reduction (%)		84%	83%	89%	85%

Table ٥. Step voltage (kV) of quadruple rods in lossy earth.

Lightning Current		**First Stroke**		**Subsequent Stroke**	
$\rho_0 (\Omega m)$		100	1000	100	1000
L(m)	10	13	19	9	16
	15	7	11	5	9
	20	2	4	1.05	5
	٢٥	2	3.43	1	2
	٣٠	1.48	2.78	0.7	1.8
	٣٥	1.38	2.28	0.53	1.44
	٤٠	1.29	2.18	0.49	1.35
Reduction (%)		90%	89%	94%	92%

From these tables, it is found that when the rod length is increased, both voltages are decreased. Moreover, depending on the earth resistivity and lightning current waveform, after a specific length, both voltages are slowly varied to reach constant values. The relative reduction is more pronounced in highly conductive earths under the subsequent stroke current. On average, when the length is increased from 10m to 40m, the relative reduction is about 85% and 91% respectively for the touch and step voltages. The significance of the two parameters, i.e., length and arm, is more pronounced in rocky earths where increasing burial depth is difficult.

IV. Two-Layer Earth

To extend the previous analyses, it is assumed that the ionization and dispersion are occurred in two-layer earth. The analysis is the same as both-affected earth except that the apparent low-frequency resistivity [33] should be used, that is

$$\rho_{0eq} = \begin{cases} \rho_{01} / \left\{1 + \left[(\rho_{01}/\rho_{02}) - 1\right]\left[1 - e^{1/k(d+2H)}\right]\right\} & \rho_{01} > \rho_{02} \\ \rho_{01} / \left\{1 + \left[(\rho_{02}/\rho_{01}) - 1\right]\left[1 - e^{1/k(d+2H)}\right]\right\} & \rho_{01} < \rho_{02} \end{cases} \quad (19)$$

Where ρ_{01} and ρ_{02} are respectively low-frequency resistivity values of the first and second layers. Also, d H and k are respectively burial depth, the thickness of the first layer and reflection factor which is defined as $k = \frac{\rho_{01} - \rho_{02}}{\rho_{01} + \rho_{02}}$. All parameters are illustrated in Fig. 11.

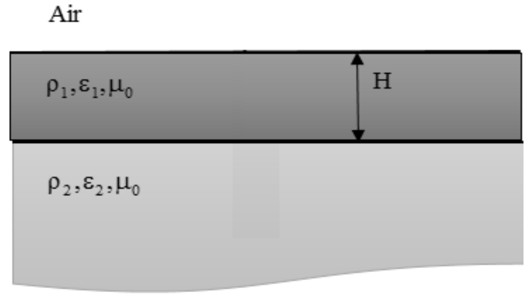
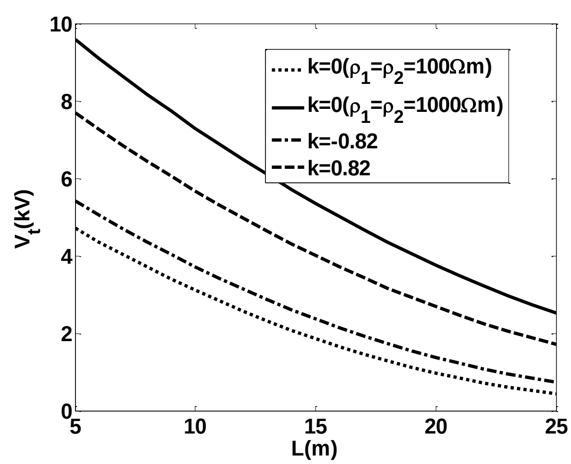


Fig. 11. Schematic of a two-layer earth.

