



# A blade-pitch controller for a large wind turbine generator in the presence of time-varying delay and polytopic uncertainty

A. Rokhsari<sup>1\*</sup>, R. Abolpour<sup>2</sup>

<sup>1</sup>Department of Electrical Engineering, Amirkabir University of Technology, Tehran, Iran

<sup>2</sup>Department of Electrical Engineering, Shiraz University, Shiraz, Iran

**ABSTRACT:** A pitch-regulated wind turbine has an exclusive pitch activator for every single blade, and it is possible to send various pitch angle demands to each blade. They possess a controller to perform this task, and the problem of delay-dependent robust stability with polytopic-type uncertainties of these time-varying delay systems must be resolved. This paper deals with the dynamic output feedback robust stabilization of the large wind turbine generator in the presence of time-varying delay and polytopic uncertainty. Two critical assumptions are considered for the turbine model involving the model's parameters are uncertain, and the blade-pitch control input actuates by a time-varying unknown delay parameter. A set of intervals is considered for the uncertain and delay parameters, which are assumed to be given and known. Then, a novel algorithm is proposed to design a proper controller for this system based on the Lyapunov-Krasovskii functional approach. The proposed controller simultaneously compensates for the effects of both delay parameters and uncertain parameters. To validate the results in this study, two simulation examples are proposed considering different turbines to compare the performance of the designed controller with previously designed controllers. The results reveal the superiority of the proposed controller compared to the existing controller.

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## 1- Introduction

Wind power is one of the fastest-growing electrical industries and owns the rapid development progress among the other renewable power generation elements. Various types of this machine are developed and implemented in both theoretical and real applications. Due to the advantages of these machines, the stability and stabilizability analysis problems of wind turbines are frequently encountered in recent studies[1].

Large wind turbines attract more interest among researchers due to their massive structures and enormous blade spans[2][3]. With the enlargement of wind turbine generating capacity, it is vital to develop feasible, reliable, and robust control strategies in wind energy conversion systems to achieve maximum power performance. Tracking the full allowable power is one of the essential topics in this regard[4].

The control objectives considerably depend on the wind speed and its variations over the nominal values. The variable speed controller is exploited when it is below the nominal value[4]. The main control objective is to extract the maximum energy from wind power at its operating point. When the wind speed is above the nominal rate, the pitch controller is utilized in which the control objective is to maintain the output power constant. Since the variable-

speed wind turbine can produce higher energy and lower component mechanical stress, this type of turbine has become a field of increasing interest[5][6]. Some exciting methods for designing simple controllers have been developed during the past fifty years[7][8]. These methods provide acceptable performances. However, extending a general method for tuning the designing parameters is impossible[9].

Consideration of the delay in the model of wind machines is not usual in previous studies. However, the time-varying delay in this model is generally apparent due to the natural properties of its dynamics[10]. Additionally, hydraulic pressure-driven unit in large power wind generation system causes a time-varying delay in the wind generation system[11][12]. Unfortunately, no study basically and directly investigates the time-varying delay in the wind model equations. Indeed, the proposed controllers, regardless of the time-varying delay in the system's model[13].

Regarding this issue, scholars have conducted so much research. In 2020, Yuan et al., in a paper called "Multivariable robust blade pitch control design to reject periodic loads on wind turbines," developed a multivariable robust IPC framework to reject periodic loads. They modeled the inter-blade coupling to provide response characteristics in the frequency domain. In this research, Systematic case investigations demonstrate that, with the proposed IPC strategy, one can achieve significant periodic load mitigation as well as fatigue alleviation in speed-varying wind fields[14].

\*Corresponding author's email: ali96@aut.ac.ir



In the same year, Civelek published a paper on optimizing fuzzy logic (Takagi-Sugeno) blade pitch angle controller in wind turbines by genetic algorithm. They designed a fuzzy controller to control the wind turbine blades is optimized with a genetic algorithm that is improved. They also added some new features to enhance Advanced Intelligent Genetic Algorithm's (AIGA's) performance. One of these is the addition of acceptable error concept (AEC). Moreover, the maximum number of crossover points in AIGA is determined as a function of the length of the chromosome. This implementation improved the algorithm. Simulation results indicate that optimization improves the output power [15]. In 2021, Khaksari et al. presented an article entitled "an observer-based blade-pitch controller for wind using finite sliding mode in high wind speed." They deal with the difficulty of designing a robust dynamic output feedback controller for the wind machine. This research exploits the designing controller problem of wind turbines in the presence of time-varying delays and uncertain parameters. This article proposes a novel algorithm that designs a proper controller based on the idea of Finsler's Lemma[16]. Again in 2021, Zhang and Plestan published a paper called "Individual/collective blade pitch control of floating wind turbine based on adaptive second order sliding mode." They applied a new control strategy based on an adaptive second-order sliding mode approach to a floating wind turbine system in the above- rated region. The proposed controller is partially based on multi-blade coordinates transformation that combines collective and individual collective blade pitch control for power regulation, platform pitch motion reduction and reduction of blades fatigue load. They implemented the proposed controller on FAST simulator and demonstrates high level of performances[17]. In the following year, 2022, Elsis and his partners presented research entitled "Robust Design of ANFIS-Based Blade Pitch Controller for Wind Energy Conversion Systems Against Wind Speed Fluctuations." They proposed an adaptive neuro-fuzzy inference system (ANFIS) as an effective control technique for blade pitch control of the WECS instead of conventional controllers. Their research also suggested an effective strategy to prepare a sufficient dataset for training and testing the ANFIS controller. They developed a new optimization algorithm named the mayfly optimization algorithm (MOA) to find the optimal parameters of the proportional integral derivative (PID) controller to find the optimal dataset for training and testing of the ANFIS controller[18].

This paper proposes an algorithm to design an output feedback controller for wind turbine machines in the presence of time-varying delays and uncertain model parameters. Indeed, the basic turbine model obtained in the literature is given. Then, the algorithm's proper controller is designed and applied to the turbine model. Therefore, the main novelty of this paper is to develop an algorithm that is used to design the controller for the uncertain model with a time-varying delay. The proposed controller is presented based on the idea of the Lyapunov Krasovskii functional. This idea is frequently encountered in previous studies[19][20].

