Grid Impedance Estimation Using Several Short-Term Low Power Signal Injections

M. M. AlyanNezhadi1*, F. Zare2, H. Hassanpour1

1 Image Processing & Data Mining Lab, Shahrood University of Technology, Shahrood, Iran
2 Power Engineering Group, the University of Queensland, Queensland, Australia

ABSTRACT: In this paper, a signal processing method is proposed to estimate the low and high-frequency impedances of power systems using several short-term low power signal injections for a frequency range of 0-150 kHz. This frequency range is very important, and thus it is considered in the analysis of power quality issues of smart grids. The impedance estimation is used in many power system applications such as power quality analysis of smart grids and grid connected renewable energy systems. The proposed impedance estimation technique is based on applying a wideband voltage signal at a Point of Common Coupling (PCC) and then a division of the voltage to a generated current signal in a frequency range of 0-150 kHz. In a noisy system, the energy of the injected signal must be sufficient for an accurate approximation. This is the main issue in proposing a new method for the impedance estimation. In this paper, our simulation error is additive white Gaussian noise which is considered as a generic measurement noise. The proposed algorithm consists of three main parts: 1) Determining several injection signals with sufficient energy using the Genetic algorithm. At least one of the determined signals should have sufficient energy in some frequencies so that the union of these ranges is the universal set of estimation. 2) Injecting individuals of the signals to the grid separately, and estimating the impedance following Ohm’s law. The width of injection signals is calculated by the best chromosome in GA. 3) The fusion of estimated impedances. The simulation results show that the proposed method can properly estimate grid impedance in a wide frequency range up to 150 kHz.

1- Introduction

Electrical impedance is measured as the opposition of the circuit to a current when a voltage signal is applied. The grid impedance consists of resistive, inductive and capacitive couplings and depends on the type and configuration of grids such as feeders and transformers. The grid impedance varies in different frequencies. Impedance estimation is useful in many applications such as increasing the stability of the current controller by online parameter selection[1], designing stable inverter by considering changes in impedance over time[2], designing passive filter [3], designing active filter [4] and islanding detection[5, 6]. Impedance estimation techniques can be categorized into two main groups, namely passive [8, 7] and active [10 ,9 ,2] methods. Passive methods estimate the impedance of a grid without changing or injecting a signal to the grid, hence with no effect on the quality of the network. In these methods, the current and voltage of the grid are measured, and then the impedance is estimated, following the Ohm’s law in the frequency domain. The techniques in this category have two main problems: I) The grid impedance can be estimated only at the frequencies that are present in the sampling time. Since usually the normal grids have one main frequency (frequencies of 50 or 60 Hz) and its harmonics in the working time, if grid distortions are not enough, these methods are unable to estimate the impedance[11]. II) In the multiple junctions, measuring the current is impossible and these methods are unable to estimate the impedance. In the active methods, some distortions are applied to the grid by signal injection. The major advantage of active methods is their high accuracy in impedance estimation. Thus, active methods have attracted much attention. On the other hand, once the energy of the injecting signal is high, these methods reduce the grid power quality [11]. For the impedance estimation in noisy systems, the energy of the injected signal must be sufficient for an accurate approximation. The energy of rectangular injection pulse is dependent on peak amplitude and pulse width[12]. The peak amplitude of the injection signal is limited up to 2kV. In addition, increasing the pulse width reduces the bandwidth of the signal and in the frequencies of the low energy (usually in high frequencies), estimation is not accurate. The accurate impedance estimation can be done in different frequencies by pulses with different widths. In this paper, a novel method is proposed for the impedance estimation in frequencies up to 150 kHz by several low power signal injections. The proposed method determines several injection signals which at least one of the injected signals has sufficient energy in some frequencies that union of these ranges is the universal set of the estimation set (frequencies up to 150 kHz). Sufficient energy depends on the level of noise. There are various types of noise in power grids [13-18], but in this paper, we assume that the grid has only measurement noise. Measurement noise in power grid can be considered as an additive white Gaussian noise[12]. The paper is organized as follows. Section 2 introduces the impedance estimation by signal injection. In section 3, the accuracy of impedance estimation is investigated. In section
4 the effect of signal injection energy in the accuracy of impedance estimation is explained. The proposed method and simulation results are given in sections 5 and 6. Finally, section 7 is allocated to the conclusion.

2- Grid Impedance Estimation

According to the Ohm’s law, the impedance of every point in a circuit is equal to the division of voltage to a current signal in frequency domain. A voltage signal can be injected into a circuit for the impedance estimation.

