# Transmission switching cost modeling and determination candidate lines for participation in joint energy and reserve markets 

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#### Abstract

There has been a great resolution calling for smart grids in recent years. The introduction of new technologies that make the network flexible and controllable is the main part of smart grid concept and a key factor to its success. A transmission network as a part of system network has drawn less attention. Transmission switching as a transmission service can release us from load shedding and remove the constraints' violations. In addition to removing the congestion and decreasing the system cost, transmission switching may damage generating units due to transient states in the instance of reconfiguration. Therefore, in optimal transmission switching, the system security, practical limitations, and possible damages should be considered. Considering dynamic constraints in the proposed model avoids the occurrence of transient instability when opening the line in transmission switching action. A network reduction method based on modified Jacobean AC Newton-Raphson technique power flow considering a switchable line in technique is used for speeding up the calculation, efficiency, and simplicity. An approach for selecting the best lines in switching operation in the network is proposed. Based upon this approach, the lines with the highest effect on cost reduction are considered as the candidate switchable line. To investigate the efficiency of the proposed strategy, IEEE 57 bus test system is studied.


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## 1- Introduction

Transmission switching studies have been of the interest to the researchers since Eighties. In primary studies, the main focus was decreasing the load shedding. Next, the efficiency of the optimal transmission switching to solve other operation issues such as voltage drop, network loss, and system security was analyzed. After restructuring in power systems and the introduction of smart grid concept, transmission switching problem was redefined in the new environment [1-7].
Switching was used in [8-9] for congestion removal. A method based on DC Optimal Power Flow (OPF) was used in these references. In [10-11] the $\mathrm{N}-1$ security criteria have been added to the model presented in [8-9].
In [12-14], heuristic methods were used to restrict the search space and therefore, to reduce the execution time. In these papers, the lines with the highest impact on congested lines were categorized based on a sensitivity analysis. In most of the studies on transmission switching, only the DC network constraints have been considered and the AC constraints voltage security constraints and reactive load flow have been neglected. Since the switching may cause a violation of voltage constraints as well as other AC constraints, the methods presented based on DC load flow are less efficient [15].
On the other hand, the AC constraints cause nonlinearity in the problem. Therefore, with these constraints, the switching problem is a Mixed Integer Non-linear Programming (MINLP) problem. These problems take so long to be solved and it is possible that no solution is found. The global optimality is also not guaranteed. Problem decomposition has been proposed to solve the issue.
Reference [15] found the switching scheme and generation

[^0]schedule using a DC OPF at the first step. The results then were tested using an AC power flow and in the case of constraint violation, the switching scheme was banned and a new switching scheme was found. As the result of the separation of DC and AC sub-problems, this method also fails to guarantee the global optimum solution.
In [15-17] Benders decomposition was used. In the main problem, the generation schedule and switching scheme were found based on DC OPF. In sub-problems, AC constraints were checked and in the case of violation, the violated constraints were linked to the main problem. These newly introduced constraints change the results of the main problem to remove the constraint violations in sub-problem.
Security constraints were included in [17] through N-1 criteria. The security checking sub-problem was not linked to the main problem. This restricts the chance of a global optimum solution. The method presented to find the order of switching has also some problems that cause the solution to deviate from the optimum solution in some cases.
Based on the results of studies that some of them have been reported in this section, transmission switching can be useful for operation cost reduction. However, this switching may cause the system instability in some instances. This increases the network security cost. This paper analyses and models the transmission switching with dynamic constraints in a probabilistic cooptimization model for energy and spinning reserve scheduling. Using this model, the safe operation considering the dynamic switching constraints has been guaranteed.
Moreover, considering dynamic constraints to reduce the computation time, the network reduction technique is used. Using the network reduction technique, available transmission lines in network are divided into two sections: switchable and nonswitchable lines.

The remainder of this paper is organized as follows. Section 2 represents transmission switching cost modeling. Section 3 represents problem formulation, section 4 presents proposed algorithm for problem solving and section 5 provides some results for a case study, and the corresponding discussions. Finally, section 6 lists some relevant conclusions drawn in this paper.

## 2- Transmission switching cost modeling and determination candidate Lines

Before providing the problem model, an approach for determining the best lines for switching operation in the network is presented. Based upon the proposed approach, the lines with the highest effect on cost reduction are selected as the candidate lines.
Assume that each generator of the network has a marginal cost denoted by ICi; and each generator before the switching operation is producing PGi. The total generation cost of the system ( $\operatorname{Cost}(\mathrm{PG})$ ) could be determined using the marginal cost of power generation of each unit. Due to the change in the transmission network structure after switching, if the power productions of generating units change by $\Delta P G_{i}$, the total generation cost will change by $\Delta P G_{i}$. Therefore, the following equation could be written:
$\operatorname{Cost}(\mathrm{PG})+\Delta \operatorname{Cost}(\mathrm{PG})=\sum_{\mathrm{i}=1}^{\mathrm{NG}} \mathrm{IC}_{\mathrm{i}} \times\left(\mathrm{PG}_{\mathrm{i}}+\Delta \mathrm{PG}_{\mathrm{i}}\right)$
$\Rightarrow \Delta \operatorname{Cost}(\mathrm{PG})=\sum_{\mathrm{i}=1}^{\mathrm{NG}} \mathrm{IC}_{\mathrm{i}} \times\left(\Delta \mathrm{PG}_{\mathrm{i}}\right)$
Moreover, the change in power production of units by the change in the power transmitted across the transmission lines is as follows:
$\Delta \mathrm{PG}_{\mathrm{i}}=\Delta \mathrm{P}_{\mathrm{i}}+\Delta \mathrm{PD}_{\mathrm{i}} \Rightarrow \mathrm{PD}_{\mathrm{i}}=$ const
$\Rightarrow \Delta \mathrm{PG}_{\mathrm{i}}=\Delta \mathrm{P}_{\mathrm{i}} \Rightarrow \Delta \mathrm{P}_{\mathrm{i}}=\Delta \mathrm{PG}_{\mathrm{i}}=\sum_{\mathrm{l}=1}^{\mathrm{NL}} \frac{\partial \mathrm{P}_{\mathrm{i}}}{\partial \mathrm{P}_{\mathrm{L}}} \times \Delta \mathrm{P}_{\mathrm{L}}$
By substituting equation (2) in equation (1), we have:

$$
\begin{align*}
& \Delta \operatorname{Cost}(\mathrm{PG})=\sum_{\mathrm{i}=1}^{\mathrm{NG}} \mathrm{IC}_{\mathrm{i}} \times\left(\Delta \mathrm{PG}_{\mathrm{i}}\right)=  \tag{3}\\
& \sum_{\mathrm{i}=1}^{\mathrm{NG}} \mathrm{IC}_{\mathrm{i}} \times\left(\sum_{\mathrm{l}=1}^{\mathrm{NL}} \frac{\partial \mathrm{P}_{\mathrm{i}}}{\partial \mathrm{P}_{\mathrm{L}}} \times \Delta \mathrm{P}_{\mathrm{L}}\right)=\sum_{\mathrm{i}=1}^{\mathrm{NG}} \sum_{\mathrm{l}=1}^{\mathrm{NL}} \mathrm{IC}_{\mathrm{i}} \times \frac{\partial \mathrm{P}_{i}}{\partial \mathrm{P}_{\mathrm{L}}} \times \Delta \mathrm{P}_{\mathrm{L}}
\end{align*}
$$

