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Interval Analysis of Controllable Workspace for Cable Robots

A. Zarif Loloei[1*], H. D. Taghirad[2], N. Kouchmeshky[3]

1- Assistant Professor, Department of Electrical Engineering, Pardis Branch, Islamic Azad University, Pardis, Tehran, Iran
2- Professor, Department of Electrical Engineering, K. N. Toosi University of Technology, Tehran, Iran
3- M.Sc. Student, Department of Computer Engineering, Arak Branch, Islamic Azad University, Arak, Iran

ABSTRACT

Workspace analysis is one of the most important issues in the robotic parallel manipulator design. However, the unidirectional constraint imposed by cables causes this analysis more challenging in the cable driven redundant parallel manipulators. Controllable workspace is one of the general workspace in the cable driven redundant parallel manipulators due to the dependency on geometry parameters in the cable driven redundant parallel manipulators. In this paper, a novel tool is presented based on interval analysis for determination of the boundaries and proper assessment of the enclosed region of controllable workspace of cable-driven redundant parallel manipulators. This algorithm utilizes the fundamental wrench interpretation to analyze the controllable workspace of cable driven redundant parallel manipulators. Fundamental wrench is the newly definitions that opens new horizons for physical interpretation of controllable workspace of general cable driven redundant parallel manipulators. Finally, the proposed method is implemented on a spatial cable driven redundant manipulator of interest.

KEYWORDS

Controllable Workspace, Interval Analysis, Cable Driven Redundant Parallel Manipulator, Fundamental Wrench, Boundary of Workspace, Unidirectional Constraint.

* Corresponding Author, Email: zarif@pardisiau.ac.ir

1- INTRODUCTION

Cable driven parallel manipulators (CDPMs) consist of a moving platform which is connected by means of actuated cables to the base. CDPMs are a special design of parallel manipulators (PMs) that heritage the advantages of PMs such as high load carrying capability and simultaneously have alleviated some of their shortcomings, such as restricted workspace. Due to several eminent features of CDPMs, they have stimulated the interest of many researchers and they are becoming the state-of-the-art in many real world applications, such as telescope [1], haptic interface [2], metrology [3], rehabilitation [4] and material handling [5].

In spite of many advantages and promising potentials, there are many challenging problems in the design and development of cable manipulators. Redundancy is an inherent requirement for CDPMs due to the fact that cables shall remain in tension for the whole workspace of the manipulator. Based on this fact, CDPMs are classified into under-constrained and fully constrained CDPMs [6], where, in this paper only fully constrained CDPMs are considered for the analysis. In such manipulators and in a non-singular posture, the moving platform can perform n Degrees-of-Freedom (DoF) motion provided that at least n+1 cables are considered in the structure of the robot to completely constrain the moving platform.

Workspace analysis is always one of the essential tasks in the design of any mechanism. However, the unidirectional constraint imposed by cables causes this analysis more challenging for CDPMs. In the literature, four different types of workspace have been introduced: (1) Wrench feasible workspace [7], (2) Dynamic workspace [8], (3) Static workspace [9] and (4) Controllable workspace [10].

In this paper, more emphasis is placed on the controllable workspace which represents the most general workspace. Controllable workspace pertains at finding the set of poses (position and orientation) of the moving platform in which any wrench can be exerted on the moving platform, while cables are all in tension. This kind of workspace depends only on the geometry of manipulator, i.e. the position of fixed and moving attachment points [11, 12], and therefore, it is important in the design of such manipulators [13, 14].

All methods including the analytical and numerical methods proposed to analyze the controllable workspace problem [10-14], suffers from a lack of physical interpretation of controllable workspace. A set of novel external wrenches called fundamental wrenches is introduced in order to provide a physical interpretation of controllable workspace. This concept is represented previously by the authors in [15]. Moreover, an analytical method is developed to determine the controllable workspace of CDPMs based on fundamental wrenches. The proposed method is generally applicable to any cable manipulators with any number of redundant cables as long as its Jacobian matrix is of full rank [15].

