



Analyzing the internal resonances and energy exchange between modes of power system considering Frequency – Energy dependence using Pseudo-Arclength and shooting algorithm

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ABSTRACT: The power system nonlinearity and its profound impact on the individual states of power system is first evaluated and the interaction between their constituent modes during the occurrence of internal mode resonance (IMR) is discussed in this paper. A typical dynamical feature of nonlinear systems is the frequency-energy dependence of their states and their corresponding constituent modes which is also underlined in this paper. At first predominant state is identified which is defined as the one with highest energy level and the internal mode resonances and energy exchange between its constituent modes are explored accordingly. However, Perturbation Techniques such as Normal Form (NF) or Modal Series (MS) and several polynomial approximation are explored and it is demonstrated that such methodologies do not lead to the acceptable results and does not work well in near-resonant conditions. For this reason, the integrated algorithm consists of Shooting and Pseudo-Arclength is employed for obtaining Frequency-Energy Plot (FEP) to estimate and evaluate the involved modes behavior during the resonance and the energy level at which the internal resonance occurs. The studies are performed on 39-bus New England test power System and the final results prove the accuracy and effectiveness of the proposed methodology and algorithm.

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1. INTRODUCTION

Nowadays, Large-scale interconnected power systems are being operated under heavy loading conditions close to their stability margin and thermal rating of the equipment particularly in peak load implying that the power system owner would rather exploit all the capacity of the system equipment in peak load scenarios than expand the transmission lines and substations which are so cost-intensive in terms of environmental and financial issues. This phenomena would result in the so-called stressed power system during the peak load which exhibits the strong nonlinearity and complex dynamic behavior [1]. In the other side, the growing increase of inverter-based renewable power plants and FACTS devices and nonlinear loads in power systems have increased the system nonlinearity and exacerbated the problem [2]. However, when a highly nonlinear and stressed power system is subject to a perturbation, the resonance among the constituent modes of the states in the linear system might occur at high energy level due to their frequency-energy dependence and therefore the frequency response function (FRF) of the nonlinear system is no longer invariant which is in contrast to the traditional linear analysis and then the linear framework must be abandoned in favor of a nonlinear modal analysis [3-5]. The interaction between modes in stressed power system due to nonlinearities have been vastly dealt

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with in several papers [4-15] wherein, different techniques for nonlinear modal analysis have been presented which are briefly described in the following. Normal Form (NF), Modal Series (MS) and Polynomial Approximation are the most renowned methods applied for nonlinear modal analysis. The method of normal form was first employed by Lamarque and Jezequel and Nayfeh using a complex formulation [4-5]. The NF method is regarded with a nonlinear change of coordinates to turn the equations of motion into the simplest possible form by eliminating as many as possible of the nonlinear terms from the governing equations. Several paper have recently employed the Normal Form theory to account for the nonlinear terms in decoupling the governing equations of motion in power system to model the impact of system nonlinearity on the interaction between modes [5-14]. In The MS method, a nonlinear system is presented as a rather straightforward generalization of the linear case and the Taylor Series are used [15-17]. The fundamental drawback of the NF and MS methods are as follows.

- They are quite analytically involved and require a careful treatment in the presence of internal resonances
- The resultant dynamics are only accurate for small amplitude motions
- The upper bound for these motions is not known a prior
- They are limited to systems with low dimensionality and restricted to polynomial nonlinearity



- Due to Mode bifurcation in higher energy levels, the nonlinear modes cannot be regarded as nonlinear continuation of the modes of linear system

Furthermore, several papers proposed the numerical or semi-analytical techniques [18-22]. Numerical techniques rely on the extensive numerical simulations and are computationally intensive. Semi-analytical techniques such as Harmonic Balance Method (HBM) and Galerkin-based approach fail to give accurate response when the turning point is encountered (Bifurcation) [23-28]. To resolve the mentioned drawbacks, the contribution and novelty of the paper is to identify the predominant states and its constituent modes using energy of the modes and then represent an integrated method including Pseudo-Arclength and shooting algorithm which is a hybrid numerical and sequential continuation method to estimate the energy level at which the resonance is more likely to occur. The nonlinear mode trajectory is estimated through the Pseudo-Arclength and also the shooting algorithm enables us to follow the trajectory [23-24]. For further clarification, the main objective is divided into two following parts.