Now, the change in the network cost regarding the change in line $L$ could be formulated as presented in the following:
$\Delta \operatorname{Cost}(\mathrm{PG})^{\mathrm{L}}=\sum_{\mathrm{i}=1}^{\mathrm{NG}} \mathrm{IC}_{\mathrm{i}} \times \frac{\partial \mathrm{P}_{\mathrm{i}}}{\partial \mathrm{P}_{\mathrm{L}}} \times \Delta \mathrm{P}_{\mathrm{L}}$
The above equation is normalized based on the transmitted power across the transmission line $\left(P f^{L}\right)$, in order to make it possible to compare the effect of different lines on cost
$\mathrm{ROC}^{\mathrm{L}}=\frac{\Delta \operatorname{Cost}(\mathrm{PG})^{\mathrm{L}}}{\mathrm{Pf}^{\mathrm{L}}}$
reduction.
Now, using the ROC ${ }^{\mathrm{L}}$ index, the transmission lines could be ranked based on their effect on cost reduction and as a result,
the candidate lines for switching could be readily determined. Now that the candidate lines are determined, the modeling of the switching cost of the transmission in the optimization of the joint energy and spinning reserve market is provided. By each switching, the insulating properties of switchgears are diminished, and after a specific number of switching the switchgear should be replaced. Therefore, the switching cost should be taken into consideration. In a short term planning period, the switching costs of the transmission lines determined by the number of switching and the cost of each switching should be considered in the optimization problem. Therefore, a cost function depicted using Fig. 1 is considered. In this study, it is assumed that a high number of switching decrease the lifetime of the switchgear to one-third of the nominal lifetime. In order to determine this figure, a $\$ 50000$ switchgear with the lifetime of 15 years is considered. Therefore, the cost of each switching is determined considering 6 number of switching per day, the lifetime of $5(15 / 3)$ years and $\$ 50000$ capital cost.

## 3- Problem Formulation

The cost of switching, including the cost of opening and closing operations and the cost related to the depreciation of the switch insulators is modeled based on the reinvestment costs for installing the new switches. In a short period, the cost of switching is considered to be proportional to the number of switching operations.
In [18-32] different MINLP problems were solved in different engineering branches, including the Unit Commitment (UC) problem. In this section, a probabilistic MINLP model is proposed for co-optimization of day-ahead energy and reserve markets. Switching capability is just considered for some network lines. The objective function of (6) is considered to minimize the energy, spinning reserve and switching costs in a 24 hour time period.
In (6), units' production costs, startup costs and reserve capacity costs are shown in the first term. The second term includes load shedding costs and the cost associate with reserve applications (change in production schedule). Switching cost is shown in the third term.
The network and units' constraints should be considered for both pre- and post-contingency states. The constraints can be divided into two groups, post-contingency, and precontingency constraints. A complete list of constraints is found in [31-33].
In order to model the dynamic constraints, synchronous machine classical model has been used. In this model,
$\dot{\gamma}(\mathrm{i})=\Omega_{\mathrm{b}} \times(\mathrm{w}(\mathrm{i})-1)$
$\mathrm{w}(\mathrm{i})=\frac{1}{\mathrm{M}(\mathrm{i})} \times\left(\mathrm{P}_{\mathrm{G}}(\mathrm{i})-\mathrm{P}_{\mathrm{e}}(\mathrm{i})\right)$
considering a constant field voltage the transient stability equations are as follows:
In (20), PG(i) is the input mechanical power of the unit which is considered to be constant. $\Omega_{\mathrm{b}}$ is the rate of frequency and $w(i)$

$$
\begin{gather*}
\operatorname{Pe}(\mathrm{i}, \mathrm{t})=\mathrm{E}(\mathrm{i}) \sum_{\mathrm{j}} \mathrm{E}(\mathrm{j})\left[\mathrm{B}_{\mathrm{ij}}(\mathrm{t}) \times \sin (\gamma(\mathrm{i}, \mathrm{t})-\gamma(\mathrm{j}, \mathrm{t}))\right.  \tag{8}\\
\\
\left.+\mathrm{G}_{\mathrm{ij}}(\mathrm{t}) \times \cos (\gamma(\mathrm{i}, \mathrm{t})-\gamma(\mathrm{j}, \mathrm{t}))\right]
\end{gather*}
$$




Fig. 1. Switching cost for switchable line
and $\gamma(i)$ are the rotor speed in per unit and rotor angle of unit $I$, respectively. The unit inertia constant is $M(i)$. The electrical power output of the unit can be written as equation (8).

$$
\begin{align*}
& \gamma^{\mathrm{n}+1}(\mathrm{i}, \mathrm{t})=\gamma^{\mathrm{n}}(\mathrm{i}, \mathrm{t})+\frac{\Delta \mathrm{t}}{2} \times\left(\mathrm{w}^{\mathrm{n}+1}(\mathrm{i}, \mathrm{t})+\mathrm{w}^{\mathrm{n}}(\mathrm{i}, \mathrm{t})-2\right) \\
& \mathrm{w}^{\mathrm{n}+1}(\mathrm{i}, \mathrm{t})=\mathrm{w}^{\mathrm{n}}(\mathrm{i}, \mathrm{t})+\frac{\Delta \mathrm{t}}{2 \mathrm{M}(\mathrm{i}, \mathrm{t})} \times\left(\mathrm{PG}(\mathrm{i}, \mathrm{t})-\mathrm{Pe}^{\mathrm{n}}(\mathrm{i}, \mathrm{t})\right.  \tag{9}\\
& \left.+\mathrm{PG}(\mathrm{i}, \mathrm{t})-\mathrm{Pe}^{\mathrm{n}+1}(\mathrm{i}, \mathrm{t})\right) \\
& \mathrm{n}=1, \ldots, \text { Nend } \quad \mathrm{i}=1, \ldots, N G
\end{align*}
$$

In (8), $E(i)$ is the electrical motive force of stator field. $B i j(t)$ and $G i j(t)$ are the element of row $i$ and column $j$ of reduced susceptance and reduced conductance matrices, respectively. The rotor angle and speed can be found by dividing the time span of the transient state into Need steps using (9). In these equations $\Delta t$ is the length of time steps and Nend is the number of time steps indexed by $n$. Considering the switchable lines $\mathrm{Bn}(\mathrm{t})$ and $\mathrm{Gn}(\mathrm{t})$ can be defined using (11).

$$
\begin{array}{r}
\operatorname{Pe}^{\mathrm{n}}(\mathrm{i}, \mathrm{t})=\mathrm{E}(\mathrm{i}) \sum_{\mathrm{j}} \mathrm{E}(\mathrm{j})\left[\mathrm{B}_{\mathrm{ij}}^{\mathrm{n}}(\mathrm{t}) \times \sin \left(\gamma^{\mathrm{n}}(\mathrm{i}, \mathrm{t})-\gamma^{\mathrm{n}}(\mathrm{j}, \mathrm{t})\right)\right.  \tag{10}\\
\left.+\mathrm{G}_{i \mathrm{ij}}^{\mathrm{n}}(\mathrm{t}) \times \cos \left(\gamma^{\mathrm{n}}(\mathrm{i}, \mathrm{t})-\gamma^{\mathrm{n}}(\mathrm{j}, \mathrm{t})\right)\right]
\end{array}
$$

## 4- Proposed algorithm for problem-solving

Switching problem in a joint energy and spinning reserve market are problems with nonlinear constraints and binary variables. This nonlinearity leads to great complexities andincreases the problem execution time and even in some cases causes convergence failure. In order to reduce the execution time, in this section, Benders Decomposition, and branch based network reduction methods are used.
As the result of transmission switching and consideration of the stability constraints regarding these switching, the execution time has increased.
Benders Decomposition technique is applied to solve the joint energy and spinning reserve market. This problem is decomposed into a master problem and three sub-problems. Units' status, generation schedule, and switching decision are determined in master problem considering the available data. The results of the master problem are used in sub-problems. The first and second sub-problems handle the network constraints. In this section, the branch based network reduction is used and the network lines are divided into two groups. The lines of the first group have the switching capability with undetermined switching state. The switching decisions for these lines are determined using the master problem and the first sub-problem. In the second sub-problem, the constraints regarding the lines without switching capability are determined considering the results of the master problem and the first sub-problem.
Using the results of the master problem, the sub-problems of switchable lines, fixed lines, and network stability are analyzed and finally the optimal results of units' status, generation schedule and switching decisions are determined. In this section, a branch-based network reduction technique based on the AC Newton-Raphson power flow algorithm is proposed. This method is proposed for the networks with undetermined switching decisions.
Firstly, the following linear problem is considered:
$\mathrm{AX}=\mathrm{b}$
Variable $X$ is divided into two groups of variables named $X 1$ and $X 2$. Equation (13) is rewritten as (14) based on this division.