The controller should guarantee the model's robust stability for all possible values of model coefficients and consider the time-varying delay. For this purpose, a set of mathematical tools is used to design the controller, as mentioned in the following[21][22].

This paper is organized as follows: Section 2 proposes the transfer function model from pitch to tower fore- aft deflection, including a time-varying delay in the hydraulic pressure-driven unit of the wind generation system. In Section 3, the main idea of this paper is mentioned, which is an algorithm to design a proper controller for the uncertain model in the presence of a time-varying delay. The simulation examples are presented in section 4, consisting of two samples with different dimensions turbines. The simulation results reveal the superiority of the proposed controller. Finally, Section 5 concludes the paper.

## 2- System model

At the operating point, the linear model of blade-tower dynamics is considered to be the following equation[21]:

$$f(s) = G_p(s)\beta(s) \quad (1)$$

where  $f$  is the tower fore-aft modal deflection and  $\beta$  is the deviation of the pitch angle from its nominal value. Indeed, equation (1) states a causal linear equation exists for the tower deflection and the pitch angle deviation.

The transfer model  $G_p(s)$  is assumed to have the following form:

$$G_p(s) = \frac{a_2s^2 + a_1s + a_0}{s^4 + b_3s^3 + b_2s^2 + b_1s + b_0} \quad (2)$$

The transfer function coefficients  $\{a_i\}_{i=0}^2$  and  $\{b_i\}_{i=0}^3$  represents the time constant of the wind generation model.

The pitch-driven model is influenced by the hydraulic pressure-driven, which causes a time delay in the generation model [21]. It complexes the controller design and stability analysis of the model. According to this effect, the model equation (2) will be modified as follows:

$$G_p(s) = \frac{a_2s^2 + a_1s + a_0}{s^4 + b_3s^3 + b_2s^2 + b_1s + b_0} e^{-\tau s} \quad (3)$$

where  $\tau$  is the time delay parameter.

Usually, the time delay parameter is assumed to be constant and time-invariant in the previous studies [21]. However, this assumption can significantly affect the model's stability due to the time-varying nature of the delay parameter. Hence, the delay parameter is assumed to be time-varying as given below:

$$\forall t: \tau(t) \in [\tau_l, \tau_u] \quad (4)$$

where  $\tau_l$  and  $\tau_u$  are the values of the lower and upper bounds of the time-varying delay, respectively.

It is worth mentioning that the time derivation of the delay should be finite due to some physical constraints. Hence, the following assumption is considered:

$$\forall t: |\dot{\tau}(t)| \leq \tau_D \quad (5)$$

where  $\tau_D$  is the upper bound of the time derivation of the mentioned delay parameter.

According to the model deviations and time and physical dependency of the model, the transfer function coefficients (3) have fixed but unknown values[23]. Thus, the following equations describe the uncertainty bounds of these coefficients:

$$a_i \in [\underline{a}_i, \bar{a}_i] \text{ for } i = 0,1,2 \quad (6)$$

$$b_i \in [\underline{b}_i, \bar{b}_i] \text{ for } i = 0,1,2,3$$

The values of these coefficients depend on the physical specification of the wind machine and environment parameters. Hence, the lower and upper bounds in equations (6) have been determined to cover the reasonable deviations of the physical and environmental parameters[24].

Before presenting the main idea of this paper, the following lemmas are needed to present.

Lemma 1 [25]. Assume  $g(\theta): \mathbb{R} \rightarrow \mathbb{R}^n$  is a vector function,  $Q$  is a symmetric positive definite matrix and  $c_1$  and  $c_2$  are positive numbers ( $c_2 > c_1$ ). Then, the following inequality is satisfied:

$$\begin{aligned} & (c_2 - c_1) \int_{c_1}^{c_2} \dot{g}(\alpha)^T Q \dot{g}(\alpha) d\alpha \geq \\ & (g(c_2) - g(c_1))^T Q \int_0^h g(\alpha)^T Q g(\alpha) d\alpha \end{aligned} \quad (7)$$

Lemma 2 (Finsler's Lemma) [26]. Let  $x \in \mathbb{R}^n$ ,  $Q \in \mathbb{R}^{n \times n}$  be a symmetric positive definite matrix and  $B \in \mathbb{R}^{m \times n}$  such that  $\text{rank}(B) < n$ . Then, the following statements are equivalent:

- i)  $\forall x: Bx = 0 \rightarrow x^T Qx < 0$
- ii)  $\exists X \in \mathbb{R}^{n \times m}: Q + XB + B^T X^T < 0$
- iii)  $B^{\perp T} Q B^{\perp} < 0$

where  $B^{\perp}$  is the null matrix of the matrix  $B$ , which means  $B B^{\perp} = 0$ .

### 3- Controller design

This section presents the main idea of this paper, which is an algorithm to design an output feedback controller for the model (3).

This section consists of three subsections: extracting the closed-loop system, output feedback stabilizability analysis, and proposing the design algorithm. The controller structure is presented in the first subsection, and the closed-loop model is extracted based on the controller and open-loop model (3). The closed-loop model is an LTI system with polytopic uncertainty and a state delay. The second subsection investigates the stability analysis of the closed-loop system and proposes a theorem for this purpose based on the idea of the Lyapunov Krasovskii functional. Finally, the third subsection proposes the design algorithm for this paper.