To analysis of such earths, two scenarios as listed in Table .6 are first applied, and corresponding touch and step voltages are then computed versus the arm length as shown in Fig. 12. With the aim of comparison, both voltages of the single-layer earth (k=0) are also included in the same figure. From this figure, it can be inferred that the same as single-layer earth, when the length is increased both voltages are decreased. In addition, in two-layer earth both voltages are placed between the individual ones in single-layer earth. This fact shows significance of including the second layer especially when the first layer is more solid than the second layer (k<0).

Table 6. Electrical parameters of two layers in two-layer earth.

Layer	**First**			**Second**		
Parameter	ρ_{01}	ε_{r1}	H	ρ_{02}	ε_{r2}	H
First scenario	1000	10	10	100	10	∞
Second scenario	100	10	10	1000	10	∞



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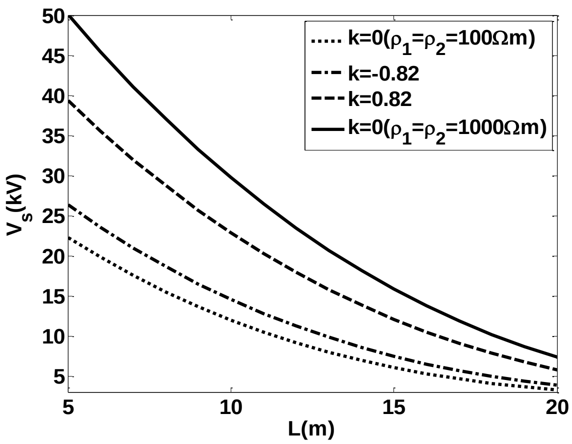

(b)

Fig. 12. Variations of the (a): touch and (b): step voltages under first stroke current versus length in two-layer earth for three values of k.

V. Conclusion

In this study, different properties of the earth consisting of ionization, dispersion, porosity level, and water content on the touch and step voltages of multiple rods were investigated. Acoording to the simulation results based on the improved multi-conductor transmission line model (IMTL) , the following key findings are inferred:

1- In ionized and dispersive earth, when the number of arms is increased from one to four arms, both voltages are reduced. Reduction is on average 81.5% and 69.5% respectively for the touch and step voltages. In addition, increasing the arm length from 10m to 40m results in reducing 85% and 91% for the touch and step voltages respectively. This reduction is continued up to a starting length beyond which it is stopped.

2- Integrating the apparent low-frequency resistivity in two-layer earth with the improved MTL results in computation of the touch and step voltages in such earths. simulation results show practical significance of the two layers epecially when the second layer is more resistive than the first layer.

The above notes can be benefit for power and telecommunication engineers in exact design of multiple grounding rods.

REFERENCES

[1] L. Grcev, “Impulse efficiency of Ground Electrodes”, *IEEE Transaction on Power Delivery*, vol. 24, pp. 441-451, 2009.

[2] R. Alipio, S. Visacro, “Impulse Efficiency of Grounding Electrodes: Effect of Frequency-Dependent Earth Parameters,” *IEEE Transaction on Power Delivery, vol.* 29, pp. 716-723, 2014.

[3] Jinliang He, “Progress in Lightning Impulse Characteristics of Grounding Electrodes With Earth Ionization,” *IEEE Transaction on Industry Application*, vol. 51, pp. 4924-4933, 2015.

[4] S. Visacro, “What Engineers in Industry Should Know About the response of Grounding Electrodes Subjected to Lightning Currents”, *IEEE Transaction on Industry Application*, vol. 51,pp. 4943-4951, 2015.

[5] M. Akbari, K. Sheshyekani, M. Reza Alemi, “The Effect of Frequency Dependence of Earth Electrical Parameters on the Lightning Performance of Grounding Systems”, *IEEE Transactions on Electromagnetic Compatibility*, vol. 55, pp. 739-746, 2013.

[6] J. Wu, B. Zhang, J. He, R. Zeng, “A Comprehensive Approach for Transient Performance of Grounding System in the Time Domain”, *IEEE Transactions on Electromagnetic Compatibility*, vol. 57, pp. 250-256, 2015.