1-The predominant state is identified through Frequency-Energy analysis of its constituent modes in highly stressed condition (it should be noted that the energy of the state is defined as the sum of the energy of its constituent modes), 2- The Frequency – Energy Plot (FEP) of the predominant mode (obtained from previous part) is computed through the proposed integrated method and the critical energy level at which the internal resonance might occur, is computed and presented.

The remainder of the paper is organized as follows. The integrated approach (Shooting and Pseudo-Arclength methods) is described in Section II. Section III studies the methodology and the frameworks. The case study and the simulation results considering aforementioned methods under different operation scenarios is presented in section IV and finally the conclusion is given in section V.

2. AN INTEGRATED APPROACH FOR TRACING THE NONLINEAR MODE TRAJECTORY (SHOOTING AND PSEUDO-ARLENGTH METHODS)

To follow the mode trajectory for the change of energy level in a strongly nonlinear power system, a shooting procedure combined with the so-called Pseudo-Arclength continuation method is employed in this paper. In this section a brief review of the mentioned methods is carried out and the integrated method is finally characterized by an algorithm.

2-1- Shooting Method

Consider the following swing equation of the SMIB power system in the classical model.

$$M\ddot{\delta} + D\dot{\delta} - \omega_0(P_m - P_e) + f_{nl}\{\delta, \dot{\delta}\} = 0$$

$$P_m = P_{\max} \sin(\delta_s) \quad (1)$$

$$P_e = P_{\max} \sin(\delta)$$

Where M is the inertia matrix; D is the damping constant, ω_0 is the synchronous frequency, δ is the rotor angle and δ_s is the angle at SEP (Stable Equilibrium Point). P_m , P_e represent the mechanical power and electrical power of the machines respectively. f_{nl} is the nonlinear function in a strong nonlinear power system. The equation of motion in (1) can be recast into state space form.

$$\dot{z} = g(z) \quad (2)$$

Where $z = [\delta^* \ \dot{\delta}^*]^*$ is the 2n-dimensional state vector, and star implicates the transpose operation and thus

$$g(z) = \begin{pmatrix} \dot{\delta} \\ -M^{-1}(D\dot{\delta} - \omega_0(P_m - P_e) + f_{nl}\{\delta, \dot{\delta}\}) \end{pmatrix} \quad (3)$$

It is assumed that the inertia matrix is invertible and $z^{(0)} = z_0 = [\delta_0^* \ \dot{\delta}_0^*]^*$ is the initial condition of the dynamical system. Assume that T is the minimal period of the periodic solution of the autonomous system (2) then it can be exhibited that $z(t, z_0) = z(t+T, z_0)$. It is to be noted that periodic solution of the governing nonlinear equation of motion (2) is key to estimating the nonlinear mode trajectory. To pursue the aim, a numerical technique called “shooting algorithm” is exploited. The boundary-value problem is solved numerically through the shooting algorithm. The shooting function H is defined as following.

$$H(z_0, T) = z(T, z_0) - z_0 = 0 \quad (4)$$

According to (4), the shooting function is defined as the difference between the system response at time T and the initial condition. The function is solved by a continuation method (Pseudo-Arclength method) which is described later. The next step is to expand the nonlinear function in Taylor series (6) considering the corrections $(\Delta z_0, \Delta T_0)$ based on (5), whereas the higher order term (H.O.T) is neglected.