$$
\left(\begin{array}{ll}
\mathrm{A}_{11} & \mathrm{~A}_{12}  \tag{14}\\
\mathrm{~A}_{21} & \mathrm{~A}_{22}
\end{array}\right) \times\binom{\mathrm{x}_{1}}{\mathrm{x}_{2}}=\binom{\mathrm{b}_{1}}{\mathrm{~b}_{2}}
$$

Variables $X 2$ can be found using the second line of (14):
$\mathrm{x}_{2}=\mathrm{A}_{22}^{-1} \times\left(\mathrm{b}_{2}-\mathrm{A}_{21} \times \mathrm{x}_{1}\right)$
In order to find $X 1$ one can write (16).
$\left(\mathrm{A}_{11}-\mathrm{A}_{12} \times \mathrm{A}_{22}^{-1} \times \mathrm{A}_{21}\right) \times \mathrm{x}_{1}=\mathrm{b}_{1}-\mathrm{A}_{12} \times \mathrm{A}_{22}^{-1} \times \mathrm{b}_{2}$
Equation (16) can be used for network reduction. The linear model of Newton-Raphson power flow method can be expressed as follows:

$$
\left(\begin{array}{cc}
\mathrm{J}_{1} & \mathrm{~J}_{2}  \tag{17}\\
\mathrm{~J}_{3} & \mathrm{~J}_{4}
\end{array}\right) \times\binom{\Delta \delta}{\Delta \mathrm{V}}=\binom{\mathrm{dP}_{0}}{\mathrm{dQ}_{0}}
$$

Considering the flow of lines with undetermined switching
decisions as the injected power, these equations can be written as follows:

$$
\left(\begin{array}{ll}
\mathrm{J}_{1} & \mathrm{~J}_{2}  \tag{18}\\
\mathrm{~J}_{3} & \mathrm{~J}_{4}
\end{array}\right)^{-\mathrm{M}} \times\binom{\Delta \delta}{\Delta \mathrm{V}}=\binom{\mathrm{dP}_{0}}{\mathrm{dQ}_{0}}-\mathrm{A}^{\mathrm{M}} \times\binom{\Delta \mathrm{P}_{\mathrm{L}}}{\Delta \mathrm{Q}_{\mathrm{L}}}^{\mathrm{M}}
$$

In fact, in (18) the power flows of the lines with undetermined switching decisions are added to the equations and the effects of these lines are modeled in this way. Now the network lines are divided into two different groups. The lines in the first group are the switchable lines (the lines with undetermined switching decision). The lines of the second group are the fixed lines (without switching capability). The set of switchable lines is called (M) and the set of other lines is denoted by ( -M ). Now, by omitting the reactance of switchable lines from $\mathrm{Y}^{\text {bus }}$ matrix, the Jacobin matrix of $\mathrm{J}^{-\mathrm{M}}$ is achieved. The matrix $\mathrm{A}^{\mathrm{M}}$ is the busbranch incidence matrix for the lines of the first group and the sending and receiving buses of these lines. $\Delta P_{L}$ and $\Delta Q_{L}$ are the incremental vectors of active and reactive power flow of the switchable lines. By categorizing the network lines into these two groups (18) is rewritten as follows:

$$
\begin{align*}
& \left(\begin{array}{llll}
\mathrm{J}_{11} & \mathrm{~J}_{12} & \mathrm{~J}_{13} & \mathrm{~J}_{14} \\
\mathrm{~J}_{21} & \mathrm{~J}_{22} & \mathrm{~J}_{23} & \mathrm{~J}_{24} \\
\mathrm{~J}_{31} & \mathrm{~J}_{32} & \mathrm{~J}_{33} & \mathrm{~J}_{34} \\
\mathrm{~J}_{41} & \mathrm{~J}_{42} & \mathrm{~J}_{43} & \mathrm{~J}_{44}
\end{array}\right)^{-\mathrm{M}} \times\left(\begin{array}{l}
\Delta \delta^{\mathrm{M}} \\
\Delta \mathrm{~V}^{\mathrm{M}} \\
\Delta \delta^{-\mathrm{M}} \\
\Delta \mathrm{~V}^{-\mathrm{M}}
\end{array}\right)  \tag{19}\\
& =\left(\begin{array}{l}
\mathrm{dP}_{0}{ }^{\mathrm{M}} \\
\mathrm{dQ}_{0}{ }^{\mathrm{M}} \\
\mathrm{dP}_{0}{ }^{-\mathrm{M}} \\
\mathrm{dQ}_{0}{ }^{-\mathrm{M}}
\end{array}\right)-\left(\begin{array}{llll}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}\right) \times\left(\begin{array}{l}
\Delta \mathrm{P}_{\mathrm{L}}{ }^{\mathrm{M}} \\
\Delta \mathrm{Q}_{\mathrm{L}}{ }^{\mathrm{M}} \\
0 \\
0
\end{array}\right)
\end{align*}
$$