[7] O. Kherif, S. Chiheb, M. Teguar, A. Mekhaldi, N. Harid, “Time-Domain Modeling of Grounding Systems’ Impulse Response Incorporating Nonlinear and Frequency-Dependent Aspects”, *IEEE Transactions on Electromagnetic Compatibility*, vol. 60,pp. 907-916, 2018.

[8] M. Moradi, “Analysis of Transient Performance of Grounding System Considering Frequency-Dependent Earth Parameters and Ionization”, *IEEE Transactions on Electromagnetic Compatibility*, vol. 62, pp. 785-797, 2019.

[9] Jalil Ghayur Safar, Reza Shariatinasab, Jinliang He, “Comprehensive Modeling of Grounding Electrodes Buried in Ionized Earth Based on MoM-HBM Approach”, *IEEE Trans. Power. Del*. vol. 57,pp. 1627-1636, 2019.

[10] R. Shariatinasab, J. Ghayur Safar, J. Gholinezhad, and J. He, “Analysis of Lightning-Related Stress in Transmission Lines Considering Ionization and Frequency-Dependent Properties of the Earth in Grounding Systems”, *IEEE Transactions on Electromagnetic Compatibility*, vol. 62, pp. 2849-2857, 2020.

[11] Reza Shariatinasab, Javad Gholinezhad, and Keyhan Sheshyekani, “Estimation of Energy Stress of Surge Arresters Considering the High-Frequency Behavior of Grounding Systems”, *IEEE Transactions on Electromagnetic Compatibility*, vol. 60, no. 4, pp.917-925, 2018.

[12] S. R. Ostadzadeh, and S. S. Sajjadi, “Effective area of grounding grids in frequency-variant earths: causality versus non-causality”, *Electrical Engineering*, vol. 104, pp. 2123-2131, 2022.

[13] S. S. Sajjadi, V. Aghajani and S.-R.-Ostadzadeh “Comprehensively Formulae for Effective Length of Multiple Rods Considering Different Aspects of Earth: Improved MTL-Intelligent Water Drop Algorithm”, *International Journal of Numerical Modelling, Electronics, Network Devices, and Fields*, vol. 33 , no. 4, pp. 1-20, *2020.*

[14] S. R. Ostadzadeh, and S. S. Sajjadi, “Predicting formulae for effective length of counterpoise

10.22060/EEJ.2026.25015.5818

wires buried in ionized, dispersive and inhomogeneous earths", *COMPEL-The international journal for computation and mathematics in electrical and electronic engineering*, vol. 39, no. 6, pp. 1375-1391, 2020.

[15] S. R. Ostadzadeh, S. S. Sajjadi, S. H. H. Sadeghi, "GPR of Multiple Vertical Rods under Lightning Strokes Considering Ionization, Dispersion, and Non-Homogeneity of Lossy Soils", *AUT J. Elec. Eng.*, vol. 55, no. 1, pp. 107-114, 2023.

[16] Anton Habjanic and Mladen Trlep, "The Simulation of the Soil Ionization Phenomenon Around the Grounding System by the Finite Element Method", *IEEE Tran. On Magn*. vol. 42, no. 4, pp. 867-870, 2006.

[17] K. Sheshyekani, S.H.H. Sadeghi, R. Moini, F. Rachidi, "Frequency-domain analysis of ground electrodes buried in an ionized soil when subjected to surge currents: A MoM–AOM approach", *Electric Power Systems Research*, vol. 81, pp. 290–296, 2011.

[18] Leonid D. Grcev, "Transient Electromagnetic Fields Near Large Earthing Systems", *IEEE Tran. On Magn*. vol. 12, no. 3, pp. 1525-1528, 1996..

