Tight formation flight based on visual sensory systems was exchange for a relative positioning of unmanned aircraft. In the scope of formation flight problem, [13] investigates a limited information exchange among the robots is assumed and the distances should be estimated [7]. Moreover, usually, control of the mobile robots using an onboard camera. Only [12], researchers have focused on vision-based formation problem is solved based on the concept of the leader-follower system is obtained. Absence of an active communication link between the leader and the follower is another feature that the authors of this paper pursue. This paper is mainly intended to employ seekers as sensors that provide relative measurements for the formation keeping in the leader-follower structure. In general, seekers are categorized into two classes, namely, three axes seeker and two axes seeker [18]. In this paper, elevate-azimuth seekers which are of two-axes type and can provide relative measurements in elevation and azimuth axes are adopted. Generally, LOS angles and LOS rate angles with respect to the leader are the information provided by seekers [19]-[21]. With considering the seeker dynamics, a model for the related mechanism is accomplished. In this model, the noise of the onboard sensor in measuring the LOS angle rate is considered. In the literature, this noise is considered as the main uncertainty in seeker measurement and is called glint noise (see [22]-[27] for instance). Simulation results are given to study the effect of the seeker dynamics and sensor noise on the accuracy of the formation keeping. It is worth mentioning that application of the proposed idea is tested by the authors via a hardware in the loop simulation test-bed in [28] and more analytical studies by considering uncertainties in seeker measurements and the leader maneuvering are presented in [29] and [30]. The rest of the paper is organized as follows. The leader-
follower kinematic equations are formulated in section 2. Modeling of the airborne seeker is presented in section 3. In section 4, the formation control structure is proposed. Section 5 provides simulation results and finally, section 6 concludes the paper.

2-leader-follower kinematic equations

Leader-follower formation can be achieved via regulation of the relative angles \( \psi_{r}, \theta_{r} \) and the relative distance \( r_{L} \) to maintain these quantities at the desired values, as depicted in Fig. 1. To solve the leader-follower formation problem, at first, a kinematic formulation for the follower motion equations and the leader-follower relative kinematics must be derived. For this purpose, three coordinate frames \( I, L \) and \( V \) are defined as the inertial reference frame, the line of sight frame and the follower velocity frame, respectively.

To derive the motion equations of the follower, by introducing \( V_{F} = \begin{bmatrix} v_{x} & 0 & 0 \end{bmatrix}^{\top} \) as the follower velocity vector \( V_{F} \) with respect to the frame \( V \), we have

\[
V_{F} = J^{C} V_{P} = \begin{bmatrix} v_{x} \cos \psi_{r} \cos \theta_{r} & v_{x} \sin \psi_{r} \cos \theta_{r} & -v_{y} \sin \theta_{r} \end{bmatrix}^{\top},
\]

where \( J^{C} \) denotes rotation matrix of \( I \) with respect to \( V \); \( V_{P} \) is the follower speed. The variables \( \psi_{r} \) and \( \theta_{r} \) are the angles of follower velocity vector with respect to the reference frame.

\[
\dot{V}_{F} = J^{C} \dot{V}_{P} + \Omega_{\psi_{r} \theta_{r}} \times V_{F}
\]

Now, let us define \( \begin{bmatrix} a_{x} & a_{y} & a_{z} \end{bmatrix}^{\top} \) as the acceleration components in the frame \( V \). Then, \( \dot{D}_{F} V_{F} \) as the derivative of \( V_{F} \) is expressed in term of the frame \( I \), \( \Omega_{\psi_{r} \theta_{r}} = \begin{bmatrix} \omega_{x} & \omega_{y} & \omega_{z} \end{bmatrix}^{\top} \) as the angular velocity components in the frame \( V \) and using the following Coriolis formula,

\[
\dot{V}_{F} = D_{V} V_{F} + \Omega_{\psi_{r} \theta_{r}} \times V_{F}
\]

in combination with (1), the motion equation of the follower can be obtained as it follows,

\[
\begin{bmatrix}
\dot{x}_{F} = v_{x} \cos \psi_{r} \cos \theta_{r} \\
\dot{y}_{F} = v_{x} \sin \psi_{r} \cos \theta_{r} \\
\dot{z}_{F} = -v_{y} \sin \theta_{r}
\end{bmatrix},
\]

\[
\begin{bmatrix}
\dot{x}_{F} = a_{x} \\
\dot{y}_{F} = a_{y} \\
\dot{z}_{F} = a_{z}
\end{bmatrix},
\]

where \( \Omega_{\psi_{r} \theta_{r}} = \begin{bmatrix} \omega_{x} & \omega_{y} & \omega_{z} \end{bmatrix}^{\top} \) is position vector of the follower with respect to the reference frame.