The incremental vectors of the voltage magnitudes and angles of the buses directly connected to the switchable lines are given by $\Delta \mathrm{V}^{\mathrm{M}}$ and $\Delta \delta^{\mathrm{M}}$, respectively. For the other buses, these incremental vectors are denoted by $\Delta \mathrm{V}^{-\mathrm{M}}$ and $\Delta \delta^{-\mathrm{M}}$.
$\mathrm{dP}_{0}{ }^{\mathrm{M}}$ and $\mathrm{dP}_{0}{ }^{-\mathrm{M}}$ are the incremental vectors of active and reactive power injected into the buses connected to the switchable lines. For the other buses, these vectors are given bydP${ }_{0}^{-\mathrm{M}}$ and $\mathrm{dQ}_{0}{ }^{-\mathrm{M}}$. Finally, (19) can be divided as given in (20).
$\left(\begin{array}{lllll}\mathrm{J}_{11} & \mathrm{~J}_{12} & \mathrm{~J}_{13} & \mathrm{~J}_{14} \\ \mathrm{~J}_{21} \mathrm{~J}_{22} & \mathrm{~J}_{23} \mathrm{~J}_{24} \\ \mathrm{~J}_{31} & \mathrm{~J}_{32} & \mathrm{~J}_{33} & \mathrm{~J}_{34} \\ \mathrm{~J}_{41} & \mathrm{~J}_{42} & \mathrm{~J}_{43} & \mathrm{~J}_{44}\end{array}\right)^{-\mathrm{M}} \times\left(\begin{array}{l}\Delta \delta^{\mathrm{M}} \\ \Delta \mathrm{V}^{\mathrm{M}} \\ \Delta \delta^{-\mathrm{M}} \\ \Delta \mathrm{V}^{-\mathrm{M}}\end{array}\right)=\left(\begin{array}{l}\mathrm{dP}_{0}{ }^{\mathrm{M}} \\ \mathrm{dQ}_{0}{ }^{\mathrm{M}} \\ \mathrm{dP}_{0}{ }^{-\mathrm{M}} \\ \mathrm{dQ}_{0}{ }^{-\mathrm{M}}\end{array}\right)-\binom{\mathrm{A}^{\mathrm{M}}}{0} \times\left(\begin{array}{l}\Delta \mathrm{P}_{\mathrm{L}}{ }^{\mathrm{M}} \\ \Delta \mathrm{Q}_{\mathrm{L}}{ }^{\mathrm{M}} \\ 0 \\ 0\end{array}\right)$
To solve (20) for $\Delta \mathrm{V}^{\mathrm{M}}$ and $\Delta \delta^{\mathrm{M}}$ one may write:
$\left.\left\{\left(\begin{array}{ll}J_{11} & J_{12} \\ J_{21} & J_{22}\end{array}\right)^{-M}-\left(\left(\begin{array}{ll}J_{13} & J_{14} \\ J_{23} & J_{24}\end{array}\right)^{-M} \times\left(\begin{array}{lll}J_{33} & J_{34} \\ J_{43} & J_{44}\end{array}\right)^{-M}\right)^{-1} \times\left(\begin{array}{ll}J_{31} & J_{32} \\ J_{41} & J_{42}\end{array}\right)^{-M}\right)\right\} \times$
$\binom{\Delta \delta^{M}}{\Delta V^{M}}=\left(\binom{d P_{0}{ }^{M}}{\mathrm{dQ}_{0}{ }^{M}}-\mathrm{A}^{\mathrm{M}} \times\binom{\Delta \mathrm{D}_{\mathrm{L}}{ }^{M}}{\Delta \mathrm{Q}_{\mathrm{L}}{ }^{M}}\right)-\left(\begin{array}{ll}\mathrm{J}_{13} \mathrm{~J}_{14} \\ \mathrm{~J}_{23} & \mathrm{~J}_{24}\end{array}\right)^{-\mathrm{M}}$
$\times\left(\left(\begin{array}{ll}\mathrm{J}_{33} & \mathrm{~J}_{34} \\ \mathrm{~J}_{43} & \mathrm{~J}_{44}\end{array}\right)^{-\mathrm{M}}\right)^{-1} \times\binom{\mathrm{dP}_{0}{ }^{-\mathrm{M}}}{\mathrm{dQ}_{0}{ }^{-\mathrm{M}}}$

Solving (21), $\Delta \mathrm{V}^{\mathrm{M}}$ and $\Delta \delta^{\mathrm{M}}$ can be found using (22).

$$
\begin{align*}
& \binom{\Delta \delta^{-\mathrm{M}}}{\Delta \mathrm{~V}^{-\mathrm{M}}}=\left(\left(\begin{array}{ll}
\mathrm{J}_{33} & \mathrm{~J}_{34} \\
\mathrm{~J}_{43} & \mathrm{~J}_{44}
\end{array}\right)^{-\mathrm{M}}\right)^{-1}  \tag{22}\\
& \times\left(\binom{\mathrm{dP}_{0}^{-\mathrm{M}}}{\mathrm{dQ}_{0}{ }^{-\mathrm{M}}}-\left(\left(\begin{array}{ll}
\mathrm{J}_{31} & \mathrm{~J}_{32} \\
\mathrm{~J}_{41} & \mathrm{~J}_{42}
\end{array}\right)^{-\mathrm{M}} \times\binom{\Delta \delta^{\mathrm{M}}}{\Delta \mathrm{~V}^{\mathrm{M}}}\right)\right)
\end{align*}
$$

Fig. 2 shows the application of Benders Decomposition and branch based network reduction techniques to model the transmission network switching in joint energy and spinning reserve market optimization. In this figure, a master problem is shown along with three sub-problems. As can be seen at the first stem, the master problem is solved to find the units' status, generation schedule, committed reserves, load shedding and switching decisions. Then the network reduction technique is applied. The first sub-problem finds the switchable lines. In this sub-problem, the fixed lines are not considered. After finding the switchable lines, the second sub-problem analyzes the fixed lines' constraints. In the case of constraint violation in each sub-problem, a feedback (cut) is formed and introduced to the master problem. In order to make sure that this switching decision does not lead to instability, the third sub-problem analyzes the network stability considering this switching decision. In the case of violation in the third subproblem, a feedback is formed, including the proper switching decision which is introduced to the master problem.
According to branch based network reduction technique based on Newton-Raphson power flow, the objective function of the first sub-problem is the minimization of the incremental active and reactive power flow for convergence of the power flow for lines with switching ability considering optimization constraints and is expressed as follows:
Subproblem $1=\min w^{1 c}(\operatorname{PG}(i, t), \operatorname{SR}(i, t), L C(b, t), Z(1, t))$
$=\sum_{b=1}^{\mathrm{Nb}}\left(\mathrm{P}^{\mathrm{uplc}}+\mathrm{P}^{\mathrm{dnlc}}\right)+\left(\mathrm{Q}^{\text {uplc }}+\mathrm{Q}^{\text {drlc }}\right)$
Constraints of this sub-problem are as follows:
$\mathrm{Z}(1, \mathrm{t})=\mathrm{Z}(\hat{1}, \mathrm{t}) \leftrightarrow \eta(1, \mathrm{t})$
$\binom{F \times \Delta P^{c}(i, t)}{F \times \Delta Q^{c}(i, t)}^{M}+\binom{Z \times \Delta L C P^{c}(i, t)}{Z \times \Delta L C Q^{c}(i, t)}^{M}+\binom{P^{\text {uplc }}}{Q^{\text {uplc }}}^{M}-\binom{P^{\text {dnlc }}}{Q^{\text {dnlc }}}^{M}+$
$\left\{\left(\begin{array}{ll}J_{11} & J_{12} \\ J_{21} & J_{22}\end{array}\right)^{-M}-\left(\left(\begin{array}{ll}J_{13} & J_{14} \\ J_{23} & J_{24}\end{array}\right)^{-M} \times\left(\left(\begin{array}{ll}J_{33} & J_{34} \\ J_{43} & J_{44}\end{array}\right)^{-M}\right)^{-1} \times\left(\begin{array}{ll}J_{31} & J_{32} \\ J_{41} & J_{42}\end{array}\right)^{-M}\right)\right\}^{c} \times\binom{\Delta \delta^{M}}{\Delta V^{M}}^{c}=$

$\binom{\Delta \mathrm{P}_{\text {Line }}^{\text {swithable }}}{\Delta \mathrm{Q}_{\text {Line }}^{\text {switchable }}}^{\mathrm{c}}-\left(\begin{array}{ll}\frac{\partial \mathrm{P}_{\text {line }}^{\text {switchable }}}{\partial \delta} & \frac{\partial \mathrm{P}_{\text {line }}^{\text {switchable }}}{\partial \mathrm{V}} \\ \frac{\partial \mathrm{Q}_{\text {line }}^{\text {switchable }}}{\partial \delta} & \frac{\partial \mathrm{Q}_{\text {line }}^{\text {switchable }}}{\partial \mathrm{V}}\end{array}\right)^{\mathrm{M}} \times\binom{\Delta \delta^{\mathrm{M}}}{\Delta \mathrm{V}^{\mathrm{M}}}^{\mathrm{c}} \leq \mu^{\mathrm{c}}(1, \mathrm{t}) \times(1-\mathrm{Z}(1, \mathrm{t})) \times \mathrm{M}$