[19] SAEID GHOLAMI, FARKOUSH , ABDUL WADOOD CHANG-HWAN KIM , TAHIR KHURSHAID , MUHAMMADIRFAN, AND SANG-BONG RHEE, "Reducing the Effect of Lightning on Step and Touch Voltages in a Grounding Grid Using a Nature-Inspired Genetic Algorithm With ATP-EMTP", *IEEE ACCESS*, vol. 7, pp. 81903-81910, 2019.

[20] ADEL Z. EL DEIN, S. YASSEN, "Comparison Between Square and Circular Designs of Grounding Grid", WSEAS TRANSACTIONS on POWER SYSTEMS, vol. 13, 2018.

[21] S. S. Sajjadi, S. R. Ostadzadeh, "Lightning response of Multi-port Grounding Grids Buried in Dispersive Soils: An Approximation versus Full Wave Methods and Experiment", Advanced Electromagnetics, vol. 8, no. 1, pp. 43-50, March 2019.

[22] Std80-2013 - IEEE Guide for Safety in AC Substation Grounding.

[23] S. S. Sajjadi, V. Aghajani, S. R. Ostadzadeh, "Transient Analyses of Grounding Electrodes Considering Ionization and Dispersion Aspects of Earths Simultaneously: An Improved Multiconductor Transmission Line Model (Improved MTL)", *Applied Computational Electromagnetic Society Journal (ACES)*, vol. 34, pp. 731-737, 2019.

[24] A. Jardines, J. L. Guardado, J. Torres, J. J. Chavez, M. Hernandez, "A Multiconductor Transmission Line Model for Grounding Grid", *Electrical Power and Energy Systems*, vol. 60, pp. 24-33, 2014.

[25] Ostadzadeh Saeed Reza, "Validity of Improved MTL for Effective Length of Counterpoise Wires under Low and High-Valued Lightning Currents", *AEM Journal*, vol. 9, no. 1, pp. 71-77, 2020.

[26] S. S. Sajjadi, and S. R. Ostadzadeh, "Parametric dependence of lightning impulse behavior of grounding electrodes buried in a dispersive and ionized lossy soil under first- and subsequent-stroke currents", *COMPEL-The international journal for computation and mathematics in electrical and electronic engineering*, vol. 39, no. 4, pp. 757-773, 2020.

[27] F. Heidler, W. Zischank, Z. Flisowski, C. Bouquegneau, and C. Mazzetti, "Parameters of lightning current given in IEC 62305-background, experience and outlook," presented at the 29th Int. Conf.Light. Protect., Uppsala, Sweden, Jun. 23—26, 2008.

[28] D. Cavka, N. Mora, and F. Rachidi "A Comparison of Frequency-Dependence Earth Models: Application to the Analysis of Grounding systems", *IEEE Transactions on Electromagnetic Compatibility*, vol. 56, no. , pp. 177-187, 2013.

[29] S. R. Ostadzadeh, and S. S. Sajjadi, "Effective area of grounding grids in frequency-variant earths: causality versus non-causality", *Electrical Engineering*, vol. 104, pp. 2123-2131, 2022.

[30] S. R. Ostadzadeh, and S. S. Sajjadi,, "Effective length and earth surface voltage of grounding electrodes in causal-dispersive earths", *International Journal of Applied Electromagnetics and Mechanics*, vo78, no. 4, pp. 360-371, 2025.

[31] Rafael Alipio, and S. Visacro, "Modeling the Frequency Dependence of Electrical Parameters of Earth", *IEEE Transactions on Electromagnetic Compatibility*, vol. 15, no. 1, pp.1163-1171, 2014.

[32] E. E. Oettle, "A new general estimation curve for predicting the impulse impedance of concentered earth electrodes," in *Proc. IEEE Power Eng. Soc. Summer Meeting*, San Francisco, CA, USA, pp. 2980–2982, Jul. 1987.

[33] Chang CN, Lee CH. "Computation of ground resistances and assessments of ground grid safety at 161/23.9 kV indoor-type substation", IEEE Trans. Power Del., vol. 21, pp. 873-878, 2006.