$$H(z_0^{(0)} + \Delta z_0^{(0)}, T^{(0)} + \Delta T^{(0)}) = 0 \quad (5)$$

$$H(z_0^{(0)}, T^{(0)}) + \left. \frac{\partial H}{\partial z_0} \right|_{(z_0^{(0)}, T^{(0)})} \Delta z_0^{(0)} + \left. \frac{\partial H}{\partial T} \right|_{(z_0^{(0)}, T^{(0)})} \Delta T^{(0)} + H.O.T = 0 \quad (6)$$

The periodic solution is featured by the initial conditions (z_0) and the period (T) which are computed by means of an iterative procedure.

$$\begin{aligned} z_0^{(k+1)} &= z_0^{(k)} + \Delta z_0^{(k)} \\ T^{(k+1)} &= T^{(k)} + \Delta T^{(k)} \end{aligned} \quad (7)$$

With

$$\left. \frac{\partial H}{\partial z_0} \right|_{(z_0^{(k)}, T^{(k)})} \Delta z_0^k + \left. \frac{\partial H}{\partial T} \right|_{(z_0^{(k)}, T^{(k)})} \Delta T_0^k = -H(z_0^{(k)}, T^{(k)}) \quad (8)$$

K is the shooting iteration index. To achieve the convergence together with desired accuracy, the constraints $H(z_0, T) \approx 0$ and $h(z_0) = \mathbf{0}$ must be fulfilled where, $h(z_0)$ is the phase condition. In summary, the nonlinear computation is carried out by solving (9).

$$F(z_0, T) = \begin{cases} H(z_0, T) = 0 \\ h(z_0) = 0 \end{cases} \quad (9)$$

By derivation of (4), $\partial H/\partial T$ is presented by

$$\frac{\partial H}{\partial T}(z_0, T) = \left. \frac{\partial z(t, z_0)}{\partial t} \right|_{t=T} = g(z(T, z_0)) \quad (10)$$

$\partial H/\partial z_0$ is provided by

$$\frac{\partial H}{\partial z_0}(z_0, T) = \left. \frac{\partial z(t, z_0)}{\partial z_0} \right|_{t=T} - I \quad (11)$$

In (11), I is the identity matrix. As described previously, the nonlinear mode trajectory must be evaluated through numerical continuation of periodic motions. Newton-Raphson method due to its strong dependence on the initial guess and the fixed value of the period, is unable to represent the turning points in the mode trajectory and for this reason Pseudo-Arclength continuation method is employed in this paper as a continuation method to avoid the mentioned drawbacks of the Newton-Raphson method.

2-2- Pseudo-Arclength Method

Pseudo-Arclength is a continuation method which enables us to consider the predictions and correction of the period simultaneously in the shooting process. However, predictor and corrector steps are considered to find the next periodic solution $(z_{0,(j+1)}, T_{(j+1)})$ from the known solution $(z_{0,(j)}, T_{(j)})$. At step j , assume that $(\tilde{z}_{0,(j+1)}, \tilde{T}_{(j+1)})$ is the prediction of which is generated along the tangent vector to the branch at the current point. We define

$$\begin{bmatrix} \tilde{z}_{0,(j+1)} \\ \tilde{T}_{(j+1)} \end{bmatrix} = \begin{bmatrix} \tilde{z}_{0,(j)} \\ \tilde{T}_{(j)} \end{bmatrix} + s_{(j)} \begin{bmatrix} p_{z,(j)} \\ p_{T,(j)} \end{bmatrix} \quad (12)$$

Where $s_{(j)}$ is the predictor stepsize and $\begin{bmatrix} p_{z,(j)} \\ p_{T,(j)} \end{bmatrix}$ is the tangent vector to the branch according to (9). Finally, the solution to the problem is achieved as follows.

$$\begin{bmatrix} \frac{\partial H}{\partial z_0} \Big|_{(z_{0,(j)}, T_{(j)})} & \frac{\partial H}{\partial T} \Big|_{(z_{0,(j)}, T_{(j)})} \\ \frac{\partial h}{\partial z_0} \Big|_{(z_{0,(j)})} & 0 \end{bmatrix} \begin{bmatrix} p_{z,(j)} \\ p_{T,(j)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (13)$$