$\Delta \mathrm{P}_{\text {Line }}^{\min } \leq \Delta \mathrm{P}_{\text {Line }}^{\text {switchablec }} \leq \Delta \mathrm{P}_{\text {Line }}^{\min }$
$\Delta \mathrm{Q}_{\text {Line }}^{\min } \leq \Delta \mathrm{Q}_{\text {Line }}^{\text {switchablec }} \leq \Delta \mathrm{Q}_{\text {Line }}^{\min }$
$\Delta \mathrm{V}_{\text {min }}^{\mathrm{M}} \leq \Delta \mathrm{V}^{\mathrm{Mc}} \leq \Delta \mathrm{V}_{\max }^{\mathrm{M}}$
$\Delta \mathrm{P}^{\mathrm{cM}}(\mathrm{i}, \mathrm{t})=0$ in steadystate $\leftrightarrow \pi(\mathrm{i}, \mathrm{t})$
$\Delta \mathrm{LCP}^{\mathrm{cM}}(\mathrm{i}, \mathrm{t})=0 \quad$ in staedystate
$\Delta \mathrm{LCQ}^{\mathrm{cM}}(\mathrm{i}, \mathrm{t})=0 \quad$ in staedy state
$\Delta \mathrm{Q}^{\min } \leq \Delta \mathrm{Q}^{\mathrm{cM}}(\mathrm{i}, \mathrm{t}) \leq \Delta \mathrm{Q}^{\max } \leftrightarrow \overline{\psi(\mathrm{b}, \mathrm{t})}, \quad \psi(\mathrm{b}, \mathrm{t})$
$\Delta \mathrm{P}^{\min }(\mathrm{i}, \mathrm{t}) \leq \Delta \mathrm{P}^{\mathrm{cM}}(\mathrm{i}, \mathrm{t}) \leq \Delta \mathrm{P}^{\max }(\mathrm{i}, \mathrm{t}) \leftrightarrow \overline{\sigma(\mathrm{i}, \mathrm{t})}, \underline{\sigma(\mathrm{i}, \mathrm{t})}$
$\Delta \mathrm{P}^{\min }(\mathrm{i}, \mathrm{t})=\min \left\{\mathrm{SR}^{\mathrm{Dn}}, \operatorname{RRD}(\mathrm{i})\right\}$
$\Delta \mathrm{P}^{\max }(\mathrm{i}, \mathrm{t})=\min \left\{\mathrm{SR}^{\mathrm{Up}}, \operatorname{RRU}(\mathrm{i})\right\}$
$\Delta \operatorname{LCP}^{\min }(\mathrm{i}, \mathrm{t}) \leq \Delta \mathrm{LCP}^{\mathrm{cm}}(\mathrm{i}, \mathrm{t}) \leq \Delta \mathrm{LCP}{ }^{\max }(\mathrm{i}, \mathrm{t}) \leftrightarrow \overline{\omega(\mathrm{i}, \mathrm{t})}, \omega(\mathrm{i}, \mathrm{t})$
Linearizing the problem constraints, the linear programming is applied in an iterative framework to find the optimum value of bus perturbations using updated Jacobin matrix. If the objective function is lower than an acceptable tolerance, the results of the master problem are valid, otherwise, a violation feedback is linked to the master problem. This feedback is given in (39).


After obtaining results of the first sub-problem, the objective function of the second sub-problem that deals with the fixed lines is taken into account. This objective function is formulated as presented in the following:
Subproblem $2=\min ^{2 c}(\operatorname{PG}(i, t), \operatorname{SR}(i, t), L C(b, t))$
$=\sum_{\mathrm{b}=1}^{\mathrm{Nb}}\left(\mathrm{P}^{\mathrm{up} 2 \mathrm{c}}+\mathrm{P}^{\mathrm{dn} 2 \mathrm{c}}\right)+\left(\mathrm{Q}^{\mathrm{up} 2 \mathrm{c}}+\mathrm{Q}^{\mathrm{d} 2 \mathrm{c}}\right)$
Constraints of this sub-problem are as follows:

$\binom{\Delta \mathrm{P}_{\text {Line }}^{\text {nonswithable }}}{\Delta \mathrm{Q}_{\text {Line }}^{\text {nonsithable }}}^{c}-\left(\begin{array}{l}\frac{\partial \mathrm{P}_{\text {line }}^{\text {nonswithable }}}{\partial \delta} \\ \frac{\partial \mathrm{Q}_{\text {linser }}^{\text {nosichable }}}{\partial \delta}\end{array}\right.$
$\binom{\Delta \mathrm{P}_{\text {Line }}^{\text {nonsithable }}}{\Delta \mathrm{Q}_{\text {Line }}^{\text {nonswithable }}}^{\mathrm{c}}-\left(\begin{array}{l}\frac{\partial \mathrm{P}_{\text {line }}^{\text {nonswicthable }}}{\partial \delta} \\ \frac{\partial \mathrm{Q}_{\text {line }}^{\text {ninswitchable }}}{\partial \delta}\end{array}\right.$

$\Delta \mathrm{P}_{\text {Line }}^{\min } \leq \Delta \mathrm{P}_{\text {Line }}^{\text {nonswitchablec }} \leq \Delta \mathrm{P}_{\text {Line }}^{\min }$
$\Delta \mathrm{Q}_{\text {Line }}^{\min } \leq \Delta \mathrm{Q}_{\text {Line }}^{\text {nonswitchablec }} \leq \Delta \mathrm{Q}_{\text {Line }}^{\min }$


Fig. 2. Application of Benders Decomposition and branch based network reduction techniques to model the transmission network switching in joint energy and spinning reserve market optimization
$\Delta \mathrm{V}_{\min }^{-\mathrm{M}} \leq \Delta \mathrm{V}^{-\mathrm{Mc}} \leq \Delta \mathrm{V}_{\max }^{-\mathrm{M}}$
$\Delta \mathrm{P}^{\mathrm{c}-\mathrm{M}}(\mathrm{i}, \mathrm{t})=0$ in steadystate $\leftrightarrow \pi^{2}(\mathrm{i}, \mathrm{t})$
$\Delta \mathrm{LCP}^{\mathrm{c}-\mathrm{M}}(\mathrm{i}, \mathrm{t})=0$ in staedystate
$\Delta \mathrm{LCQ}^{\mathrm{c}-\mathrm{M}}(\mathrm{i}, \mathrm{t})=0$ in staedy state
$\Delta \mathrm{Q}^{\min } \leq \Delta \mathrm{Q}^{\mathrm{c}-\mathrm{M}}(\mathrm{i}, \mathrm{t}) \leq \Delta \mathrm{Q}^{\max } \leftrightarrow \overline{\psi^{2}(\mathrm{~b}, \mathrm{t})}, \underline{\psi^{2}(\mathrm{~b}, \mathrm{t})}$
$\Delta \mathrm{P}^{\min }(\mathrm{i}, \mathrm{t}) \leq \Delta \mathrm{P}^{\mathrm{c}-\mathrm{M}}(\mathrm{i}, \mathrm{t}) \leq \Delta \mathrm{P}^{\max }(\mathrm{i}, \mathrm{t}) \leftrightarrow \overline{\sigma^{2}(\mathrm{i}, \mathrm{t})}, \sigma^{2}(\mathrm{i}, \mathrm{t})$ (51)
$\Delta P^{\min }(i, t)=\min \left\{S^{D n}, R R D(i)\right\}$
$\Delta \mathrm{P}^{\max }(\mathrm{i}, \mathrm{t})=\min \left\{\mathrm{SR}^{\mathrm{Up}}, \operatorname{RRU}(\mathrm{i})\right\}$
$\Delta \mathrm{LCP}^{\min }(\mathrm{i}, \mathrm{t}) \leq \Delta \mathrm{LCP}{ }^{\mathrm{c}-\mathrm{m}}(\mathrm{i}, \mathrm{t}) \leq \Delta \mathrm{LCP}^{\text {max }}(\mathrm{i}, \mathrm{t}) \leftrightarrow \overline{\omega^{2}(\mathrm{i}, \mathrm{t})}, \omega^{2}(\mathrm{i}, \mathrm{t})$ (54)
Linearizing the problem constraints, the linear programing is applied in an iterative framework to find the optimum value of bus perturbations using updated Jacobin matrix. If the objective function is lower than an acceptable tolerance, the results of the master problem are valid, otherwise, if an unacceptable $\mathrm{w}^{2 \mathrm{c}}(\mathrm{PG}(\mathrm{i}, \mathrm{t}), \mathrm{SR}(\mathrm{i}, \mathrm{t}), \mathrm{LC}(\mathrm{b}, \mathrm{t}))$ is obtained, a cut feedback is linked to the master problem in the second iteration. This feedback is as follows.
$w^{2 c}(\operatorname{PG}(i, t), S R(i, t), L C(b, t))+$
$\left\{\begin{array}{l}\sum_{i=1}^{N G} \pi^{2}(i, t) \times(P(i, t) \times \mu(i, t)-P(\hat{i}, t) \times \mu(\hat{i}, t))+ \\ \sum_{i=1}^{N G}\left(\overline{\psi^{2}(i, t)} \times Q_{i}^{\max }-\underline{\psi^{2}(i, t)} \times Q_{i}^{\min }\right) \times(\mu(i, t)-\mu(\hat{i}, t))+ \\ \sum_{i=1}^{N G}\left(\overline{\sigma^{2}(i, t)} \times \Delta P^{\max }(i, t)-\underline{\sigma^{2}(i, t)} \times \Delta P^{\min }(i, t)\right)+ \\ \sum_{b=1}^{N B}\left(\overline{\omega^{2}(i, t)} \times \Delta L C P^{\max }(i, t)-\underline{\omega^{2}(i, t)} \times \Delta L C P^{\min }(i, t)\right)\end{array}\right\} \leq 0$