The workspace analysis of manipulators requires a suitable framework in order to propose a proper and systematic method. Although, the pervious proposed method by authors [15] determined the boundary equations of controllable workspace analytically, but this method is limited to 3D workspace for boundary visualization of 6-DOF mechanisms. In this paper, an interval analysis approach is examined to determine controllable workspace of CDPMs based on fundamental wrench. Interval analysis is an alternative numerical method in order to obtain practically competent results for the analysis of workspace of robotics mechanical systems [16]. Gouttefarde *et al.* presented in [16] the interval procedure to obtain the wrench feasible workspace of CDPMs for the first time. Also, a systematic interval approach is presented by Khalilpour and Zarif Loloei *et al.* in [17] for computing an applicable index, namely feasible kinematic sensitivity [17]. The drawback of the proposed method in [17] is that only applicable for planar parallel mechanism and constant orientation workspace.

The implemented procedure in this paper is provided based on fundamental wrench to determine the controllable workspace and it is more applicable than the presented method in [16] and [17] in terms of implementation. Also, the proposed procedure is applicable for spatial cable driven parallel manipulators.

The remaining of the paper is organized as follows. Brief description of fundamental wrench is given in the next section, which is followed by elaboration of an interval analysis approach in order to determine the controllable workspace in section 3. The implementation of the proposed method on the case study is given in section 4, and section 5 summarizes the concluding remarks.

2- CONTROLLABLE WORKSPACE BASED ON FUNDAMENTAL WRENCH ANALYSIS

A. BACKGROUND

For CDRPM the relationship between the tension force of cables and an external wrench $\boldsymbol{w}$ applied to the moving platform center point G is given by [18]:

$$Af = w \;\; , \;\; A = -J^T \tag{1}$$

where $\boldsymbol{f}=(f_1,\dots,f_m)$ the vector of cable tension force, $\boldsymbol{w}$ denotes the external wrench acting on the moving platform, $\boldsymbol{A}$ is the structure matrix [19], whose columns are denoted by $\boldsymbol{A}_i$ and $\boldsymbol{J}$ is the manipulator Jacobian matrix. In a general 6-DOF CDPM, the wrench vector $\boldsymbol{A}_i$ is defined as [18]:

$$\mathbf{A} = [\mathbf{A_1} \dots \mathbf{A_m}],\ \mathbf{A}_i = \begin{pmatrix} \hat{\mathbf{S}}_\mathbf{i} \\ \mathbf{E_i} \times \hat{\mathbf{S}}_\mathbf{i} \end{pmatrix} \tag{2}$$

in which, as illustrated in Figure 1, $\boldsymbol{E}_i$ denotes the position vector of the moving attachment point of the i^{th} limb measured with respect to point G and $\boldsymbol{S}_\mathrm{i}$ is the unit vector connecting moving attachment point of each cable to a fixed attachment point.

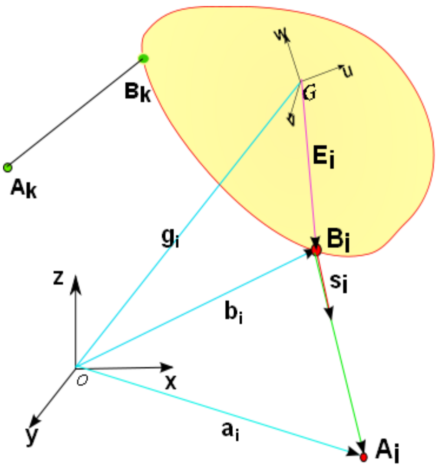

Figure 1: The general structure of a cable-driven parallel manipulator and its vectors.

In order to determine the controllable workspace of cable manipulator, the position and orientation of the manipulator through which (1) is solvable for nonnegative cable forces in the presence of external wrenches applied to the moving platform, must be determined. Note that the CDRPM structure matrix is a non-square $n \times m$ matrix. A set of all manipulator configurations (X) belongs to the controllable workspace, if and only if, a feasible solution exists for any wrench exerted on the end-effector of manipulator [10].