The general flowchart of the Pseudo-Arclength method

is illustrated in Fig.1 and the hierarchy of Pseudo-Arclength Continuation method is illustrated in Fig 2. It is to be noted that the predictor step is tangent to the branch and the corrector step is perpendicular to the predictor step. According to Fig.1 and Fig 2, the flowchart is set based on the sequence and continuation. When the mode amplitude versus frequency (Fig.2) becomes closer to the turning point, the proposed algorithm can detect the bifurcation which is defined as the internal resonance between modes of interest. It should be mentioned that the linear modal analysis doesn't work well near to the turning point.

3. METHODOLOGY AND FRAMEWORK

The FFT method (Fast Fourier Transform) has been commonly known for frequency analysis of the oscillation whilst it is restricted to discrete and predefined frequencies and is applicable for linear power system considering natural frequencies of the power system. In this paper, a given signal in nonlinear scenarios is decomposed into damped sinusoidal oscillations (so-called nonlinear normal modes of a signal) over a predefined time range in order to detect and quantify harmonic wave trends in time and analyze interharmonic oscillations. This objective is achieved through finding a sum of exponentials which fits best to a given series. However, Energy of a signal is described as the sum of the energy contents of the constituent individual modes while frequency of oscillation of a given signal strongly depends on its conserved energy. When the power system is highly nonlinear or is subject to a perturbation and during the power system transient, Energy between the individual modes involved is exchanged and this phenomenon may be interpreted as internal mode resonance (IMR). Thus, Modes of higher energy are regarded as predominant modes while those of lower energy are ignored in the computation process of internal resonances.

Let's consider that $y(t)$ is a given signal. To perform the frequency analysis, $y(t)$ could be represented as sum of individual modes. Thus

$$y(t) \approx \sum_{k=0}^{n-1} \text{mode}_k(t) \quad (14)$$

Where

$$\begin{aligned} \text{mode}_k &= \text{magnitude} \\ &\times e^{i \cdot (\text{phase})} \\ &\times e^{i \cdot (\text{damping} + 2\pi \cdot \text{frequency}) \cdot t} \end{aligned} \quad (15)$$

The energy of the mode which has no complex conjugate in the representation of $y(t)$ is represented in (16).

$$\text{Energy}_k = \int |\text{Re}(\text{mode}_k(t))|^2 dt \quad (16)$$

The energy of the mode which has complex conjugate in the representation of $y(t)$ is written as in (17).