#### 3- 1- Extracting the closed-loop system

Firstly, model (3) is transformed into the state-space model as follows:

$$\begin{aligned} \dot{x}(t) &= A(b)x(t) + B\beta(t - \tau(t)) \\ f(t) &= C(a)x(t) \end{aligned} \quad (8)$$

where  $x(t) \in \mathbb{R}^4$  is the state vector of this model. The system's matrices are obtained as given below:

$$\begin{aligned} A(b) &= \begin{bmatrix} -b_3 & -b_2 & -b_1 & -b_0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ C(a) &= [0 \quad a_2 \quad a_1 \quad a_0] \end{aligned} \quad (9)$$

The uncertain parameters  $a$  and  $b$  respectively belong to the following uncertain spaces  $\pi_a$  and  $\pi_b$  based on equation (6).

$$\begin{aligned} \pi_a &= \text{co} \{ [\underline{a}_0, \underline{a}_1, \underline{a}_2], [\underline{a}_0, \underline{a}_1, \bar{a}_2], \\ & [\underline{a}_0, \bar{a}_1, \underline{a}_2], \dots, [\bar{a}_0, \bar{a}_1, \bar{a}_2] \} \end{aligned} \quad (10)$$

$$\begin{aligned} \pi_b &= \text{co} \{ [\underline{b}_0, \underline{b}_1, \underline{b}_2, \underline{b}_3], [\underline{b}_0, \underline{b}_1, \underline{b}_2, \bar{b}_3], \\ & \dots, [\bar{b}_0, \bar{b}_1, \bar{b}_2, \bar{b}_3] \} \end{aligned} \quad (11)$$

Above, notation  $\text{co} \{ \cdot \}$  stands for the convex combination operator.

To unify the notations, assume  $\theta = [a^T \ b^T]^T$  contains the whole uncertain parameters of the model.  $\theta$  will belong to the uncertain space  $\pi$ , which is the Cartesian product of spaces  $\pi_a$  and  $\pi_b$ . In the rest of this paper, notations  $\theta$  and  $\pi$  are respectively noted by the uncertain vector and uncertain space [27][28].

Using this assumption, the model (8) can be rewritten as given below:

$$\dot{x}(t) = A(\theta)x(t) + B\beta(t - \tau(t)) \tag{12}$$

$$f(t) = C(\theta)x(t)$$

Now, consider the following controller:

$$\dot{x}_c(t) = A_c(\mu)x_c(t) + B_c(\mu)f(t) \tag{13}$$

$$\beta(t) = C_c(\mu)x_c(t)$$

where  $x_c \in R^{n_c}$  in which  $n_c$  is the controller order and  $A_c(\mu): R^{n_d} \rightarrow R^{n_c \times n_c}$ ,  $B_c(\mu): R^{n_d} \rightarrow R^{n_c}$  and  $C_c(\mu): R^{n_d} \rightarrow R^{n_c}$  are matrix functions of the design vector  $\mu \in R^{n_d}$ . System matrices consist of some known and unknown parameters where the unknown ones are denoted by  $\mu$ , including  $n_d$  entries. The design vector  $\mu$  should be precisely determined to establish the system's stability.

Remark 1. Controller (13) can include known and unknown parameters that imply its free structure. This free structure consequences the controller's flexibility that can be exploited in exceptional cases like PID.

Using (12) and (13), the closed-loop model can be obtained as follows:

$$\dot{z}(t) = \begin{bmatrix} A(\theta) & 0_{n,n_c} \\ B_c(\mu)C(\theta) & A_c(\mu) \end{bmatrix} z(t) + \begin{bmatrix} 0_{n,n} & BC_c(\mu) \\ 0_{n_c,n} & 0_{n_c,n_c} \end{bmatrix} z(t - \tau(t)) \tag{14}$$

where  $z(t) = [x^T(t) \ x_c^T(t)]^T$  is the state vector of the closed-loop model. For the convenience of the notations, consider the following matrix definitions:

$$\mathcal{A}(\theta, \mu) = \begin{bmatrix} A(\theta) & 0_{n,n_c} \\ B_c(\mu)C(\theta) & A_c(\mu) \end{bmatrix} \tag{15}$$

$$\mathcal{A}_d(\mu) = \begin{bmatrix} 0_{n,n} & BC_c(\mu) \\ 0_{n_c,n} & 0_{n_c,n_c} \end{bmatrix} \tag{16}$$

Using definitions (15-16), model (14) leads to (17):

$$\dot{z}(t) = \mathcal{A}(\theta, \mu)z(t) + \mathcal{A}_d(\mu)z(t - \tau(t)) \tag{17}$$

The stability of the above model is deeply investigated in the following subsection of this paper.

### 3- 2- Output feedback stabilizability analysis

Firstly, Definition 1 introduces the robust exponential stability of the closed-loop model (17).

Definition 1. The closed-loop model (17) is said to be robustly exponential stable if there exist continuous vector functions  $M(\theta)$  and  $c(\theta)$  that hold the following conditions:

$$\forall \theta \in \pi: c(\theta) > 0 \tag{18}$$

$$\forall \theta \in \pi, \forall t \in R_+: \|z(t)\| \leq M(\theta)e^{-c(\theta)t}\|z(0)\| \tag{19}$$

where  $z(t)$  is any possible trajectory of the closed-loop model (17).

Theorem 1 proposes a set of LMMI conditions to investigate the robust exponential stability for the closed-loop model (17) via Lyapunov Krasovskii functional approach in the sequel [29].

Theorem 1. Assume there exists a real vector  $\mu \in R^{n_d}$ , symmetric positive definite matrices  $P \in R^{m \times m}$ ,  $\{Q_i\}_{i=1}^3 \subset R^{n \times n}$  and  $\{R_i\}_{i=1}^3 \subset R^{n \times n}$  and also matrix  $Y(\theta): \pi \rightarrow R^{5n \times n}$  that satisfy the following conditions:

$$\forall \theta \in \pi:$$

$$\begin{bmatrix} \phi_1 & \frac{e^{-\gamma\tau_1}}{\tau_1}R_1 & (1 - \tau_D)\frac{e^{-\gamma\tau_u}}{\tau_u}R_2 & \frac{e^{-\gamma\tau_u}}{\tau_u}R_3 & P \\ * & \phi_2 & 0_{m,m} & 0_{m,m} & 0_{m,m} \\ * & * & \phi_3 & 0_{m,m} & 0_{m,m} \\ * & * & * & \phi_4 & 0_{m,m} \\ * & * & * & * & \phi_5 \end{bmatrix} + \tag{20}$$