The controllable workspace determined by fundamental wrench is certainly valid for any other typical wrench applied to the moving platform. Because the fundamental wrench is the worst case wrench generated at the moving platform in which r degrees of redundancy are annihilated at all configurations. The important property of fundamental wrench is that if each of fundamental wrenches of n DOF cable manipulator with r degree of redundancy is applied on the center point of the platform, the r corresponding cables become ineffective. This claim is rigorously stated in [15].

The set of fundamental wrenches ($\boldsymbol{w}_f$) of n DOF cable manipulator with both r degrees of redundancy and a full rank structure matrix refers to $m!/(r!(m-r)!)$ vectors which each equals a linear positive combination of r arbitrary opposite direction of the wrench vector ($\boldsymbol{A}_i$).

$$\mathbf{w}_f = -\sum_i n_i \mathbf{A}_i \tag{3}$$

where n_i's are the r positive elements of the null vector of structure matrix ($\boldsymbol{A}$). Each of them is an i^{th} element of the null vector of the corresponding cable [15].

Note that there exists a set of fundamental wrench if and only if at least one positive null vector of structure matrix exists. This definition is more tractable for manipulators with one degree of redundancy [20]. In this case, the direction of fundamental wrench ($\boldsymbol{w}_f$) is a fixed direction denoted in the space by removing the positive coefficient (n_i). Therefore, the fundamental wrench for the cable manipulator with one degree of redundancy is given by:

$$\mathbf{w}_f = -\mathbf{A}_i\ ,\ \ i = 1, \dots, n+1 \tag{4}$$

B. CONTROLLABLE WORKSPACE ANALYSIS

One of the distinctive advantages of introducing fundamental wrench is its physical interpretation that correspond it to a zero tension force at each cable, and therefore, this investigation is reduced to looking for the existence of such wrenches in the workspace. The controllable workspace problem of manipulator with one degree of redundancy is defined as:

$$\mathbf{A}_{n\times(n+1)} \mathbf{f}_{n+1} = \mathbf{w}_f\ ,\ \mathbf{w}_f = -\mathbf{A}_i \tag{5}$$

Based on the property of fundamental wrench, the controllable workspace conditions are obtained as:

$$\Delta_{ij} = \left| \mathbf{A}_1 \dots \mathbf{A}_{j-1}\ (-\mathbf{A}_i)\ \mathbf{A}_{j+1} \dots \mathbf{A}_{i-1}\ \mathbf{A}_{i+1} \dots \mathbf{A}_{n+1} \right| \tag{6}$$
$$\Delta_i = \left| \mathbf{A}_1 \dots\ \mathbf{A}_{i-1}\ \mathbf{A}_{i+1} \dots \mathbf{A}_{n+1} \right|$$

where Δ_i is the determinant of the structure matrix in which the i^{th} column is removed. A set of all manipulator configurations (X) belongs to the controllable workspace if and only if all Δ_ij and Δ_i have the same sign [15].

For the workspace analysis, the manipulator is divided into two categories based on the structure matrix introduced in [15], [20]. One of them is the sub-robot that is part of the robot with only one degree of redundancy. Therefore, the structure matrix of sub-robot is an arbitrary $n+1$ wrench vector of the structure matrix from the major manipulator. A combined sub-robot is the other one that represents a manipulator with a combined structure matrix whose its n column is an arbitrary n wrench vector of the structure matrix of the main manipulator, and one of the columns is obtained from the positive linear combination of the other k wrenches, $1<k<r+1$. The combined structure matrix can be written as:

$$\mathbf{A}_{cs} = [\mathbf{A}_s | \mathbf{A}_c]_{n\times(n+1)} \tag{7}$$

In which, $\boldsymbol{A}_s$ is an arbitrary n wrench vector of the structure matrix of the main manipulator and $\boldsymbol{A}_c$ is positive linear combination of other k wrenches. The combined vector ($\boldsymbol{A}_c$) is obtained from following equation:

$$\mathbf{A}_C = \sum_k \mathbf{A}_i \text{ , } 2 \le k \le r \tag{8}$$

Therefore, the sub-robot and the combined sub-robot are both representing manipulators with only one degree of redundancy. Hence, the controllable workspace conditions are obtained from (6).