In the third step, the sub-problem deals with the network stability and stability assurance when the status of the lines with switches change are solved. The third sub-problem is as follows:
subproblem $3=\min w^{3 n}(\operatorname{PG}(i, t), Z(1, t))=\sum_{i=1}^{N G}\left(\varphi^{u p n}+\varphi^{d n n}\right)$
Constraints of this sub-problem are as follows:

$$
\begin{align*}
& \gamma^{n+1}(i, t)=\gamma^{n}(i, t)+\frac{\Delta t}{2} \times\left(w^{n+1}(i, t)+w^{n}(i, t)-2\right) \\
& \begin{array}{l}
w^{n+1}(i, t)=w^{n}(i, t)+\frac{\Delta t}{2 M(i, t)} \times\left(\operatorname{PG}(i, t)-P^{n}(i, t)+P G(i, t)-\operatorname{Pe}^{n+1}(i, t)\right)
\end{array}  \tag{57}\\
& \begin{array}{l}
n=1, \ldots, \text { Nend } \quad i=1, \ldots, N G \\
P^{n}(i, t)=E(i) \sum_{j} E(j)\left[B_{i j}^{n}(t) \times \sin \left(\gamma^{n}(i, t)-\gamma^{n}(j, t)\right)\right. \\
\left.\quad+G_{i j}^{n}(t) \times \cos \left(\gamma^{n}(i, t)-\gamma^{n}(j, t)\right)\right]
\end{array}
\end{align*}
$$

$$
Y_{\text {bus }}=G_{\text {bus }}+j B_{\text {bus }}=\left[\begin{array}{cccc}
y_{11} & y_{12} \times Z^{n}(1, t) & \ldots & y_{1 n} \\
y_{12} \times Z^{n}(1, t) & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots \\
y_{n 1} & \cdots & \cdots & y_{n n}
\end{array}\right]
$$

$$
\begin{equation*}
\mathrm{Z}^{1}(1, \mathrm{t})=\mathrm{Z}(1, \mathrm{t}-1) \tag{59}
\end{equation*}
$$

$$
\mathrm{Z}^{\mathrm{n}}(1, \mathrm{t})=\mathrm{Z}^{\mathrm{n}+1}(1, \mathrm{t})=\mathrm{Z}(1, \mathrm{t}) \quad \mathrm{n}=2, \ldots, \text { Nend }-1
$$

$$
\mathrm{G}_{\text {loadi }}=\mathrm{P}_{\text {loadi }} / \mathrm{V}_{\mathrm{i}}^{2}, \mathrm{~B}_{\text {loadi }}=\mathrm{Q}_{\text {loadi }} / \mathrm{V}_{\mathrm{i}}^{2} \mathrm{i}=1, \ldots \mathrm{Nb}
$$

$\gamma^{\mathrm{n}}(\mathrm{i}, \mathrm{t})-\frac{\sum_{\mathrm{k}=1}^{\mathrm{NG}} \mathrm{H}_{\mathrm{k}} \gamma^{\mathrm{n}}(\mathrm{k}, \mathrm{t})}{\sum_{\mathrm{k}=1}^{\mathrm{NG}} \mathrm{H}_{\mathrm{k}}}+\varphi^{\mathrm{upn}}-\varphi^{\mathrm{dnn}} \leq 100$
In case there is constraint violation in the third sub-problem, a proper constraint is generated and linked to the master problem. This new constraint prevents the selection of this switching status in the optimization process.

$$
\begin{equation*}
\sum_{l \in \text { opened lines in time } \mathrm{t}}(\mathrm{Z}(\mathrm{l}, \mathrm{t})) \geq 1 \tag{61}
\end{equation*}
$$

Table 1. UC status in energy market

| Bus Generator | Hours (1-24) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 | 1 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 1 |  | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 |  | 0 | 0 | 0 | 0 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 0 | 0 |
| 12 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 | 1 |
| Table 2. UC status in spinning reserve market |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Bus Generator | Hours (1-24) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 | 1 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |  |  | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  | 0 | 0 | 0 | 0 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 |  | 0 | 0 | 0 | 0 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 0 | 0 | 0 | 0 |
| Table 3. UC status in energy market, case (b) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Bus Generator | Hours (1-24) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 | 0 |
| 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 | 1 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 | 0 |  | 0 | 1 | 1 |  | 1 | 1 | 1 | 1 | 0 | 0 |
| 9 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 | 0 | 0 |  | 0 | 1 | 1 |  | 1 | 1 | 1 | 1 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |  | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 | 0 |
| Table 4. UC status in spinning reserve market, case (b) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Bus Generator |  |  |  |  |  |  |  |  |  |  |  |  |  | urs | (1-2 | -24) |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 | 0 |  | 0 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 | 0 |
| 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 |  | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 1 |  | 1 | 1 | 1 | 1 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 | 0 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 0 | 0 | 0 |
| 9 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 0 | 0 | 0 | 0 |

## 5- Numerical Results

IEEE 57 bus system is selected to analyze the results of considering the transmission switching in a probabilistic joint energy and reserve problem. This system contains 80 lines and 7 generation units. The total load is 1250.8 MW . The other data, including the units' offers and electrical characteristics, can be found in [34]. The single phase diagram is given in Fig. 3.
Candidate lines for switching operation are:
$\{1-2,12-16,3-4,13-15,44-48,48-49,22-38,8-9,7-29$ and 1-15\} To analyze the effect of switching operations, the following case studies will be analyzed:
a) Joint energy and reserve clearing without switching.
b) Joint energy and reserve clearing with switching neglecting the transient stability constraints.
c) Joint energy and reserve clearing with switching considering the transient stability constraints.
Tables 1 and 2 show the UC status in joint energy and reserve clearing without switching operation. Energy and reserve costs are $319156 \$ 59840$ \$, respectively for this case. The probabilistic security cost (cost of applied reserve) is 62544 $\$$ in this case. The average marginal prices are given in Fig. 4 for different hours.