$$\text{He}\{Y(\theta)[\mathcal{A}(\theta, \mu) \ 0_{m,m} \ \mathcal{A}_d(\mu) \ 0_{m,m} \ -I_{m,m}]\} \leq 0$$

where  $m = n + n_c$  and matrices  $\{\phi_i\}_{i=1}^5$  are defined with the following equations:

$$\phi_1 = \gamma P + Q_1 + Q_2 + Q_3 - \frac{e^{-\gamma\tau_1}}{\tau_1} R_1 - (1 - \dot{\tau}) \frac{e^{-\gamma\tau_u}}{\tau_u} R_2 - \frac{e^{-\gamma\tau_u}}{\tau_u} R_3$$

$$\phi_2 = -e^{-\gamma\tau_1} Q_1 - \frac{e^{-\gamma\tau_1}}{\tau_1} R_1 \quad (21)$$

$$\phi_3 = -e^{-\gamma\tau_u} (1 - \dot{\tau}) Q_2 - (1 - \dot{\tau}) \frac{e^{-\gamma\tau_u}}{\tau_u} R_2$$

$$\phi_4 = -e^{-\gamma\tau_u} Q_3 - \frac{e^{-\gamma\tau_u}}{\tau_u} R_3$$

$$\phi_5 = \tau_1 R_1 + \tau R_2 + \tau_u R_3$$

Then, model (17) will be robustly exponential stable.

Proof. Consider the following Lyapunov Krasovskii functional:

$$V_1 = z^T(t) P z(t) \quad (22)$$

$$V_{2,1} = \int_{t-\tau_1}^t e^{-\gamma(t-\alpha)} z^T(\alpha) Q_1 z(\alpha) d\alpha \quad (23)$$

$$V_{2,2} = \int_{t-\tau}^t e^{-\gamma(t-\alpha)} z^T(\alpha) Q_2 z(\alpha) d\alpha \quad (24)$$

$$V_{2,3} = \int_{t-\tau_u}^t e^{-\gamma(t-\alpha)} z^T(\alpha) Q_3 z(\alpha) d\alpha \quad (25)$$

$$V_2 = V_{2,1} + V_{2,2} + V_{2,3} \quad (26)$$

$$V_{3,1} = \int_{-\tau_1}^0 \int_{t+\alpha}^t e^{-\gamma(t-\beta)} \dot{z}^T(\beta) R_1 \dot{z}(\beta) d\beta d\alpha \quad (27)$$

$$V_{3,2} = \int_{-\tau}^0 \int_{t+\alpha}^t e^{-\gamma(t-\beta)} \dot{z}^T(\beta) R_2 \dot{z}(\beta) d\beta d\alpha \quad (28)$$

$$V_{3,3} = \int_{-\tau_u}^0 \int_{t+\alpha}^t e^{-\gamma(t-\beta)} \dot{z}^T(\beta) R_3 \dot{z}(\beta) d\beta d\alpha \quad (29)$$

$$V_3 = V_{3,1} + V_{3,2} + V_{3,3} \quad (30)$$

Then, the foremost Lyapunov Krasovskii function is considered to be as follows;

$$V = V_1 + V_2 + V_3. \quad (31)$$

The time-derivation of the first Lyapunov Krasovskii functional will be obtained as follows:

$$\dot{V}_1 = -\gamma V_1 + \gamma z^T(t) P z(t) + z^T(t) P \dot{z}(t) + \dot{z}^T(t) P z(t) \quad (32)$$

Using the Leibnitz formula [30], one obtains:

$$\dot{V}_{2,1} = -\gamma V_{2,1} + z^T(t) Q_1 z(t) - e^{-\gamma\tau_1} z^T(t - \tau_1) Q_1 z(t - \tau_1) \quad (33)$$

$$\dot{V}_{2,2} = -\gamma V_{2,2} + z^T(t) Q_2 z(t) - e^{-\gamma\tau} (1 - \dot{\tau}) z^T(t - \tau) Q_2 z(t - \tau) \quad (34)$$

$$\dot{V}_{2,3} = -\gamma V_{2,3} + z^T(t) Q_3 z(t) - e^{-\gamma\tau_u} z^T(t - \tau_u) Q_3 z(t - \tau_u) \quad (35)$$

Using equations (32-34), the following equation is obtained:

$$\begin{aligned} \dot{V}_2 \leq & -\gamma V_2 + z^T(t) (Q_1 + Q_2 + Q_3) z(t) - \\ & e^{-\gamma\tau_1} z^T(t - \tau_1) Q_1 z(t - \tau_1) - \\ & e^{-\gamma\tau_u} (1 - \dot{\tau}) z^T(t - \tau) Q_2 z(t - \tau) - \\ & e^{-\gamma\tau_u} z^T(t - \tau_u) Q_3 z(t - \tau_u) \end{aligned} \quad (36)$$

Also, using the Leibnitz formula, one obtains:

$$\dot{V}_{3,1} = -\gamma V_{3,1} + \tau_1 \dot{z}^T(t) R_1 \dot{z}(t) - \int_{t-\tau_1}^t e^{-\gamma(t-\alpha)} \dot{z}^T(\alpha) R_1 \dot{z}(\alpha) d\alpha \quad (37)$$

$$\dot{V}_{3,2} = -\gamma V_{3,2} + \tau \dot{z}^T(t) R_2 \dot{z}(t) - (1 - \tau_D) \int_{t-\tau}^t e^{-\gamma(t-\alpha)} \dot{z}^T(\alpha) R_2 \dot{z}(\alpha) d\alpha \quad (38)$$

$$\begin{aligned} \dot{V}_{3,3} &= -\gamma V_{3,3} + \tau_u \dot{z}^T(t) R_3 \dot{z}(t) - \\ &\int_{t-\tau_u}^t e^{-\gamma(t-\alpha)} \dot{z}^T(\alpha) R_3 \dot{z}(\alpha) d\alpha \end{aligned} \quad (39)$$