Based on theorems in [15], the controllable workspace of major manipulator is obtained from a union of both controllable workspace of both the sub-robot and the combined sub-robot. A systematic approach is described in the following section to determine the controllable workspace of CDPM based on interval analysis and fundamental wrench properties.

3- Controllable Workspace Determination Based on Interval Analysis

A. BASIC CONCEPTS

Interval analysis is amongst the numerical method proposed in the literature that provides a guaranteed solution. The basic principles of interval analysis are simple, while efficient implementation requires expertise. In interval analysis, one deals with intervals of numbers instead of the numbers themselves [21]:

$$[X] = [\underline{x}, \overline{x}] = \left\{ x \,\middle|\, x \in R \text{ , } \underline{x} \le x \le \overline{x} \right\} \tag{9}$$

where $[\underline{x}, \overline{x}]$ are the left endpoint and the right endpoint of the interval number [X]. By an n-dimensional interval vector, we mean an ordered n-tuple of intervals:

$$\mathbf{X} = (X_1, \ldots, X_n) \tag{10}$$

In interval analysis all variables are independently investigated [22]. Thus, the output range of the interval function could be wider than the function span, but certainly the answer region lies within the output range. For a more comprehensive introduction on this, the reader is referred to [23]. It should be noted that all the interval algorithms proposed in this paper are implemented in Matlab and uses the INTLAB package supporting interval calculations.

B. WORKSPACE DETERMINATION

For workspace analysis of each sub-robots and combined sub-robots, $n+1$ closed form expressions with respect to Δ shall be analyzed. These expressions are polynomial of position variable with coefficients that depend on geometry variables of manipulator [24]. An important property of determinant is that its value depends linearly on any of its columns. This property can be used to expand the determinant and simplify its expression [24]. For manipulator with a 6 DOF and r degrees of redundancy, these expressions for each sub robot or combined sub robots as defined as:

$$\begin{aligned}\Delta = {} & F_1 x^3 + F_2 x^2 y + F_3 x^2 z + F_4 xyz + F_5 x^2 \\ & + F_6 xz + F_7 xy + F_8 x + F_9 y^3 + F_{10} y^2 x \\ & + F_{11} y^2 z + F_{12} y^2 + F_{13} yz + F_{14} y \\ & + F_{15} z^3 + F_{16} z^2 y + F_{13} z^2 x + F_{18} z^2 \\ & + F_{19} z + F_{20}\end{aligned} \tag{11}$$

in which F_i depends on rotation and geometry variables of manipulator. Using interval formulation of [Δ], with respect to its pose [X] and [Φ], for all sub-robots and combined sub-robots, the controllable workspace of the main manipulator is computed.

For this purpose, [Δ] has the same sign for at least one sub-robot or combined sub-robot. The pseudo code is given in Table 1 provides the logic workspace condition of each sub-robot and combined sub-robot. The *compute-closed-form* function is the interval function of (11) as:

$$\begin{aligned}[\Delta] = {} & [F_1 x^3 + F_2 x^2 y + F_3 x^2 z + F_4 xyz + F_5 x^2 \\ & + F_6 xz + F_7 xy + F_8 x + F_9 y^3 + F_{10} y^2 x \\ & + F_{11} y^2 z + F_{12} y^2 + F_{13} yz + F_{14} y \\ & + F_{15} z^3 + F_{16} z^2 y + F_{13} z^2 x + F_{18} z^2 \\ & + F_{19} z + F_{20}]\end{aligned} \tag{12}$$