$$\text{Energy}_k = \int |2\text{Re}(\text{mode}_k(t))|^2 dt \quad (17)$$

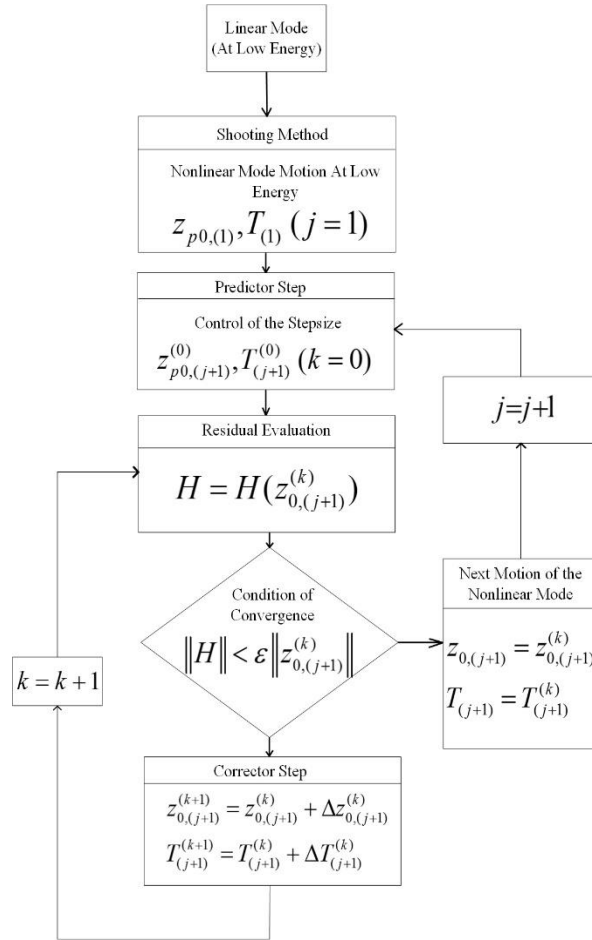


Fig. 1. Algorithm for nonlinear mode computation (integrated method)

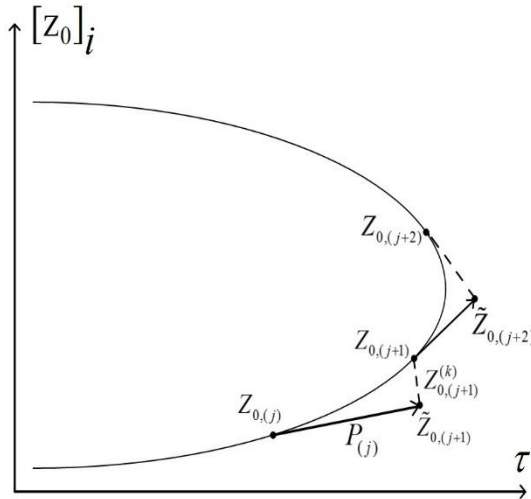


Fig. 2. Pseudo-Arclength Continuation Schematic

Predominant modes are selected from all the constituent modes of a signal by sorting all the modes according to the energy impact. As a result, mode 0 will be the most dominant one and then mode 1 and so on.

For several scenarios including no-fault and faulty condition the frequency analysis is performed and the

predominant modes are identified based on the Frequency and Energy Plot of the specified modes. The operating conditions where nonlinear modes are close to the resonance (Bifurcation) are focused and the exact resonant frequency and the modes involved are computed via Pseudo-Arclength and Shooting algorithm. Due to Frequency-Energy dependence