Fig. 3. Single phase diagram of 57-bus system


Fig. 4. Average marginal price in energy and reserve markets


Fig. 5. Instability of the unit connected to bus 1 in switching of line 15-1

Table 5. Switching status, case (b)

| switchable Lines | Hours (1-24) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1-15 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 7-29 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 8-9 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 22-38 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 48-49 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 48-44 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 13-15 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 3-4 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 12-16 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1-2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 6. Operation costs, case (b)

| costs | Before switching action | After switching action without stability limitation |
| :---: | :---: | :---: |
| Energy market cost | $319156 \$$ | $293623 \$$ |
| Spinning reserve market cost | $59840 \$$ | $55651 \$$ |
| Security network cost | $62544 \$$ | $60042 \$$ |

5-1-Joint energy and reserve clearing with switching operations neglecting the transient stability constraints The dynamic constraints are neglected in this case. The UC status are given in Tables 3 and 4 for energy and reserve markets, respectively. Comparing to the previous study, the status of some units has not been changed. However, the production of the expensive units has been decreased. The switching order is given in Table 5. Table 6 compares the costs of case (a) to those associated with this case. As can be seen, the energy, reserve, and security cost are reduced by $8 \%, 7 \%$ and $4 \%$ of the values reported in case (a). For switching operations, the switching status of hour 0 is considered to be the same as the base case.

5-2- Joint energy and reserve clearing with switching considering the transient stability constraints
Tables 7 and 8 show the UC status in energy and spinning reserve markets with stability constraints. Considering these constraints, the UC status has been changed. In fact, the system costs are higher with stability constraints included. These costs are still lower than the base case without switching operations. Table 9 shows the system energy, reserve, and security costs. As can be seen, these costs decrease by the values of $6.5 \%, 5.5 \%$ and $3 \%$ comparing to the case without switching operations. Table 10 shows the switching status for this case study. As can be seen, due to stability constraints, opening or closing operations of some switches have been performed in different hours comparing to Table 5.

Table 7. UC status in energy market, case (c)


The results show that though the transmission switching is useful for system cost reduction, it may cause transient instability in some instances. Therefore, considering the stability constraints in switching is inevitable. With the stability constraints considered in optimization, the performance of the cost reduction is lower. As can be seen, the energy cost reduction considering and neglecting the stability constraints are 6.5 and 8 percent. However, these constraints reduce the risk of instability and reduce the instability costs.
In joint energy and spinning reserve market clearing with switching operations and neglecting the stability constraints (case(b)), the status of line 15-1 has changed from open to close. Here, this
switching is modeled in PSAT. Fig. 5 shows that this switching causes the instability of unit connected to bus 1 . This shows that it is necessary to consider the stability constraints in the model.
With stability constraints included, it can be seen that the switching of line $15-1$ is performed in hours 13 to 14 instead of 16 to 17 . This switching does not cause any instability. With stability constraints considered in the model, Fig. 6 shows the rotor angle of the unit connected to bus 1 during the solution process with the time step of 0.1 second. As can be seen, during this switching in time step of 6 the rotor angle reaches the maximum value but in the next step the rotor angle decreases and finally the stability is preserved.


Fig. 6. rotor angle of the unit connected to bus 1, with changes in the status of line 15-1


Fig. 7. Average marginal price in energy and reserve markets for cases (a), (b) and (c)


Fig. 8. Switching effects on voltage at bus 18


Fig. 9. Comparison of the operation cost regarding the change in the number of lines for both continuous and decomposed models


Fig. 10. Comparison of the operation cost regarding the change in the number of lines for both continuous and decomposed models with 6 lines with switching ability


Fig. 11. Comparison of the problem execution time in both cases

To switch lines 16-12, in case (b), the status of this line is changed from close in hour 16 to open in hour 17. The simulation of this switching in PSAT shows that this switching causes system instability. However, with stability constraints considered (case (c)) this switching takes place from hour 18 to hour 19. The simulations show that the system is stable in this case.
Fig. 7 shows the average marginal price in energy and reserve markets for cases (a), (b) and (c) in 24 hours. As can be seen, these prices are higher for the case, including the stability constraints. This increase in the prices with respect to the prices in case study (b) shows that though considering the switching operations in the joint energy and reserve market neglecting the stability constraints leads to the lower prices, this causes the instability in some system units and imposes high instability cost on the system.
In addition to the reduction of energy and spinning reserve, transmission switching can improve the voltage at different system buses. Fig. 8 shows the switching effects on the voltage at bus 18 . The switching has improved the voltage at bus 18 in most of the hours. Without switching, this voltage is out of allowable range in some instances.
Figs . 9 and 10 illustrate the decrease in the operation cost with respect to the change in the number of switching for different techniques. Fig. 11 shows the operation costs, including the costs of the energy market, spinning reserve market and network security, for both continuous and decomposed models using branch-based network reduction and Bender's decomposition techniques when there are 10 lines with switching ability. As can be seen, these costs have a negligible difference due to problem decomposition. However, the impact of this difference is insignificant and is not distinguishable in Fig. 9; in Fig. 10 for the case with 6 lines with switching ability, this difference is more highlighted. It can, therefore, be concluded that the proposed methods to decrease the execution time do not render sub-optimal solutions; the accuracy of the results is the same while the execution time has significantly decreased.
Table 11 shows the execution time of the problem of energy and reserve markets' settlement with respect to the switching operation, with and without security constraints. As shown in this table, with consideration of the transmission switching operation in the second case the problem execution time is about 7.2 times of the first case; moreover, taking into account the stability constraints makes the problem execution time about 8 times regarding case 1 ; this increase in the problem execution time is because of consideration of the stability constraints in the switching operation of the transmission lines.
The above results demonstrate that due to transmission
switching operation and consideration of the stability constraints regarding this operation, the problem execution time has increased. Therefore, in order to decrease the execution time, Benders decomposition and branch based network reduction approaches are employed, thus the switching operation can be Table 11. problem execution time for cases a-c

| Case | Execution time (Sec) |
| :---: | :---: |
| Case 1 | 557 |
| Case 2 | 4026 |
| Case 3 | 4608 |

done for large-scale power systems.
In the following, the impacts of employing Benders decomposition and branch based network reduction methods for the optimization problem of joint energy and spinning reserve market considering the transmission switching operation are investigated. Using the proposed methods, the execution time for computations of the 57-bus test system has reduced from 4608 seconds to 1223 seconds. Fig. 11 depicts the impact of employing the proposed methods on problem execution time considering the variation in the number of lines with switching ability.
As depicted in the above figure, when the number of lines with switching ability is considered to be two, the problem execution time without the use of time reduction methods is 1002 seconds; this time will reduce to 690 seconds employing the proposed time reduction methods. As can be seen in the Fig. 11, with an increase in the number of lines with switching ability, when the time reduction techniques are not employed, the execution time increases considerably. However, using time reduction techniques the execution time increases in a modest way. This significant increase in the execution time when time reduction methods are not used is due to a considerable number of continuous and binary variables that should be considered simultaneously; however, when time reduction methods are used these continuous and binary variables are divided between the master and subproblems and consequently the execution time decreases.

## 6- Conclusion

In this paper, the transmission switching has been modeled in joint energy and reserve market clearing. It was observed that through the proper switching operations not only the energy cost but also the spinning reserve and security costs have been reduced. In addition, it was shown in case studies that though the switching operation can reduce the operation cost, it may cause dynamic instability and therefore can impose the additional costs to the system. Therefore, it is necessary to develop appropriate switching strategy to reduce the chance
of instability and damage to the units. This paper proposed a methodology to develop such strategies in a day-ahead market. As the results show, the reduction in system cost is lower when the dynamic stability constraints are considered in the mode, but the system stability is preserved under this setup.
Considering a large number of continuous and binary variables and also the increase in the number of lines with switching ability, the problem execution time increases significantly. In order to overcome this burden, Benders decomposition and branch based network reduction methods were proposed. Employing these methods the problem execution time decreased significantly.