To solve the leader-follower formation problem, by introducing \( ^{L}R_{L} = \begin{bmatrix} r_{L} & 0 & 0 \end{bmatrix}^{\top} \) and \( ^{L}\Omega_{L} = \begin{bmatrix} \omega_{x} & \omega_{y} & \omega_{z} \end{bmatrix}^{\top} \), we have

\[
\dot{L}_{D_{L} R_{L}} = \begin{bmatrix}
\dot{v}_{x} \\
\dot{v}_{y} \\
\dot{v}_{z}
\end{bmatrix}
\]

Time derivative of the above equation yields

\[
\dot{L}_{D_{L} R_{L}} = \dot{L}_{D_{L}} \left( D_{L} R_{L} \right) + \dot{L} \Omega_{L} \times \dot{L} R_{L} = \begin{bmatrix}
\dot{r}_{L} \\
\dot{r}_{L} \omega_{y} \\
-\dot{r}_{L} \omega_{y}
\end{bmatrix},
\]

where \( \begin{bmatrix} a_{x} & a_{y} & a_{z} \end{bmatrix}^{\top} \) is the LOS acceleration with respect to the frame \( L \). Hence, by rearranging the above equation, the relative kinematic equation can be obtained as,

\[
\begin{bmatrix}
\dot{r}_{L} = r_{L} \left( \omega_{x} + \omega_{y} \right) + a_{x} \\
\dot{\omega}_{y} = -2 \frac{\dot{r}_{L}}{r_{L}} \omega_{y} + \frac{a_{y}}{r_{L}} \omega_{x} \\
\dot{\omega}_{y} = -2 \frac{\dot{r}_{L}}{r_{L}} \omega_{y} + \frac{a_{y}}{r_{L}} \omega_{x}
\end{bmatrix},
\]

Equations (2) and (3) describe the leader-follower system kinematics.

3- Airborne Seeker Modeling

In this section, a mathematical model for seeker dynamics and kinematics is expressed. A typical gimbaled seeker contains a two-degree-of-freedom gimbal in which the external gimbal is fixed to the body as shown in Fig. 2.

To derive kinematic equations, rotation matrix \( ^{S}C \) is considered as follows:

\[
^{S}C = C_{2} \left( \psi_{SV}, \theta_{SV} \right) C_{1} \left( \psi_{SV}, \theta_{SV} \right)
\]

\[
= \begin{bmatrix}
\cos \psi_{SV} \cos \theta_{SV} & -\sin \psi_{SV} \cos \theta_{SV} & \sin \psi_{SV} \sin \theta_{SV} \\
\sin \psi_{SV} \cos \theta_{SV} & \cos \psi_{SV} \cos \theta_{SV} & \sin \psi_{SV} \sin \theta_{SV} \\
-\sin \theta_{SV} & 0 & \cos \theta_{SV}
\end{bmatrix}
\]

Defining \( \Omega_{O} = \begin{bmatrix} \omega_{x} & \omega_{y} & \omega_{z} \end{bmatrix}^{\top} \) as angular rate vectors. Now, using the following equation,
\[ \begin{align*}
\omega_{sy} &= \omega_{sy} \cos \psi_{SY} + \omega_{sy} \sin \psi_{SY}, \\
\omega_{sy} &= \omega_{sy} - \psi_{SY}, \\
\omega_{sy} &= -\omega_{sy} \sin \psi_{SY} + \omega_{sy} \cos \psi_{SY}.
\end{align*} \]

(4)

where \( I_s \) is the moment of inertia of the external gimbal. Consider the equation (4). Then, it holds that

\[ \begin{align*}
\left( I_s + I_s \right) \dot{\psi}_{SY} + c_s \dot{\psi}_{SY} &= -T_{sy} + \left( I_s + I_s \right) \omega_{sy} \\
&+ I_s \dot{\psi}_{SY} \left( \omega_{sy} \cos \theta_{SL} + \omega_{sy} \sin \theta_{SL} \right) \\
&+ I_s \dot{\psi}_{SY} \left( \omega_{sy} \sin \theta_{SL} + \omega_{sy} \cos \theta_{SL} \right).
\end{align*} \]