In a passive grid, there is no power generator (at 50 or 60 Hz), and we can assume the whole impedance seen from a Point of Common Coupling (PCC) as an element with impedance \( Z \). For the passive grid impedance estimation, we can connect the impedance estimation block to the grid as shown in Fig. 1(top). The capacitor is used to decouple the system pulse generator from the grid while it can transfer the pulse signal at the PCC.

**Fig. 1: Schematics of impedance estimation for passive/active grid networks.**

According to the Ohm’s law, the impedance seen from the PCC is equal to:

\[
Z(\omega) = \frac{V_{PCC}(\omega)}{I_{PCC}(\omega)}
\]

In the above equation, \( V_{PCC}(\omega) \) and \( I_{PCC}(\omega) \) are respectively the voltage and current through PCC in frequency domain. For the impedance estimation at frequency \( \omega_i \), the injection signal must contain frequency component \( \omega_i \). In other words, in the above equation \( V_{PCC}(\omega_i) \neq 0 \). Therefore, the selection of injection signal is important.

We can assume the whole impedance seen from PCC in active grids, as an element with impedance \( Z \) and a Thevenin voltage source [19]. For an active grid impedance estimation, we can connect the impedance estimation block to the grid. The schematic diagram is shown in Fig. 1(down). According to Ohm’s law, the impedance of PCC is equal to:

\[
Z(\omega) = \frac{V_{PCC}(\omega) - V_{\text{Thevenin}}(\omega)}{I_{PCC}(\omega)},
\]

where \( V_{PCC}(\omega) \) and \( I_{PCC}(\omega) \) are the voltage and current through PCC in frequency domain, respectively. \( V_{\text{Thevenin}}(\omega) \) is the Thevenin voltage of grid in frequency domain. Measuring Thevenin voltage of the grid is difficult[20], therefore the above equation is not effective[21]. The grid parameters usually change in sparse steps, (e.g. Thevenin voltage of grid)[22]. In other words,

\[
V_{\text{Thevenin}}(t_1, \omega) = V_{\text{Thevenin}}(t_2, \omega), \text{if } t_2 \cong t_1
\]

We can estimate the grid impedance by combining (2) and (3). Active grid impedance estimation can be done by two separate injections (where \( t_2 \cong t_1 \)) using below equation:

\[
Z(\omega) = \frac{V_{PCC,n_1}(\omega) - V_{PCC,n_2}(\omega)}{I_{PCC,n_1}(\omega) - I_{PCC,n_2}(\omega)},
\]

where \( V_{PCC,n_i}(\omega) \) and \( I_{PCC,n_i}(\omega) \) are respectively the voltage and current through PCC for injections \( i = 1, 2 \) in frequency domain. However, for the impedance estimation at frequency \( \omega_1 \), \( V_{PCC,n_1}(\omega) \sim V_{PCC,n_2}(\omega) \).

2-1- Impedance Estimation with Additive Noise

Suppose that the measured \( V_{PCC}(t) \) and \( I_{PCC}(t) \) signals have the additive noise \( e(t) \) and \( r(t) \), respectively. According to Ohm’s law, the impedance in PCC is:

\[
Z(\omega) = \frac{V_{\text{REC}}(\omega) - V_{\text{REC}}(\omega)}{I_{\text{REC}}(\omega) + E(\omega)} = \frac{V_{\text{REC}}(\omega)}{I_{\text{REC}}(\omega) + E(\omega)}\]

Hence the Absolute Percentage Error (APE) would be:

\[
\text{APE}_n(\omega) = \frac{|Z(\omega) - Z_n(\omega)|}{Z(\omega)} \times 100
\]

In the above equations, \( Z(\omega) \) and \( \text{APE}_n(\omega) \) are the estimated impedance and APE in frequency \( \omega \). Estimation is more accurate when \( \text{APE}(\omega) \) is close to zero. Due to the noise, using Ohm’s law by injecting a single pulse has an error. When the injection signal is deterministic, we can apply signal to grid repeatedly and calculate the average of estimated impedance in each injection [23,9]. The arithmetic and logarithmic averages of impedances can be calculated by:

\[
Z(\omega) = \frac{1}{P} \sum_{k=1}^{P} V_{PCC,k}(\omega) I_{PCC,k}(\omega)
\]

\[
Z(\omega) = \left( \prod_{k=1}^{P} V_{PCC,k}(\omega) I_{PCC,k}(\omega) \right)^{\frac{1}{2}}
\]

where \( P \) is the number of injections[9].