According to Lemma 1, the following equations will be obtained:

$$\begin{aligned} \dot{V}_{3,1} &\leq -\gamma V_{3,1} + \tau_l \dot{z}^T(t) R_1 \dot{z}(t) - \\ &\frac{e^{-\gamma\tau_l}}{\tau_l} (z(t) - z(t - \tau_l))^T R_1 (z(t) - z(t - \tau_l)) \end{aligned} \quad (40)$$

$$\begin{aligned} \dot{V}_{3,2} &\leq -\gamma V_{3,2} + \tau \dot{z}^T(t) R_2 \dot{z}(t) - \\ (1 - \tau_D) \frac{e^{-\gamma\tau_u}}{\tau_u} (z(t) - z(t - \tau))^T R_2 (z(t) - z(t - \tau)) \end{aligned} \quad (41)$$

$$\begin{aligned} \dot{V}_{3,3} &\leq -\gamma V_{3,3} + \tau_u \dot{z}^T(t) R_3 \dot{z}(t) - \\ &\frac{e^{-\gamma\tau_u}}{\tau_u} (z(t) - z(t - \tau_u))^T R_3 (z(t) - z(t - \tau_u)) \end{aligned} \quad (42)$$

Using equation (39-41), the following equation is obtained:

$$\begin{aligned} \dot{V}_3 &\leq -\gamma V_3 + \dot{z}^T(t) (\tau_l R_1 + \tau R_2 + \tau_u R_3) \dot{z}(t) - \\ &\frac{e^{-\gamma\tau_l}}{\tau_l} (z(t) - z(t - \tau_l))^T R_1 (z(t) - z(t - \tau_l)) - \\ (1 - \tau_D) \frac{e^{-\gamma\tau_u}}{\tau_u} (z(t) - z(t - \tau))^T R_2 (z(t) - z(t - \tau)) \\ &- \frac{e^{-\gamma\tau_u}}{\tau_u} (z(t) - z(t - \tau_u))^T R_3 (z(t) - z(t - \tau_u)) \end{aligned} \quad (43)$$

Using equations (31), (35), and (44), the time derivation of the primary Lyapunov function can be written as follows:

$$\begin{aligned} \dot{V} &\leq -\gamma V + \\ \zeta^T &\begin{bmatrix} \Phi_1 & \frac{e^{-\gamma\tau_l}}{\tau_l} R_1 & (1 - \tau_D) \frac{e^{-\gamma\tau_u}}{\tau_u} R_2 & \frac{e^{-\gamma\tau_u}}{\tau_u} R_3 & P \\ * & \Phi_2 & 0_{n \times n} & 0_{n \times n} & 0_{n \times n} \\ * & * & \Phi_3 & 0_{n \times n} & 0_{n \times n} \\ * & * & * & \Phi_4 & 0_{n \times n} \\ * & * & * & * & \Phi_5 \end{bmatrix} \zeta \end{aligned} \quad (44)$$

where

$$\zeta(t) = [z^T(t) \quad z^T(t - \tau_l) \quad z^T(t - \tau) \quad z^T(t - \tau_u) \quad \dot{z}^T(t)]^T$$

Using condition (43) and based on Finsler's Lemma, one has:

$$\dot{V} \leq -\gamma V \quad (45)$$

The above equation establishes the robust, exponentially stable closed-loop model (17).

It must be noted that the conditions of Theorem 1 are not LMI for two crucial reasons. First, the matrix function  $Y(\theta): \pi \rightarrow \mathbb{R}^{5n \times n}$  is not explicitly in this theorem and has a general form. Second, there exist some coupling terms between the Lyapunov and design parameters in the  $\text{He}\{Y(\theta)[\mathcal{A}(\theta, \mu) 0_{m,m} \quad \mathcal{A}_d(\mu) 0_{m,m} - I_{m,m}]\}$ . Theorem 2 is proposed to cope with these issues, and an ILMI-based methodology is exploited in the next section.

**Theorem 2.** Let vector  $\mu \in \mathbb{R}^{n_d}$ , symmetric positive definite matrices  $P \in \mathbb{R}^{m \times m}$ ,  $\{Q_i\}_{i=1}^3 \in \mathbb{R}^{n \times n}$ ,  $\{R_i\}_{i=1}^3 \in \mathbb{R}^{n \times n}$  and matrix  $\{Y_{\theta'}\}_{\theta' \in \partial_c(\pi)} \in \mathbb{R}^{5n \times n}$  that satisfy the following conditions:

$$\forall \theta', \theta'' \in \partial_c(\pi):$$

$$\begin{bmatrix} \Phi_1 & \frac{e^{-\gamma\tau_l}}{\tau_l} R_1 & (1 - \tau_D) \frac{e^{-\gamma\tau_u}}{\tau_u} R_2 & \frac{e^{-\gamma\tau_u}}{\tau_u} R_3 & P \\ * & \Phi_2 & 0_{m,m} & 0_{m,m} & 0_{m,m} \\ * & * & \Phi_3 & 0_{m,m} & 0_{m,m} \\ * & * & * & \Phi_4 & 0_{m,m} \\ * & * & * & * & \Phi_5 \end{bmatrix} + \quad (46)$$

$$\text{He}\{Y_{\theta'}[\mathcal{A}(\theta'', \mu) 0_{m,m} \quad \mathcal{A}_d(\mu) 0_{m,m} - I_{m,m}]\} \leq 0$$

where  $\partial_c(\delta)$  is the set of corner points of  $\delta$  and  $\{\phi_i\}_{i=1}^5$  are given in the statement of Theorem 1. Then, model (17) will be robustly exponential stable.

**Proof.** Conditions (45) imply (20) through considering  $\mathfrak{Y}(\cdot)$  as the convex combination of the  $\{Y_{\theta'}\}_{\theta' \in \partial_c(\delta)}$  in Theorem 1. Hence, robust exponential stability is achieved according to Theorem 1.

### 3- 3- Proposing the Design Algorithm

As stated earlier, the conditions of Theorem 1 are not LMI. Thus, these conditions cannot be directly solved by LMI solvers. To cope with this issue, an algorithm is developed that exploits an ILMI methodology. Briefly, the steps of the proposed algorithm are presented in the following:

#### Algorithm:

Set  $k = 0$ .