(7)

The above-mentioned equations describe the kinematics and the dynamics of the seeker mechanism. For the sensor measurement by considering \( (\psi_{SL}, \theta_{SL}) \) as the seeker beam angles with respect to the leader, we have

\[ \begin{align*}
L_{\Omega_{SL}} &= \frac{\omega_{SL}}{\omega_{sL}} = L_{\Omega_{SL}} + L_{\Omega_{SL}} = L_{\Omega_{SL}} + L_{\Omega_{SL}} + L_{\Omega_{SL}} \\
&+ L_{\Omega_{SL}} \left( \theta_{SL} - \theta_{SL} \right) \left( \theta_{SL} \right).
\end{align*} \]

Moreover, the internal gimbal torque can be expressed as it follows,

\[ \begin{align*}
T_s &= I_s \omega_{sL} = T_{sy} + T_{sy} + c_s \dot{\psi}_{SY}, \\
T_s &= T_{sy} + T_{sy} + T_{sy} - c_s \dot{\psi}_{SY}, \\
T_s &= T_{sy} + T_{sy} + T_{sy} - c_s \dot{\psi}_{SY}.
\end{align*} \]

(5)

Moreover, the internal gimbal torque can be expressed as it follows,

\[ \begin{align*}
I_s \dot{\psi}_{SY} + c_s \dot{\psi}_{SY} &= -T_{sy} + I_s \left[ -\omega_{sy} \sin \psi_{SY} + \omega_{sy} \cos \psi_{SY} \\
&- \theta_{SY} \left( \omega_{sy} \cos \theta_{SY} + \omega_{sy} \sin \theta_{SY} \right) \right].
\end{align*} \]

(6)

In a similar manner, for the external gimbal, we have

\[ \begin{align*}
T_y &= \left( I_s + I_s \right) \omega_{sy} + I_s \omega_{sy} \psi_{SY} \\
&= T_{sy} + T_{sy} + T_{sy} + c_s \dot{\psi}_{SY},
\end{align*} \]

\[ \begin{align*}
-\omega_{sy} \sin \psi_{SL} - \omega_{sy} \cos \psi_{SL}.
\end{align*} \]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Structure of two-degree-of-freedom gimbal.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Block diagram of seeker model in yaw channel}
\end{figure}
\[ w_{\text{yaw}} = -\dot{\theta}_{SV} \left( \omega_{xv} \cos \theta_{SV} + \omega_{zv} \sin \theta_{SV} \right) + \left( \omega_{xv} \cos \theta_{SV} - \omega_{zv} \sin \theta_{SV} \right), \]
\[ w_{\text{pitch}} = \frac{I_z}{I_z + I_y} \psi_{SV} \left( \omega_{xv} \cos \theta_{SV} + \omega_{zv} \sin \theta_{SV} \right) + \dot{\omega}_{sv}, \]
\[ d_{\text{yaw}} = -\omega_{xv} \sin \theta_{SV} + \omega_{zv} \cos \theta_{SV}, \]
\[ d_{\text{pitch}} = \omega_{xv} \sin \psi_{SV} \cos \theta_{SV} + \omega_{zv} \cos \psi_{SV} + \omega_{xv} \sin \psi_{SV} \sin \theta_{SV}. \]

In the above-mentioned models, a cascade control structure is used to control the gimbaled mechanism in the pitch and the yaw channels in which \( K_1, K_2, K_3 \) and \( K_4 \) are positive gains and \( \tau \) is a time constant. This control structure is generally useful when multiple measurements with only one control variable are required for a better response to disturbances in a system. Note that the inner loop should include the major disturbances and react faster than the outer loop in order to achieve a significantly improved system performance [31].