3- Accuracy of Impedance Estimation

In this section, we prepare a simulation to show the effect of noise in the accuracy of impedance estimation. The simulation is done on the grid shown in Fig. 2 and its parameters are given in Table 1. This grid is a simple example of a typical topology (Fig. 3) of a low voltage distribution line[24]. Each building is modeled by a parallel RLC. Injection signals are ideal impulses (rectangular pulse with 1 μs width) with 1 V and 1 kV peak amplitudes. In the whole of this paper, sampling frequency \( f_s \) is 1 MHz. Estimated impedance and error of impedance are given in Fig. 4 and Table 2.

To evaluate the results of this paper, we used the Signal
to Noise Ratio (SNR) in decibel (dB) and Mean Absolute Percentage Error (MAPE) in percent.

\[
\text{SNR}_{\text{dB}} = 10 \log \left( \frac{\sum_{\omega} Z_{\text{ref}}(\omega)^2}{\sum_{\omega} Z_{\text{est}}(\omega)^2} \right)
\]

\[
\text{MAPE in dB} = \frac{1}{\omega_2 - \omega_1} \sum_{\omega_1}^{\omega_2} \text{MAPE}_{\omega}\n
\text{where } Z_{\text{ref}}(\omega) \text{ and } Z_{\text{est}}(\omega) \text{ are the reference impedance and the estimated impedance in frequency } \omega, \text{ respectively, and } [\omega_1, \omega_2] \text{ is the evaluation range that is assumed [0150kHz] in this paper.}

In this simulation, each of the two signals estimates active grid impedance with a high accuracy. In noiseless condition, the grid impedance can be estimated with a low voltage signal (rectangular pulse with 1 µs width). Although power grid impedance estimation with a low voltage signal is impossible, it can be used for the impedance estimation in the new manufacturing cable.

In noisy condition, when the grid impedance increases or the injection voltage decreases, the accuracy of impedance estimation is not changed. However, in noisy condition, it is not true. This is the main issue in proposing a new method for the impedance estimation in this paper. It is good to note that, the estimation error is caused by the capacitor in the impedance estimation block (Fig. 1).

### Table 1: Parameters of the grid shown in Fig. 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_r, R_g, R_C )</td>
<td>15Ω</td>
</tr>
<tr>
<td>( L_r, L_g, L_C )</td>
<td>1mH</td>
</tr>
<tr>
<td>( C_r, C_g, C_C )</td>
<td>1µF</td>
</tr>
<tr>
<td>RLCLR Power Line (left)</td>
<td>0.5Ω, 100µH, 10nF, 100µH, 0.5Ω</td>
</tr>
<tr>
<td>RLCLR Power Line (middle)</td>
<td>0.1Ω, 10µH, 10nF, 10µH, 0.1Ω</td>
</tr>
<tr>
<td>RL Power Line (right)</td>
<td>1Ω, 1mH</td>
</tr>
<tr>
<td>Grid Voltage Source</td>
<td>220V, 50Hz</td>
</tr>
</tbody>
</table>

### Table 2: Error of impedance estimation in noiseless condition

<table>
<thead>
<tr>
<th>Peak Amplitude</th>
<th>MAPE(%)</th>
<th>SNR(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1v</td>
<td>3.9795</td>
<td>14.0018</td>
</tr>
<tr>
<td>1kv</td>
<td>3.9795</td>
<td>14.0018</td>
</tr>
</tbody>
</table>

In noiseless condition, when the grid impedance increases or the injection voltage decreases, the accuracy of impedance estimation is not changed. However, in noisy condition, it is not true. This is the main issue in proposing a new method for the impedance estimation in this paper. It is good to note that, the estimation error is caused by the capacitor in the impedance estimation block (Fig. 1).

### 4- Energy of Injecting Signal in Impedance Estimation Accuracy

One of the important signals for the impedance estimation is a rectangular shaped pulse. The energy of a rectangular pulse depends on its peak amplitude and pulse width. In this section, two simulations are performed to investigate the effects of these two parameters, pulse amplitude, and width. In the next section, a novel method is presented following the obtained results. Simulations are done on a grid in which the schematic and parameters are shown in Fig. 2 and in Table 1, respectively.