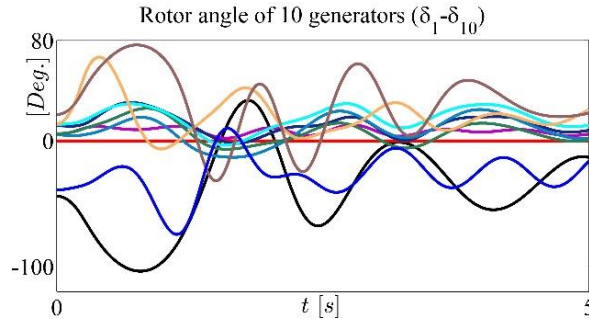


Fig. 4. The Rotor Angle of 10 Generators Following the Fault

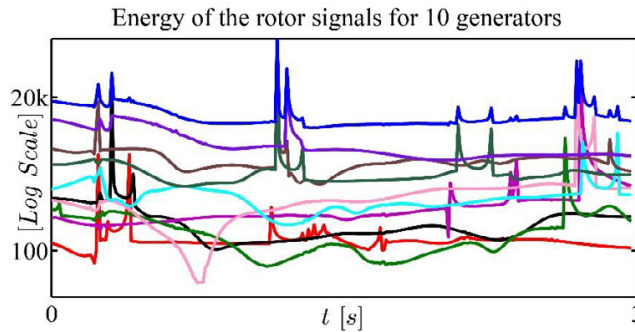


Fig. 5. Total Energy of each Generators' Rotor Angle signal

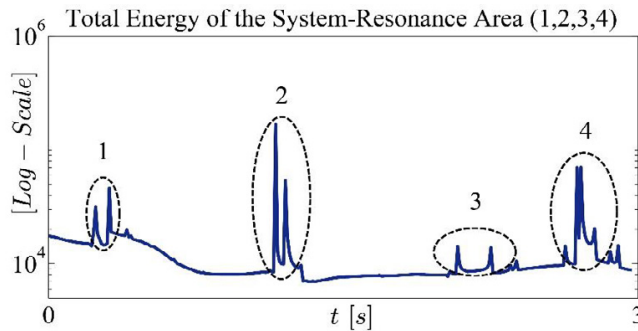


Fig. 6. Total Energy of all Generators' Rotor Angle signal

rotor angle is mostly affected by the predominant mode. The key point is that, in linear state, the energy of the predominant mode is constant and the mode can be referred to as Linear Normal Mode (LNM). Thus, in such scenarios, the traditional LNM studies (Modal Analysis) is viable.

4-2 - Scenario b – Nonlinear Modal Analysis

According to what mentioned before, in case of stressed power system (heavily loaded) or in the presence of nonlinear phenomena and especially beyond a specified energy level, the traditional linear analysis must be abandoned in favor of nonlinear modal analysis even though it depends on the type of nonlinearity and the excitation level. In this section, it is revealed that when the power system undergoes a nonlinear phenomenon, the oscillation frequency of the rotor angle will change in accordance with the total conserved energy in its

constituent modes. For this reason, the normal modes are no longer invariant and a trajectory of normal modes is formed. Not to be mentioned that the Frequency-Energy Plot (FEP) of the predominant modes could be computed through the proposed integrated method which exhibits the predominant modes behavior. This facilitates the control process which must be exerted on the power system during the heavy loading and in case of fault occurrence in order to die down the resonances.

Rotor angle of all generators are depicted in Fig 4. The total energy content of each signal is computed through (14) to (17) and the results are presented in Fig 4. According to Fig 5, at some time intervals, the energy deviation in the individual signals occur implying that energy is being exchanged among the constituent modes of the signals. Total energy in the rotor angle signals of the generators is computed and depicted in

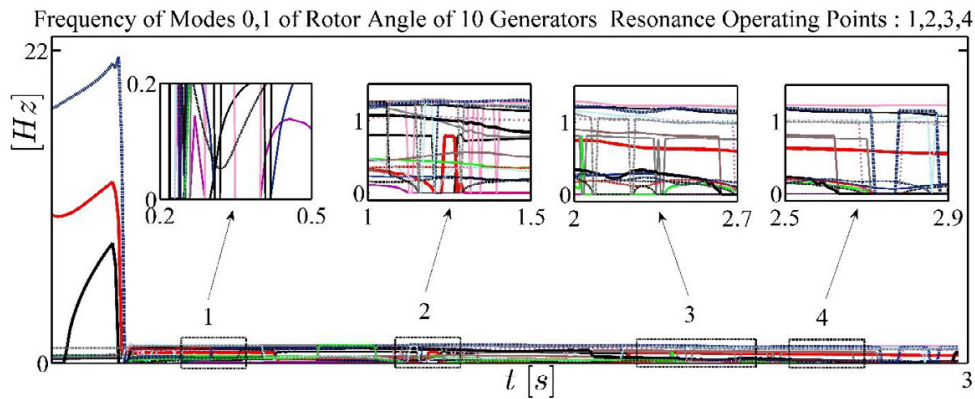


Fig. 7. The Oscillation Frequency and its closer viewpoints during the Internal Resonance (Points 1, 2, 3, 4)

Table 3. The resonant modes of power system

Fig 7	Resonant Modes
Point 1 (Fig 7)	Mode 0, G 1 Mode 0, G 3 Mode 0, G 9 Mode 0, G 10 Mode 1, G 1 Mode 1, G 4 Mode 1, G 5 Mode 1, G 8
Point 2 (Fig 7)	Mode 0, G 3 Mode 0, G 9 Mode 0, G 10 Mode 1, G 9
Point 3 (Fig 7)	Mode 0, G 3 Mode 1, G 5 Mode 0, G 10 Mode 1, G 10 Mode 0, G 5
Point 4 (Fig 7)	Mode 0, G 5 Mode 0, G 10 Mode 1, G 10

Fig 5 which could be described as the sum of the total energy of each signal. According to Fig 6, the time of interest is the interval when the energy deviation is abruptly changed which can be observed as the points 1 to 4.