The *sign()* function (in Table 1) specifies the sign of interval function [Δ], when the left endpoint (right endpoint) of [Δ] is nonnegative (negative), the sign is positive (negative) otherwise it remains unknown. These results can be shown as:

$$[\Delta] = [\underline{\Delta}, \overline{\Delta}], \; sign(\Delta) = \begin{cases} positive & \underline{\Delta} >= 0 \\ negative & \overline{\Delta} < 0 \\ unknown & otherwise \end{cases} \tag{13}$$

Moreover, the *Δ .Condition* (in Table 1) is *feasible* if [Δ] has the same sign for sub-robot or combined sub-robot, *unknown* if the sign of [Δ] is unknown and *unfeasible* otherwise.

Therefore, the interval poses [X] and [Φ] boxes are fully inside the controllable workspace if at least one sub-robot or combined sub-robot with *feasible Δ .Condition* exists. If the *unfeasible Δ .Condition* occurs for all sub-robots and combined sub-robots, the intervals pose ([X] and [Φ]) is located outside of the controllable workspace. For *unknown Δ .Condition*, the interval pose ([X] and [Φ]) is bisected and the procedure is repeated until the width of interval pose [X] and [Φ] because less than the desired ε value. In this pseudo code, the fully inside and fully outside boxes are obtained.

The procedure of controllable workspace is provided in Table 2, where L_{in} and L_{out} represent respectively the feasible and unfeasible intervals of the controllable workspace and L_{edge} indicates the bound intervals calculated according to the ε value.

In the proposed method represented by Gouttefarde *et al.* [16], the linear programming is applied to find the feasible condition. In this case, the sign of polynomial expressions is considered in our approach. This result is more applicable than the method presented in [16].

TABLE 1
THE PROPOSED PSEUDO-CODE FOR CALCULATIONS OF *Δ.CONDITION*

Input: $[x,y,z],[\alpha,\beta,\gamma]$
Output: Δ.*Condition*

```
[X] ← [x, y, z]
[Φ] ← [α, β, γ]
[Δ] = comput−closed−form([X],[Φ])
S = sign([Δ])
feas_Δ = 0
unfeas_Δ = 0

for i=1:n
 [Δ]_i = comput−closed−form([X],[Φ])
 S_i = sign([Δ]_i)
   if    S_i × S is positive
           feas_Δ = feas_Δ + 1
   else if  S_i × S is negative
           unfeas_Δ = unfeas_Δ + 1
   end if
end for

if  feas_Δ == n
    Δ.Condition ← feasible
else if  unfeas_Δ == n
    Δ.Condition ← unfeasible
else
    Δ.Condition ← unnknown
end if
```

TABLE 2
THE PROPOSED PSEUDO-CODE FOR CONTROLLABLE WORKSPACE

Input: $([X],[\Phi],[\varepsilon])$
Output: $(L_{in}, L_{out}, L_{edge})$

```
L ← ([X],[Φ])
while  L ≠ 0
    workspacecon = Δ.condition([X],[Φ])
    if  workspacecon == feasible
          L_in ← ([X],[Φ])
    else if  workspacecon == unfeasible
          L_out ← ([X],[Φ])
    else if  size([X],[Φ]) ≥ ε
          L ← Bisect([X],[Φ])
    else
          L_edge ← ([X],[Φ])
    end if
end while
```

4- CASE STUDY

In this section, the proposed analytic method and interval procedure are used to determine the controllable workspace of a special CDPM. A CDPM is designed by an eight actuated, six degrees of freedom fully constrained cable manipulator. This manipulator is under investigation for possible high speed and wide workspace applications at K. N. Toosi University (KNTU). There exist different designs for KNTU CDPM based on different approach such as collision avoidance scheme, force feasibility, and dexterity. A special design of KNTU CDPM is shown in Figure 2, which is called Galaxy. In this design the fixed and moving attachment points are carefully located at suitable locations to increase the rotation workspace of the robot.