Consider  $\mu^{(0)}$  is a random vector.

Solve the optimization problem P1 by considering  $A_c = A_c(\mu^{(k)})$ ,  $B_c = B_c(\mu^{(k)})$  and  $C_c = C_c(\mu^{(k)})$  to obtain  $P^{(k)}$ ,  $\{Q_i^{(k)}\}_{i=1}^3$ ,  $\{R_i^{(k)}\}_{i=1}^3$  and  $\{Y_{\theta'}^{(k)}\}_{\theta' \in \partial_c(\pi)}$  and  $h^{(k)}$ .

If  $h^{(k)} \leq 0$  returns  $A_c(\mu^{(k)})$ ,  $B_c(\mu^{(k)})$  and  $C_c(\mu^{(k)})$  as the solutions of the algorithm.

Solve the optimization problem P1 by considering  $P = P^{(k)}$ ,  $\{Q_i\}_{i=1}^3 = \{Q_i^{(k)}\}_{i=1}^3$ ,  $\{R_i\}_{i=1}^3 = \{R_i^{(k)}\}_{i=1}^3$  and  $\{Y_{\theta'}\}_{\theta' \in \partial_c(\pi)} = \{Y_{\theta'}^{(k)}\}_{\theta' \in \partial_c(\pi)}$  to obtain  $A_c^{(k+1)}$ ,  $B_c^{(k+1)}$  and  $C_c^{(k+1)}$  and  $h^{(k+1)}$ .

**Table 1. Configuration parameters of the first wind turbine**

parameter	value
rotor diameter	70m
tower height	90m
rated power	1.5MW
wind speed	15m/s
pitch angle	0°

If  $\left| h^{(k+1)} - h^{(k)} \right| \leq \epsilon$ , it terminates and returns null.

Set  $k = k + 1$  and go to step 3.

In the third step of the proposed algorithm, the optimization problem P1 is mentioned, which is entirely described in the following:

**P1:**

$$\min_{h, P, \{Q_i\}_{i=1}^3, \{R_i\}_{i=1}^3, Y, \mu} \mathbf{h}$$

$$P > 0$$

$$R_i > 0, Q_i > 0 \text{ for } i = 1, 2, 3$$

$$\forall \theta', \theta'' \in \partial_c(\pi):$$

$$\begin{bmatrix} \phi_1 & \frac{e^{-\gamma\tau_1}}{\tau_1} R_1 & (1 - \tau_D) \frac{e^{-\gamma\tau_u}}{\tau_u} R_2 & \frac{e^{-\gamma\tau_u}}{\tau_u} R_3 & P \\ * & \phi_2 & 0_{m,m} & 0_{m,m} & 0_{m,m} \\ * & * & \phi_3 & 0_{m,m} & 0_{m,m} \\ * & * & * & \phi_4 & 0_{m,m} \\ * & * & * & * & \phi_5 \end{bmatrix} +$$

$$\text{He}\{Y_{\theta'}, [\mathcal{A}(\theta''), \mu] \ 0_{m,m} \ \mathcal{A}_d(C_c) \ 0_{m,m} \ -I_{m,m}\} \leq \mathbf{h} I_{5n \times 5n}$$

$$\phi_1 = \gamma P + Q_1 + Q_2 + Q_3 - \frac{e^{-\gamma\tau_1}}{\tau_1} R_1$$

$$-(1 - \tau_D) \frac{e^{-\gamma\tau_u}}{\tau_u} R_2 - \frac{e^{-\gamma\tau_u}}{\tau_u} R_3$$

$$\phi_2 = -e^{-\gamma\tau_1} Q_1 - \frac{e^{-\gamma\tau_1}}{\tau_1} R_1$$

$$\phi_3 = -e^{-\gamma\tau_u} (1 - \tau_D) Q_2 - (1 - \tau_D) \frac{e^{-\gamma\tau_u}}{\tau_u} R_2$$

$$\phi_4 = -e^{-\gamma\tau_u} Q_3 - \frac{e^{-\gamma\tau_u}}{\tau_u} R_3$$

$$\phi_5 = \tau_1 R_1 + \tau_u R_2 + \tau_u R_3$$

$$G_p(s) = \frac{2.426s^2 - 4.6345s - 147.3}{s^4 + 4.857s^3 + 126.2s^2 + 266.4s + 3659} e^{-0.25s}$$

Remark 2. The parameter  $\epsilon$  is a threshold value considered to guarantee convergence and stop the algorithm if there is no feasible solution.

Therefore, the proposed algorithm will design a proper output feedback controller for model (14) under conditions (4-6).

#### 4- Simulation

Two examples are considered in this section to convey the efficiency and performance of the design algorithm compared to the other previous methods. It is worth mentioning that the considered model assumptions of the model (14) are not directly used in the earlier methods. Hence, the other methods are designed to compare the results by removing some assumptions. However, the obtained controllers are applied to the original model.

The simulation results of this section reveal the superiority of the proposed algorithm compared to the previous methods.

**Example 1.** This model is given in the paper [12]. The following table presents some physical and environmental parameters of this model (Table 1):

Using the parameter values of Table 1, the transfer function of the wind machine is obtained as follows [12]:

$$G_p(s) = \frac{2.426s^2 - 4.6345s - 147.3}{s^4 + 4.857s^3 + 126.2s^2 + 266.4s + 3659} e^{-0.25s} \quad (47)$$

It has been supposed that the coefficients of the above model can have up to 10% error from the nominal values in equation (46). Also, the delay-dependent parameters are assumed to be  $\tau_1 = 0.2$ ,  $\tau_u = 0.3$  and  $\tau_D = 0.1$ .