To further illustrate the resonance point especially in term of frequency behavior, the points 1 to 4 in Fig 7 which are the same as the points in Fig 6, are shown. According to Fig 7, as the fault occurs the frequency of oscillations increases and after the fault clearance, it retains its normal value. As it is observed, during some time intervals, the frequency of the constituent modes are nearly commensurate which leads to the energy exchange among the modes and subsequently the abrupt changes in the total conserved energy of the signal. Fig. 7a to Fig. 7d exhibit the closer viewpoint of the resonance points (points 1 to 4). In Fig 7, the resonant modes for each part is detected and listed according to Table 3.

The constituent modes of a signal in nonlinear state of

the system are called nonlinear normal mode (NNM). The Frequency-Energy Plot (FEP) is the oscillation frequency of NNM for different energy levels. It should be mentioned that each point in the FEP is analyzed and computed through predefined proposed integrated method including Pseudo-Arclength and Shooting algorithm. The points in FEP is computed sequentially and continuously so that abrupt changes are detectable.

Frequency Energy Plot of Mode 0, 1 (δ_2) is presented in Fig 8a, b and the Frequency Energy Plot of individual Modes are illustrated in Figs 9a. It is observable that the critical energy levels at which the frequency of oscillation undergoes sudden changes, must be focused and highlighted as shown in Figs 9b and 9c. However, according to critical energy range, the proper control process must be adopted.

As mentioned previously, the FEP could be drawn based on the proposed integrated algorithm as follows.