## References

[1] H. J. Koglin and H. Muller, "Overload reduction through corrective switching actions," IEE International Conference on Power System Monitoring and Control, London, pp. 159-164, 1980.
[2] R. V. Amerongen and H. V. Meeteren, "Security control by real power rescheduling network switching and Load shedding," CIGRE Report 32-02, France, 1980.
[3] V. H. Quintana and N. Miiller,"Overload and voltage control of power systems by line switching and generation rescheduling"Canadian Journal Elect. <br>\& Comp. Eng., vol. 15, no.4, pp. 167-173, 1990.
[4] G. Schnyder, H. Glavitsch, "Security enhancement using an optimal switching power flow," IEEE Trans. on Power Syst., vol. 5, no. 2, pp. 674-681, May 1990.
[5] H. Glavitsch, H. Kronig and R. Bacher, "Combined use of linear programming and load flow techniques in determining optimal switching sequences," Power Systems and Computation Conference (PSCC), Denmark, pp. 627-637, 1984.
[6] W. Shao and V. Vittal, "Corrective switching algorithm for relieving overloads and voltage violations," IEEE Trans. Power Syst., vol. 20, no. 4, pp. 1877-1885, Nov. 2005.
[7] M. Li, Q. Luh and L. Miche, "Corrective line switching with security constraints for the base and contingency cases," IEEE Trans. Power Syst., vol. 27, no. 1, pp. 125133, Feb. 2012.
[8] E. B. Fisher, R. P. O'Neill, and M. C. Ferris, "Optimal transmission switching," IEEE Trans. Power Syst., vol. 23, no. 3, pp. 1364-1355, Aug. 2008.
[9] K. W. Hedman, R. P. O’Neill, E. B. Fisher, and S. S. Oren, "Optimal transmission switching-sensitivity analysis and extensions," IEEE Trans. Power Syst., vol. 23, no. 3, pp. 1469-1479, Aug. 2008.
[10] K. W. Hedman, R. P. O’Neill, E. B. Fisher, and S. S. Oren, "Optimal transmission switching with contingency analysis," IEEE Trans. Power Syst., vol. 24, no. 3, pp. 1577-1586, Aug. 2009.
[11] K. W. Hedman, R. P. O’Neill, E. B. Fisher, and S. S. Oren, "Co-Optimization of Generation Unit Commitment and Transmission Switching With N-1 Reliability," IEEE Trans. Power Syst., vol. 25, no. 2, pp. 1052-1063, May. 2010.
[12] J. D. Fuller, R. Ramasra and A. Chai, "Fast heuristics for transmission line switching," IEEE Trans. Power Syst., vol. 27, no. 3, pp. 1377-1386, Aug. 2012.
[13] C. Liu, J. Wang and J. Ostrowski, "Heuristic prescreening
switchable branches in optimal transmission switching," IEEE Trans. Power Syst., vol. 27, no. 4, pp. 2289-2290, Nov. 2012.
[14] P. A. Ruiz, J. M. Foster, A. Rudkevich and M. C. Caramanis, "Tractable transmission topology control using sensitivity analysis," IEEE Trans. Power Syst., vol. 27, no. 3, pp. 1550-1559, Aug. 2012.
[15] M. Khanabadi and H. Ghasemi; "Transmission congestion management through optimal transmission switching"; Power and Energy Society General Meeting, May 2011.
[16] E. Nasrolahpour, H. Ghasemi and M. khanabadi, "Optimal transmission congestion management by means of substation reconfiguration," in Iranian Conference Electrical Engineering (ICEE), 2012.
[17] M. Khanabadi, H. Ghasemi and M. Doostizadeh, "Optimal transmission switching considering voltage security and N-1 contingency analysis," IEEE Trans. Power Syst., vol. 28, no. 1, pp. 542-550, Jan. 2013.
[18] Affonso CM. Silva LCPd. Potential benefits of implementing load management to improve power system security. Int J Electr Power Energy Syst 2010;32:704-10.
[19] Fu X, Wang X. Determination of load shedding to provide voltage stability. Int J Electr Power Energy Syst 2011;33:515-21.
[20] L. Wu, M. Shahidehpour, and T. Li, "Stochastic securityconstrained unit commitment," IEEE Trans. Power Syst., vol. 22, no. 2, pp. 800-811, May 2007.
[21] Y. Fu, M. Shahidehpour, and Z. Li, "Security-constrained unit commitment with AC constraints," IEEE Trans. Power Syst., vol. 20, no. 3, pp. 1538-1550, Aug. 2005.
[22] F. Bouffard, F. D. Galiana, and A. J. Conejo, "Marketclearing with stochastic security-Part I: Formulation," IEEE Trans. Power Syst., vol. 20, no. 4, pp. 1818-1826, Nov. 2005.
[23] C. Corchero, F. Javier Heredia, "Two-stage Stochastic Programming Model for the Thermal Optimal DayAhead Bid Problem with Physical Future Contracts ," IEEE Heredia, Corchero - DR 2008/11 - EIO, UPC. Copies of this report may be downloaded at http://wwweio.upc.es/~corchero/
[24] Q. P. Zheng, J. Wang Panos, M. Pardalos, Y. Guan, "Stochastic Security Constrained Unit Commitment Models and Solution Algorithm", (www2.cemr.wvu. edu/~zheng/paper/ebd4scuc.pdf)
[25] C. C. CARØE, R. SCHULTZ, "A Two-Stage Stochastic Program for Unit Commitment Under Uncertainty in a Hydro-Thermal Power System",Konrad-Zuse-Zentrum fur Informationstechnik Berlin Feb 1998.
[26] TAKRITI, S, B. KRASENBRINK, L. S.Y. WU, "Incorporating fuel constraints and electricity spot prices into the stochastic unit commitment problem", IBM Research Report RC 21066, Yorktown Heights, New York, 1997.
[27] K. Saenchai, L. Benedicenti and G. H. Huang, "A MixedInteger two-stage Interval Stochastic Programming Model for Regional Air Quality Management "Environmental Informatics Archives, vol 5, 2007, pp. 168-176.
[28] S. J. Stoyan, R. H. Kwon, "A two-stage stochastic mixed-integer programming approach to the index tracking problem " OptimEng Journal, pp. 247- 275,

DOI 10.1007/s11081-009-9095-1.
[29] M. Shahidehpour and Y. Fu, "Benders decomposition," IEEE Power and Energy Mag., vol. 3, no. 2, pp. 20-21, Mar. 2005.
[30] S. S. Nielsen, S. A. Zenios," Scalable parallel Benders decomposition for stochastic linear programming", Parallel Computing journal, vol. 23, 1997, pp. 1069-1088.
[31] R. Aazami, S. Daniar, Vali Talaeizadeh, "Physical and Stability Limitations of Transmission Switching in Electricity Market", Iran. J. Sci. Technol. Trans. Electr. Eng. (2016) 40:1-11; DOI 10.1007/s40998-016-0002-4.
[32] R. Aazami, Mahmoud Reza Haghifam, Farzad Soltanian, Masoud Moradkhani, "A comprehensive strategy for transmission switching action in simultaneous clearing of energy and spinning reserve markets", International Journal of Electrical Power and Energy Systems, Vol. 64; 2015; pp. 408-418.
[33] R. Aazami, Mahmoud Reza Haghifam, Kaveh Aflaki, "Stochastic energy and spinning reserve market with considering smart transmission switching action", ISGT conference 2012, Chicago, USA.
[34] N. Hazrati, M. Rashidi-Nejad, A.A. Gharaveisi, "Pricing and Allocation of Spinning Reserve and Energy in Restructured Power Systems via Memetic Algorithm" Large Engineering Systems Conference on Power Engineering, pp. 234-238, 2007.

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