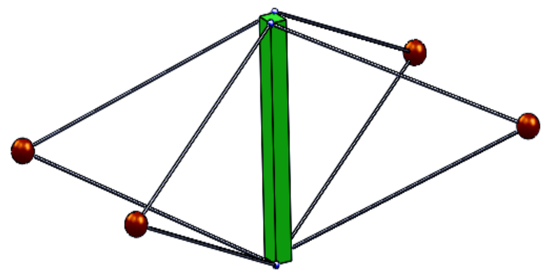

Figure 2: The KNTU CDRPM Galaxy design.

As it is shown in Figure 2, the fixed and moving attachment points are coincident at one point in pairs, whose geometric parameters are given in Table 3. The controllable workspace of the cable manipulator is obtained from the union of controllable workspaces of sub-robots and combined sub-robots corresponding to the wrench vector of main manipulator structure matrix.

TABLE 3
GEOMETRIC PARAMETERS OF GALAXY DESIGN

DESCRIPTION	QUANTITY
fa: fixed cube length	2 m
fb: fixed cube width	4 m
a: moving platform's length	0.14 m
b: moving platform's width	0.28 m
h: moving platform's height	1 m

In the proposed method, the controllable workspace of cable redundant manipulator is obtained from the union of controllable workspace of the sub-robots and the combined sub-robots that are depended on wrench vector of major manipulator structure matrix. Considering the vector definitions $\boldsymbol{S}_i$ and $\mathbf{E}_i$ as illustrated in Figure **1**, the structure matrix of KNTU CDRPM is given as [25]:

$$\mathbf{A} = \begin{pmatrix} \hat{\mathbf{S}}_1 & \dots & \hat{\mathbf{S}}_8 \\ \mathbf{E}_1 \times \hat{\mathbf{S}}_1 & \dots & \mathbf{E}_8 \times \hat{\mathbf{S}}_8 \end{pmatrix}_{6\times 8} \tag{14}$$

There exist 8 sub-robots and 28 combined sub-robots for KNTU CDRPM. With loss of generality, the 1th sub-robot workspace is determined by using the proposed method. The structure matrix of the 1th sub-robot due to removing the 1th cable is obtained from following:

$$\mathbf{A}_{s1} = \begin{pmatrix} \hat{\mathbf{S}}_1 & \dots & \hat{\mathbf{S}}_8 \\ \mathbf{E}_1 \times \hat{\mathbf{S}}_1 & \dots & \mathbf{E}_8 \times \hat{\mathbf{S}}_8 \end{pmatrix}_{6\times 7} \tag{15}$$

According to the structure matrix of other sub-robots is similar to (15) with different wrench vectors in the corresponding cable.

Moreover, the controllable workspace of the major robot for constant rotation [α,β,γ]=[0,0,0] and constant position z=0.15 is determined by the interval procedure illustrated in Figure 3 and Figure 4 for different epsilon (ε=0.1 and 0.05). In Figure 4, the boundary of workspace obtained in [15] is also plotted by solid white line for the purpose of comparison of performances these two methods. MATLAB is used to implement both methods and the computations are performed on a Acer ASPIRE laptop (Core 2 Duo CPU T6600, 2.20 GHz). The computation time is about 147 seconds to obtain Figure 4 for ε=0.05. As expected the computation time increases significantly by reducing ε.

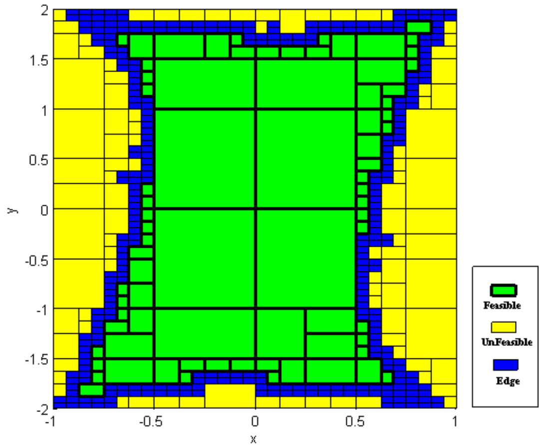

Figure 3: Controllable workspace CDRPM ([α,β,γ]=[0,0,0], z=0.15) by interval analysis procedure for ε=0.1.