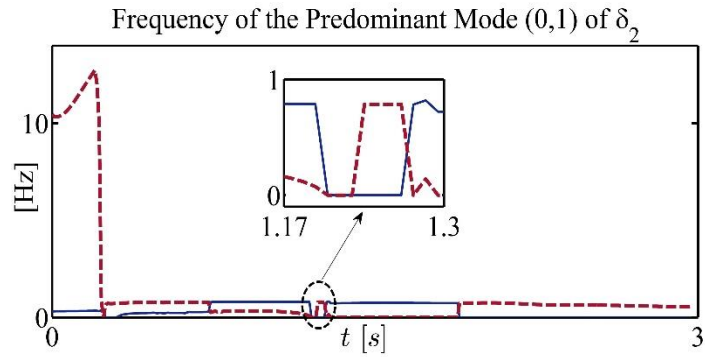


Fig. 8a. The Oscillation Frequency of the predominant modes (0, 1) of δ_2

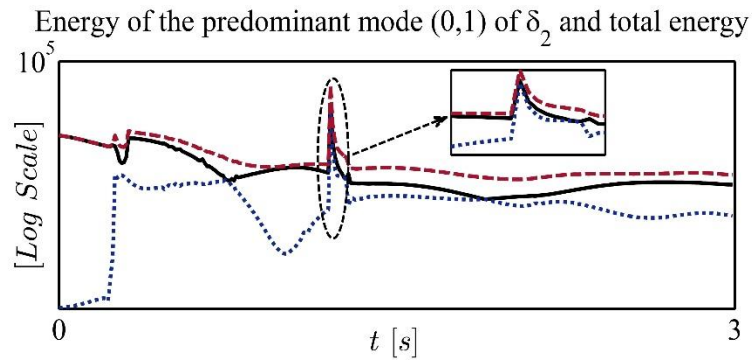


Fig. 8b. The Energy of the predominant modes (0, 1) of δ_2

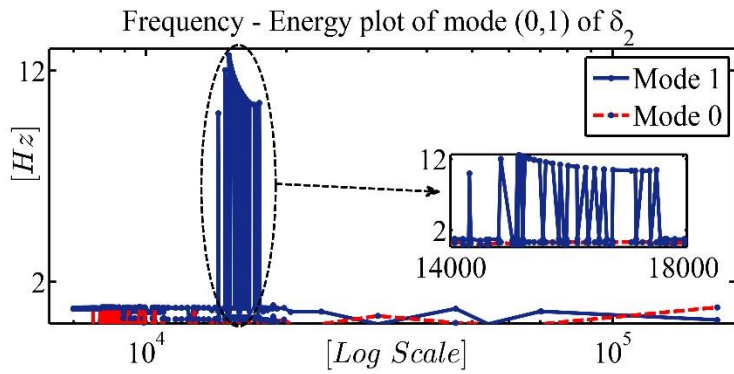


Fig. 9a. Frequency Energy Plot (FEP) of predominant modes of δ_2

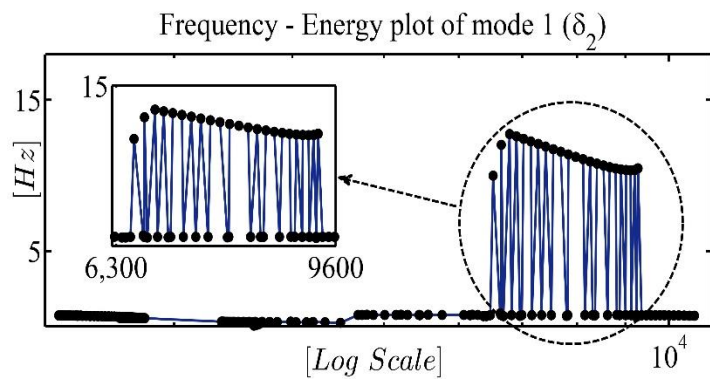


Fig. 9b. FEP of mode 1 of δ_2

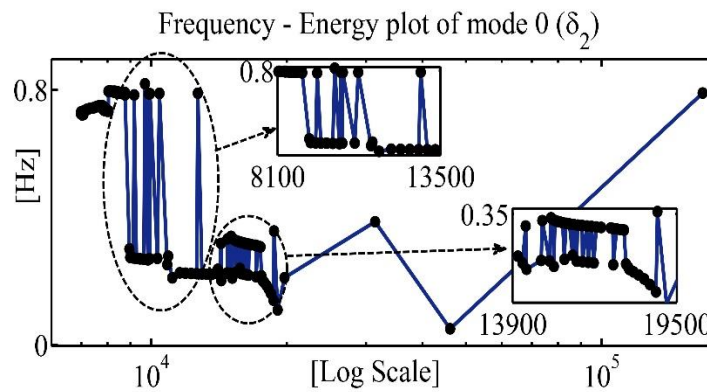


Fig. 9c. FEP of mode 0 of δ_2

5. CONCLUSION

The center of attention in this paper was on the highly stressed and nonlinear power system where abrupt change in signal behavior might take place. The Pseudo Arclength together with shooting algorithm as a sequential continuation method was applied in this paper to identify and examine the nonlinear normal mode (NNM) behavior and their internal resonances for a highly stressed power system. Analysis was carried out to exhibit the frequency-energy dependence of the modes and estimate the energy range during which this phenomenon might occur. In this way, at first the most influential signals in the power system in terms of energy were identified and secondly, the critical energy range for those signals was computed sequentially using the proposed algorithm. The obtained results explain that for linear state of the system, the linear modal analysis works well but for a predefined fault scenario, in the nonlinear state of the power system, the linear studies must be abandoned in favor of nonlinear modal analysis. Finally, the results demonstrates the accuracy and efficiency of the proposed algorithm.

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