**4- FORMATION CONTROL STRUCTURE**

Considering the kinematic equations presented in section 2 and the seeker model in section 3. Now, we need a control structure to complete the leader-follower formation control model. The proposed control method is a cascade loop control as follows:
\[ a_{st} = k_{12} \left[ r_{td} - r_{sl} \right] - v_{st}, \]
\[ a_{gl} = k_{22} \left[ \psi_{LV} - \psi_{LVd} \right] - v_{gl}, \]
\[ a_{sl} = k_{32} \left[ \theta_{LVd} - \theta_{LV} \right] - v_{sl}, \]
where \( k_{11}, k_{12}, k_{21}, k_{22}, k_{31} \) and \( k_{32} \) are positive gains that must be properly selected, and \( (\psi_{LVd}, \theta_{LVd}) \) and \( r_{td} \) are the desired values of the relative angles (orientation) and the relative distance (range). Using this control structure, the relative orientation and the relative range can be regulated to maintain these quantities at the desired values. Thus, the desired formation is achieved.

Cascade control is designed to allow the outer loop controller to respond to the slow changes in the relative distances and the relative angles, while the inner loop controller controls disturbances that happen quickly in speed or angle rates loops.

**Fig. 4. Block diagram of seeker model in pitch channel**

\[ \begin{align*}
\omega_{sl} &\rightarrow \sin \psi_{sl} \\
\omega_{gl} \rightarrow 1/s &\rightarrow \theta_{sl} \\
\cos \psi_{sl} &\rightarrow 1/s \\
\theta_{st} &\rightarrow K_1 \\
\psi_{st} &\rightarrow K_2 \\
\psi_{sv} &\rightarrow 1/s + 1/\tau s + 1 \\
\psi_{sv} &\rightarrow 1/s \\
\cos \psi_{sv} &\rightarrow \omega_{sv} \\
\end{align*} \]

**Fig. 5. Block diagram of the leader-follower formation control structure**
Finally, the schematic diagram of the leader-follower formation control structure is obtained as depicted in Fig. 5. This model can be used to simulate the leader-follower formation in a proper simulation environment.

5- simulation results
In order to verify the effectiveness of the proposed control structure, a simulation is carried out for a V-shaped leader-follower formation of three UAVs. Simulation parameters are set as listed below,

\[ r_{L1d} = r_{L2d} = 250, \theta_{LV1d} = \theta_{LV2d} = -30^\circ; \]
\[ \psi_{LV1d} = -45^\circ, \psi_{LV2d} = 45^\circ; \]
\[ K_1 = K_2 = 1000, K_3 = K_4 = 200; \]
\[ \sigma_1 = 5, \sigma_2 = 0.5, c = 0.3; \]
\[ I_s = 0.02 \text{kg} \cdot \text{m}^2, I_r = 0.01 \text{kg} \cdot \text{m}^2; \]
\[ \tau = 0.1, c_s = c_r = 1; \]
\[ k_{11} = 0.5, k_{12} = 1; \]
\[ k_{21} = 50, k_{22} = 1; \]
\[ k_{31} = 50, k_{32} = 1. \]

The initial conditions of UAVs position, speed and angles are given in Table 1.

![Fig. 6. The leader-follower 3D trajectories.](image)

![Fig. 7. Relative angles and distances regulation.](image)

![Fig. 8. Follower1 seeker angle rate measurements.](image)

![Fig. 9. Follower1 seeker angle measurements.](image)

Table 1. Initial conditions of UAVs

<table>
<thead>
<tr>
<th>UAVs</th>
<th>Position ((x, y, z)) [m]</th>
<th>Speed ([m/s])</th>
<th>Angles ([\theta, \psi]) [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leader</td>
<td>((250, 150, -100))</td>
<td>50</td>
<td>((30, 20))</td>
</tr>
<tr>
<td>Follower1</td>
<td>((0, 0, 0))</td>
<td>20</td>
<td>(d_{\text{pitch}})</td>
</tr>
<tr>
<td>Follower2</td>
<td>((100, 20, 0))</td>
<td>20</td>
<td>((0, 0))</td>
</tr>
</tbody>
</table>


51-58, DOI: 10.22060/eej.2017.12493.5083

415-427.


[24] A. K. Bhattacharyya, S. Bhattacharya, T. Garai, S. Mukhopadhyay, Modeling of RF seeker dynamics and noise characteristics for estimator design in homing guidance applications, IEEE Region 10 Colloquium and


Please cite this article using:


DOI: 10.22060/eej.2017.12493.5083

57