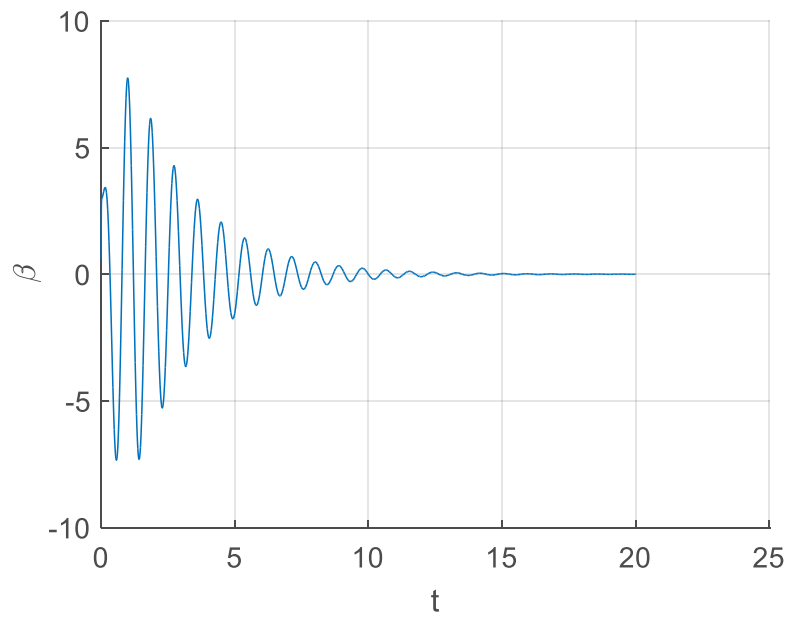
Using the proposed algorithm of this paper, the proper controller is obtained, which is presented in the following:

$$A_c = \begin{bmatrix} 24.0363 & -121.8684 \\ 883.4176 & -444.4847 \end{bmatrix},$$

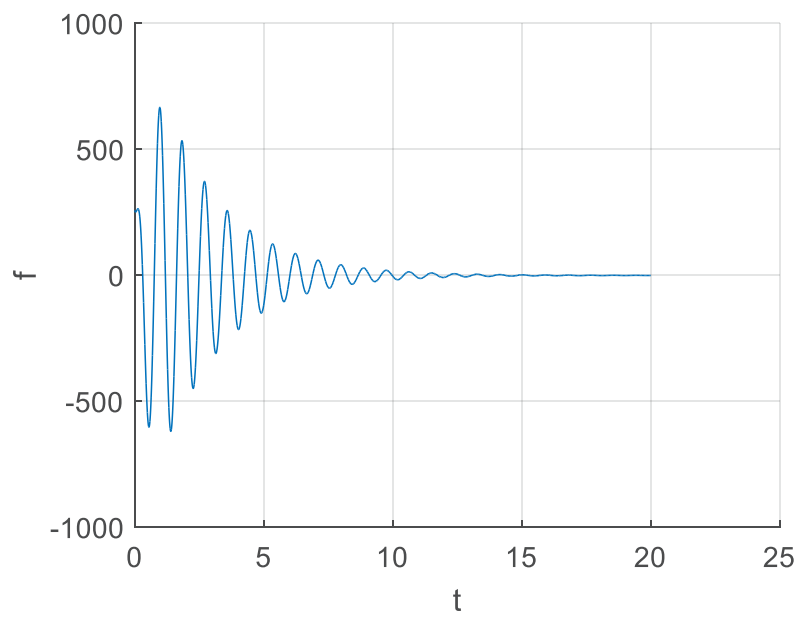
$$B_c = \begin{bmatrix} -1.2180 \\ -4.4264 \end{bmatrix} \quad (48)$$

$$C_c = [-.7260 \quad -.3313]$$

The following figures show the results of this simulation example:

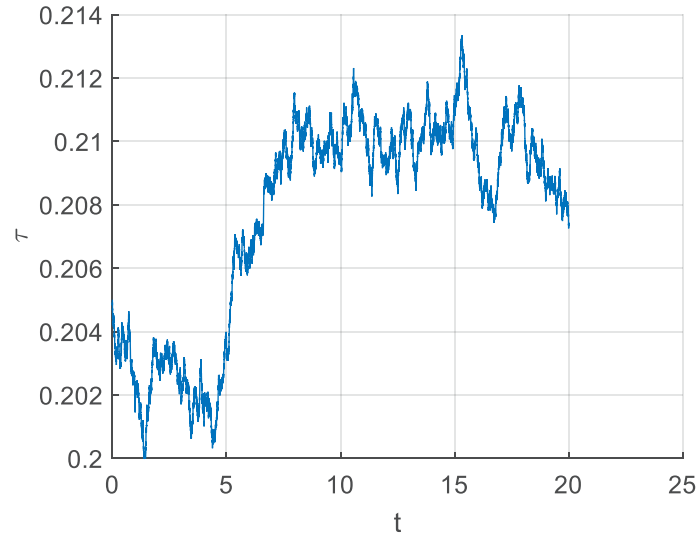


**Fig. 1. The pitch angle of the first simulation example**



**Fig. 2. Modal deflection of the first simulation example**

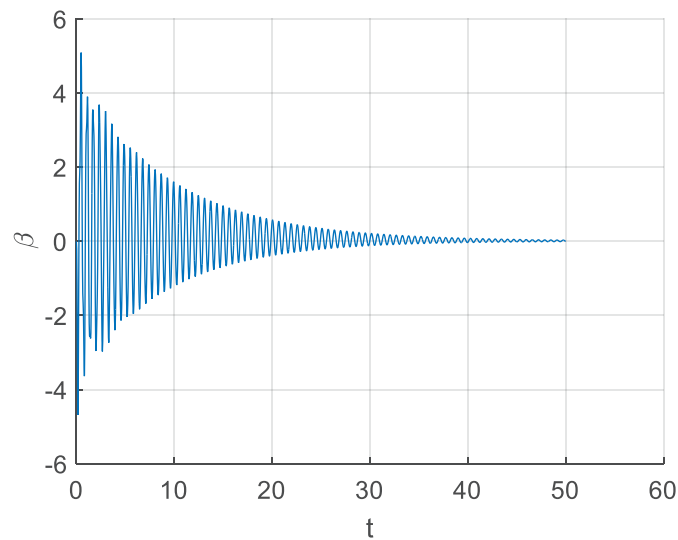




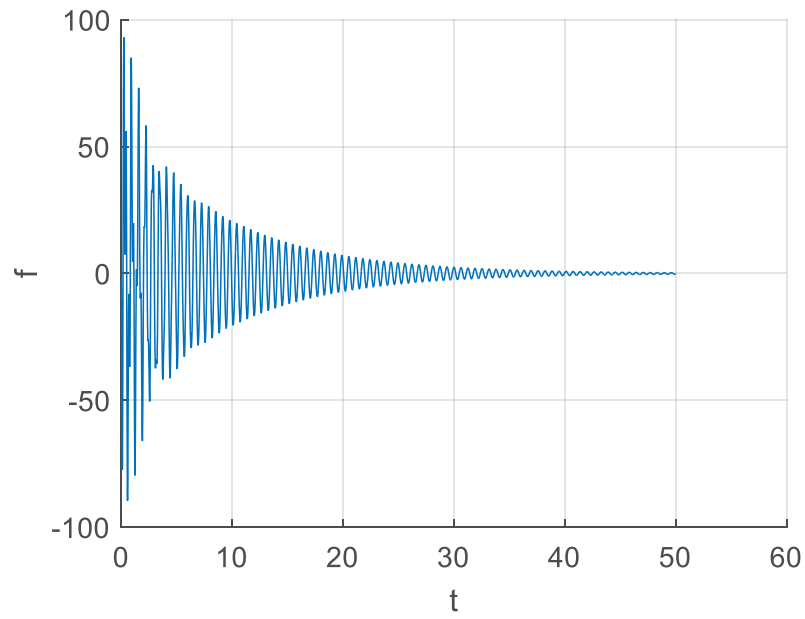
**Fig. 3. Time-varying delay of the first simulation example**

**Table 2. Configuration parameters of the second wind turbine**

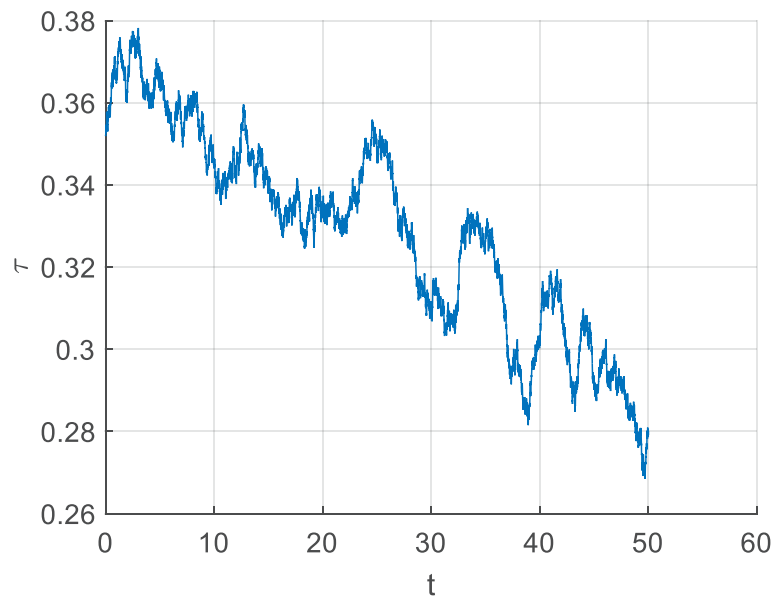
parameter	value
rotor diameter	15m
tower height	25m
rated power	50kW
wind speed	15m/s
pitch angle	0.75°



**Fig. 4. The pitch angle of the second simulation example**



**Fig. 5. Modal deflection of the second simulation example**



**Fig. 6. Time-varying delay of the second simulation example**

As can be seen, the designed controller stabilizes the uncertain time-varying delay model (43).

**Example 2.** Consider a wind machine with the following parameters [12] (Table 2),

Then, the wind machine has the following transfer function equation, which is given from [12]:

$$G_p(s) = \frac{-0.2545s^2 - 0.0647s + 0.9384}{s^4 + 2.28s^3 + 878.5s^2 + 437.7s + 7.7 \times 10^4} e^{-0.25s} \quad (49)$$

Assume the numerator coefficients have a 10% error concerning the nominal values. Also, assume  $\tau_i = 0.1$ ,  $\tau_u = 0.4$  and  $\tau_D = 0.5$ .

Using the proposed algorithm of this paper, the proper controller is obtained, which is presented in the following:

$$A_c = \begin{bmatrix} -0.0919 & 0.1601 \\ 0.1224 & -0.8797 \end{bmatrix},$$

$$B_c = \begin{bmatrix} 0.4916 \\ 0.3892 \end{bmatrix} \quad (50)$$

$$C_c = [0.9089 \quad 0.5857]$$

The following figures show the results of this simulation example:

This example also shows the performance of the designed controller to stabilize the closed-loop model.

## 5- Conclusion

Wind power is one of the fastest-growing electrical industries and owns the rapid development progress among the other renewable power generation elements. Various types of this machine are developed and implemented in both theoretical and real applications. The control objectives considerably depend on the wind speed and its variations over the nominal values. Since the variable-speed wind turbine can produce higher energy and lower component mechanical stress, this type of turbine has become a field of increasing interest. Consideration of the delay in the model of wind machines is not usual in previous studies. However, the time-varying delay in this model is generally apparent due to the natural properties of its dynamics. Unfortunately, no study basically and directly investigates the time-varying delay in the wind model equations. Indeed, the proposed controllers, regardless of the time-varying delay in the system's model.

This study investigates the problem of designing a controller for the wind turbine model in the presence of time-varying delays and uncertain parameters. The proposed controller is based on the idea of the Lyapunov Krasovskii functional and guarantees the globally exponential stability of the wind turbine model. Two examples are considered to convey the efficiency and performance of the design algorithm compared to the other previous methods. The other

methods are designed to compare the results by removing some assumptions. However, the obtained controllers are applied to the original model.

In the first example, the designed controller stabilizes the uncertain time-varying delay model. The second one demonstrates the performance of the designed controller to stabilize the closed-loop model. The simulation results reveal the proposed algorithm's superiority, efficiency, and performance compared to the previous methods.

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