In low voltage grid systems, the peak amplitude of the injection signal is controlled up to 2kV. The energy level of the pulse is not significant (short pulse and non-repetitive), and it cannot affect power quality of the grids. In this part, several simulations are carried out with an ideal impulse (pulse with only one sample in \( f_s = 1µs \)). In each simulation, the pulse peak amplitude is selected from 1V, 110V, 220V, 500V, 1kV and, 2kV. In simulations, it is supposed that we have white Gaussian additive measurement noise with SNR=50 dB in both current and voltage signal acquisitions as measurement noise.

### Table 3: Error of impedance estimation due to peak amplitude of injected signal

<table>
<thead>
<tr>
<th>Peak Amplitude of the injected signal(v)</th>
<th>MAPE(%)</th>
<th>SNR(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100% &lt;</td>
<td>-0.7919</td>
</tr>
<tr>
<td>110</td>
<td>16.4611%</td>
<td>7.8354</td>
</tr>
<tr>
<td>220</td>
<td>9.0740%</td>
<td>10.4220</td>
</tr>
<tr>
<td>500</td>
<td>5.6223%</td>
<td>12.5009</td>
</tr>
<tr>
<td>1000</td>
<td>4.5919%</td>
<td>13.3801</td>
</tr>
<tr>
<td>2000</td>
<td>4.1943%</td>
<td>13.7734</td>
</tr>
</tbody>
</table>

Fig. 2. Schematic diagram of the simulated grid.

Fig. 3. Typical topology of an electric power system.

Fig. 4: Estimated impedance for the grid shown in Fig. 2.
Table 3 shows the error of impedance estimation with the signal to noise ratio and relative error. It is shown that the error of estimation is decreased by increasing the peak amplitude of the injected signal.

In addition, to increase the energy of rectangular pulse, we can increase the pulse width. In this part, several simulations are done using rectangular pulse with 1kV peak amplitude. In each simulation, the pulse width is selected from 1 µs, 10 µs, 50 µs, 100 µs, 150 µs and 200 µs. Sampling frequency is 1MHz. It is supposed that we have white Gaussian additive measurement noise with SNR=30 dB in the current and voltage signal acquisitions.

The Ohm’s law in polar coordinates can be written as follows:

\[
Z(e^{j\omega}) = \frac{V(e^{j\omega})}{I(e^{j\omega})} = \frac{V_{re}(e^{j\omega})}{I_{re}(e^{j\omega})} + j\frac{V_{im}(e^{j\omega})}{I_{im}(e^{j\omega})} = \frac{\theta_k(e^{j\omega}) - \theta_l(e^{j\omega})}{\theta_k(e^{j\omega}) - \theta_l(e^{j\omega})}
\]

(11)

\[
\theta_k(e^{j\omega}) = \arctan \frac{\text{Im}(V_{re}(e^{j\omega}))}{\text{Re}(V_{re}(e^{j\omega}))}
\]

(12)

\[
\theta_l(e^{j\omega}) = \arctan \frac{\text{Im}(I_{re}(e^{j\omega}))}{\text{Re}(I_{re}(e^{j\omega}))}
\]

(13)

In Fig. 5, the magnitude of the injected signal is shown in the frequency domain. The accuracy of impedance estimation (in a specific frequency \(\omega\)) is directly related to the magnitude of current signal in frequency \(\omega\) (and according to Ohm’s law, the magnitude of the injection signal in frequency \(\omega\)). Fig. 6. (e) shows that the rectangular pulse with 150µs width cannot estimate the impedance in frequencies about 6.6 kHz. On the other hand, the rectangular pulse with 200µs width cannot estimate the impedance in frequencies about 5 kHz and 10 kHz (Fig. 6. (f)). In this paper, we select a few injection signals that can estimate the impedance up to 150 kHz.

With respect to the obtained results, the most important part of the impedance estimation is the optimum selection of the two (or more) pulses with different widths for injection. A rectangular signal with \(\alpha\) seconds pulse width has a local minima in frequencies, below:

\[
F_\alpha = \left\{ \frac{1}{\alpha}, \frac{2}{\alpha}, \frac{3}{\alpha}, \ldots \right\}
\]

(14)

5- The Proposed Method

In this section, a novel method is proposed by considering the effect of signal energy on the accuracy of impedance estimation. The proposed algorithm consists of three main parts:

1) Determining several injection signals with sufficient energy.