Table 4, summarizes the computation times obtained for different accuracies (ε) in obtaining 2D workspace, while the runtime for simulation of analytic method is about 291 sec. Although, the computations time of interval analysis are less than the runtime simulation of analytic method for low accuracies ε=0.1 and ε=0.05, these is significantly increased for high accuracy band ε=0.01 and more. Therefore, it is suitable to use interval analysis approach in cases where high accuracies in numerical calculation are not demanded.

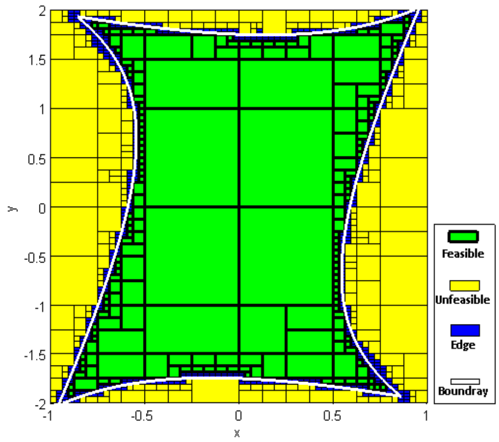

Figure 4: Controllable workspace CDRPM ([α,β,γ]=[0,0,0], z=0.15) by interval analysis procedure for ε=0.05.

TABLE 4
COMPUTATION TIME OF VARIOUS CONTROLLABLE WORKSPACES

ε	COMPUTATION TIME
0.1	32 sec
0.05	147 sec
0.01	2200 sec

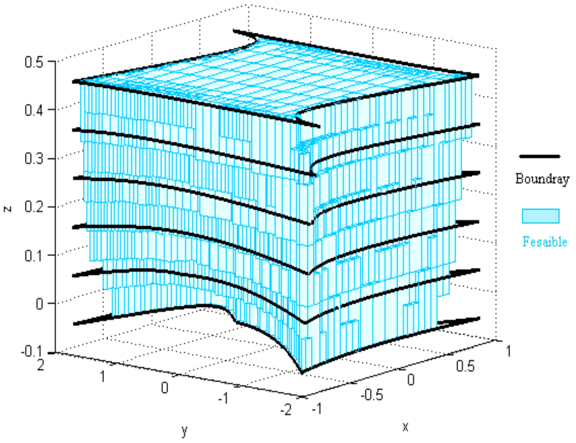

Figure 5: Constant orientation Controllable workspace of KNTU CDRPM([α,β,γ]=[0,0,0]).

Furthermore, the constant orientation controllable workspace for the robot for zero orientation in all three angles is determined and shown in Figure 5. It is observed that the controllable workspace of KNTU CDRPM is more than 40% of its overall workspace. Moreover, the controllable workspace provides a continuum volume in which there is no cables collision and singularity in the whole workspace. The constant orientation controllable workspace is also computed by the interval procedure and it is shown in this figure for ε=0.05. As it is seen in this figure the workspace obtained by interval analysis is completely inside the boundaries obtained by analytical method.

5- Conclusions

This paper introduced an applicable algorithm for the analysis of controllable workspace of spatial cable driven parallel manipulators based on interval analysis. Moreover, the fundamental wrench as used in this approach. The fundamental wrench defined a physical interpretation of controllable workspace that led to a significant reduction in the complexity of controllable workspace analysis. Therefore, a practicable interval analysis tool was provided by using this novel concept. The proposed method is generally applicable to any cable manipulator with any redundant actuation as long as its Jacobian matrix is of full rank. This method was also applied to a spatial manipulator as a case study, and controllable workspace is determined for that manipulator. Due to the physical interpretation, this approach was added to the analysis of controllable workspace and it is believed that this representation can be used as the basis of multi-objective optimization routines to further enlarge the controllable workspace of cable-driven manipulators.

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