We determine \(K\) injection signals that at least one of the injected signals has sufficient energy in some frequencies that the union of these ranges is the universal set of estimation. Universal set involve frequencies up to 150 kHz in this research. We used Genetic algorithm (GA) [25, 26] to obtain suitable signals. GA parameters were listed in Table 4. Each chromosome has \(K\) gens, and they are integer numbers between 1 µs up to 600 µs as the width of injection signals. The most important parameter of GA is its fitness function. The goal of GA is the optimization of the fitness function. We need several signal injections with sufficient energy in each frequency. Therefore, fitness function of GA is:

\[
\hat{f} = \sum_{\omega=\omega_0}^{\omega_n} f(\omega)
\]

(15)

\[
f(\omega) = \begin{cases} T - \max_{1 \leq k \leq K} \left| \hat{V}_g(\omega) \right|, & \max_{1 \leq k \leq K} \left| \hat{V}_g(\omega) \right| < T \\ 0, & \text{otherwise} \end{cases}
\]

(16)

where \(f(\omega)\) is the partial fitness function. \(\hat{V}_g(\omega)\) is \(g\)th injection signal in chromosome. Also \(T\) is threshold value.

2) Injection of signals to grid separately and estimation of the impedance based on Ohm’s law. The width of injection signals is calculated by the best chromosome in GA.

3) Fusion of estimated impedances. Finally, the impedance of grid is estimated by:

\[
Z(\omega) = \frac{V_g(\omega)}{I_g(\omega)}, k = \arg \max_{1 \leq k \leq K} \left| \hat{V}_g(\omega) \right|
\]

(17)

The proposed method is a generic solution for both single phase and three phase grids. In the unbalanced system and asymmetrical load distribution, the impedance of phases can be different. It is the main reason that we measure the impedance of each phase separately.

6- Simulation Results

In this section, we apply the proposed method on grid shown in Fig. 2 and its parameters are given in Table 1. It is supposed that we have the white Gaussian additive measurement noise with SNR=30 dB in current and voltage signal acquisitions. The traditional results with different pulse widths are shown in Fig. 2 and its parameters are given in Table 1. It is supposed that we have the white Gaussian additive measurement noise with SNR=30 dB in current and voltage signal acquisitions.

In this section, we apply the proposed method on grid shown in Fig. 2 and its parameters are given in Table 1. It is supposed that we have the white Gaussian additive measurement noise with SNR=30 dB in current and voltage signal acquisitions.

![Fig. 5: Magnitude of the injected signal in frequency domain.](image-url)
Fig. 8 and Table 5 show that the proposed method is powerful in impedance estimation. The results of classic impedance estimation method with rectangular pulse are shown in Fig. 6. They are unable to estimate impedance in frequencies up to 150 kHz, in contrast to the proposed method which can estimate the grid impedance accurately.

Table 5: Error of impedance estimation

<table>
<thead>
<tr>
<th></th>
<th>MAPE(%)</th>
<th>SNR(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed method</td>
<td>10.2775</td>
<td>13.3292</td>
</tr>
<tr>
<td>Rectangular pulse with width=7 µs</td>
<td>15.0008</td>
<td>11.2806</td>
</tr>
<tr>
<td>Rectangular pulse with width=11 µs</td>
<td>12.6452</td>
<td>11.0499</td>
</tr>
<tr>
<td>Rectangular pulse with width=14 µs</td>
<td>14.4845</td>
<td>9.8403</td>
</tr>
<tr>
<td>Rectangular pulse with width=26 µs</td>
<td>12.2104</td>
<td>11.0582</td>
</tr>
</tbody>
</table>

Fig. 7 and Table 5 show that the proposed method is powerful in impedance estimation. The results of classic impedance estimation method with rectangular pulse are shown in Fig. 6. They are unable to estimate impedance in frequencies up to 150 kHz, in contrast to the proposed method which can estimate the grid impedance accurately.

Fig. 7. Proposed injection signals by GA.
7- Conclusion
In this paper, a new method for the grid impedance estimation using rectangular signal injection is proposed. In noiseless condition, the accuracy of estimation is independent of the energy of signal. However, in terms of noise, the energy of injected signal must be sufficient. To increase the energy of rectangular pulse, peak amplitude or pulse width can be increased. Although theoretically increasing the peak amplitude pulse has good results for low-voltage power network (220v), the peak amplitude is limited up to 24v. Also by increasing pulse width, signal bandwidth is decreased. In this paper, a new method is proposed based on the data fusion of several rectangular signal injections to deal with the noise. The proposed method focuses on this important issue that a single rectangular pulse cannot estimate all frequency ranges. By employing several rectangulars, a wider frequency band can be estimated. The simulations show that the proposed method is powerful in the grid impedance estimation.

References


